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Strength of Materials: Problems and Detailed Solutions

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Preface

Strength of Materials is a fundamental and core course in engineering disciplines, and understanding its concepts is essential for students in various engineering fields, particularly civil, mechanical, mining, aerospace engineering, and more. In recent years, numerous books on Strength of Materials have been published. Among the primary reference texts for this subject are books by Beer & Johnston, Popov, Timoshenko, and others.

The present book, titled "Strength of Materials: Problems and Detailed Solutions", provides numerous solved exercises in each chapter based on the topics covered in standard Strength of Materials reference books. Readers can complete their understanding of each chapter's concepts by reviewing these problems or by proposing alternative appropriate solutions. Additionally, unsolved questions have been included at the end of each chapter to help students enhance their skills through correct solutions. Essentially, this book teaches Strength of Materials through various solved examples.

Since this book focuses solely on solving specific problems, it is necessary for readers to first learn the related concepts and formulas of each chapter by studying standard Strength of Materials references, and then proceed to solve these problems.

There may be different methods to solve a given problem. However, the approaches presented in this book reflect solely the authors' perspective, and readers are encouraged to discover other methods, thereby gaining greater proficiency in problem-solving.

The content of this book is organized into 8 chapters, each comprising various sections. Every effort has been made to align the presentation of problems in each chapter and section with the sequence typically followed in standard Strength of Materials textbooks. These chapters are as follows:

1- Effects of Axial Loads in Statically Determinate Structures
2- Effects of Axial Loads in Statically Indeterminate Structures
3- Pure Shear Stresses and Strains
4- Stress–Strain Relations
5- Beams under Bending Moments
6- Shear Stress in Members under Bending Moments
7- Torsion
8- Stresses in Thin-Walled Shells

We have strived to present a collection with minimal flaws. However, despite repeated corrections and revisions, some shortcomings may still exist in certain parts of the book. Therefore, the authors welcome all constructive comments and suggestions from our valued readers to help improve this work.

S. Karimi

Table of Contents

Chapter 1: Axial Load Effects in Determinate Structures

Section 1: Stable, Unstable, Isostatic and Hyperstatic Trusses

The objective of truss analysis is to determine the unknown support reactions and the internal forces in the members. Since only axial forces exist in truss structures, each truss member is treated as an unknown. To solve for these unknowns, a sufficient number of equilibrium equations is required. From statics, it is known that two independent equilibrium equations can be written at each joint of a truss. Therefore, if the number of joints is j, a total of $2j$ equilibrium equations are available. Now, if the number of truss members is m and the number of reaction components is r, the relationship between the unknowns and the available equations can be expressed as follows.

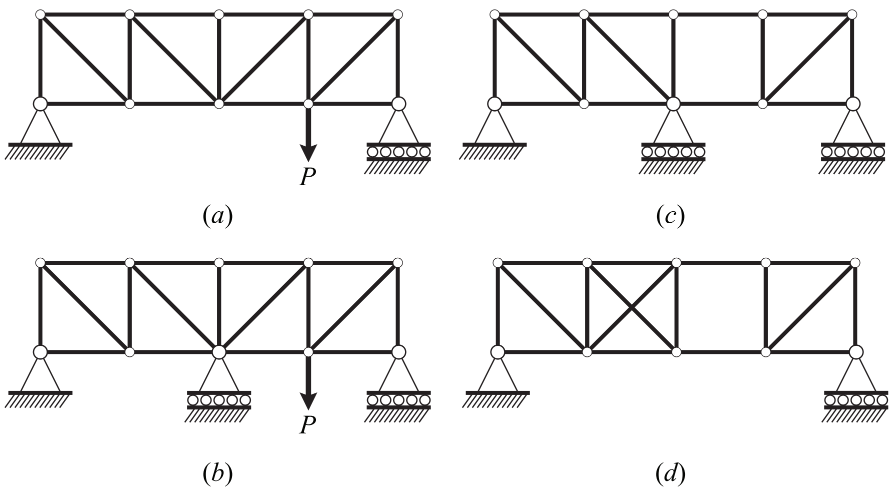
$(a)\quad 2j = m + r$

$(b)\quad 2j < m + r$

$(c)\quad 2j > m + r$

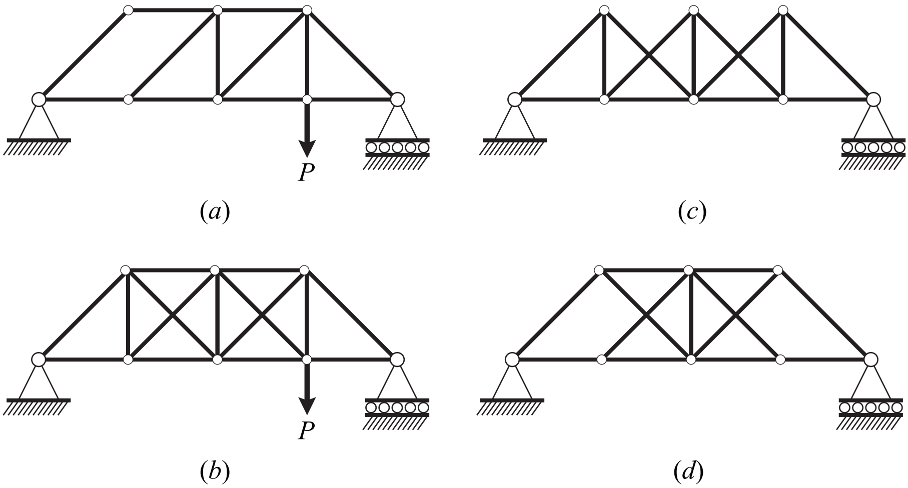
If condition (a) is satisfied, the truss is statically determinate (isostatic) and solvable using the equations of equilibrium. If condition (b) is satisfied, the truss is statically indeterminate (hyperstatic). If condition (c) is satisfied, the truss is unstable, and the structure is geometrically unstable even before any force calculations are considered. Moreover, trusses must also be examined with respect to stability. Solvability by the static equilibrium equations alone is not a sufficient condition for the stability of a truss. In what follows, the concepts of statically determinate and statically indeterminate trusses are discussed through clear examples, and their stability and instability are also examined.

1- In the figures below, identify the trusses that are unstable, stable, isostatic and hyperstatic.



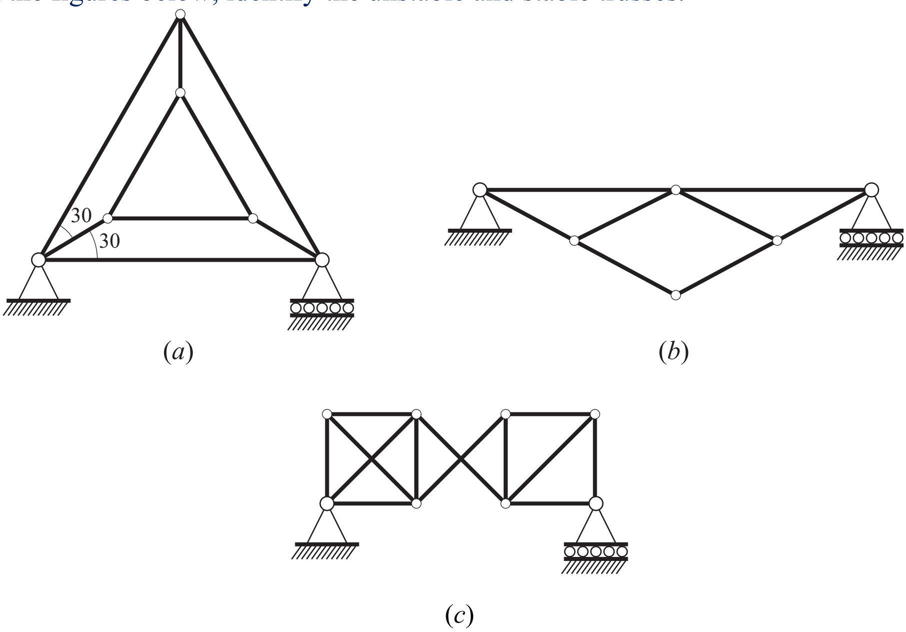
(a): The truss is stable and isostatic.
(b): The truss is stable and hyperstatic.
(c): The truss is stable and isostatic.
(d): The truss is unstable.

2- In the figures below, identify the trusses that are unstable, stable, isostatic, and hyperstatic.



(*a*): The truss is unstable.
(*b*): The truss is stable and hyperstatic.
(*c*): The truss is stable and isostatic.
(*d*): The truss is stable and isostatic.

3- In the figures below, identify the unstable and stable trusses.

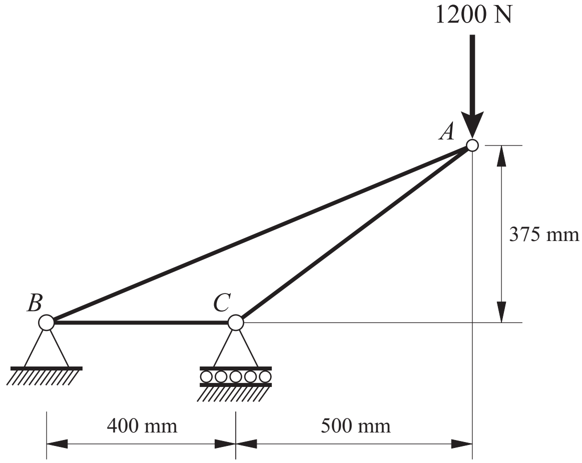


(*a*): The truss is unstable.
(*b*): The truss is unstable.
(*c*): The truss is unstable.

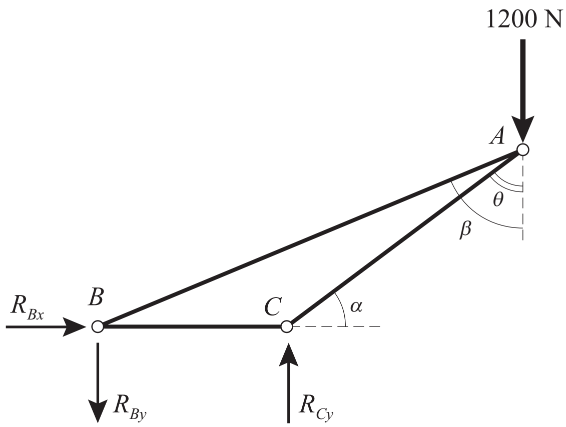
Chapter 1: Axial Load Effects in Determinate Structures

Section 2: Truss Analysis

1- Using the method of joints, determine the internal forces in the truss shown in the figure below.



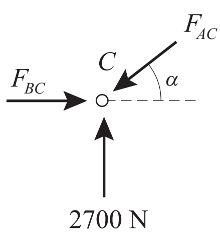
To determine the member forces in the truss, the support reactions are first obtained, and then the internal forces of the members are evaluated by writing the equilibrium equations at each joint. The free body diagram of truss is shown below.



$$\sum M_B = 0 \rightarrow (1200)(900) = (R_{Cy})(400) \rightarrow R_{Cy} = 2700\ N$$

$$\sum F_y = 0 \rightarrow 2700 - 1200 = R_{By} \rightarrow R_{By} = 1500\ N$$

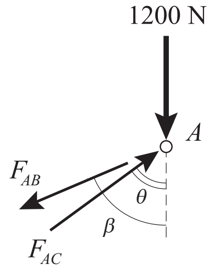
Equilibrium at joint *C*:



$$\alpha = \tan^{-1}(\frac{375}{500}) = 36.87^\circ$$

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \rightarrow \begin{cases} F_{AC}\cos 36.87 = F_{BC} \\ F_{AC}\sin 36.87 = 2700 \end{cases} \rightarrow \begin{cases} F_{AC} = 4500\ N\ (C) \\ F_{BC} = 3600\ N\ (C) \end{cases}$$

Equilibrium at joint A:



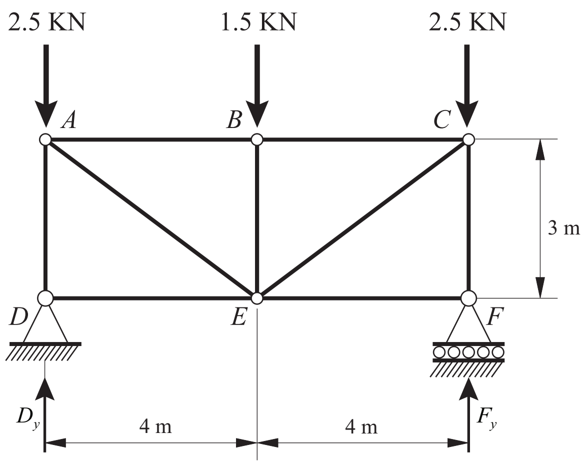
$$\theta = 90 - \alpha = 53.13^\circ$$

$$\beta = \tan^{-1}(\frac{900}{375}) = 67.38^\circ$$

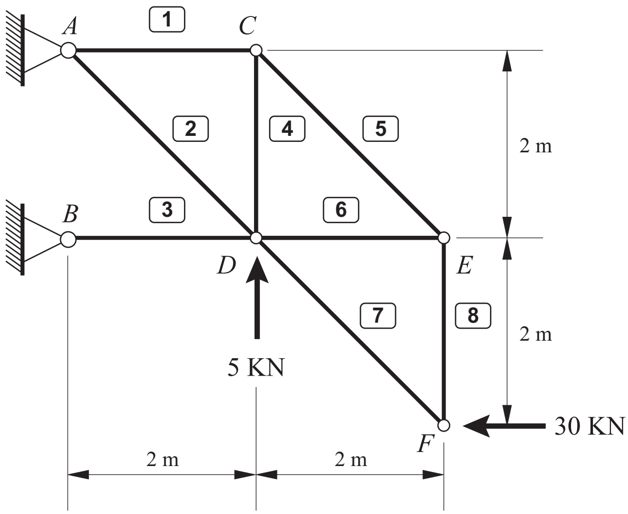
$$\sum F_x = 0$$

$$\rightarrow\ F_{AC}\sin 53.13 = F_{AB}\sin 67.38\ \rightarrow\ F_{AB} = 3900\ N\ \ (T)$$

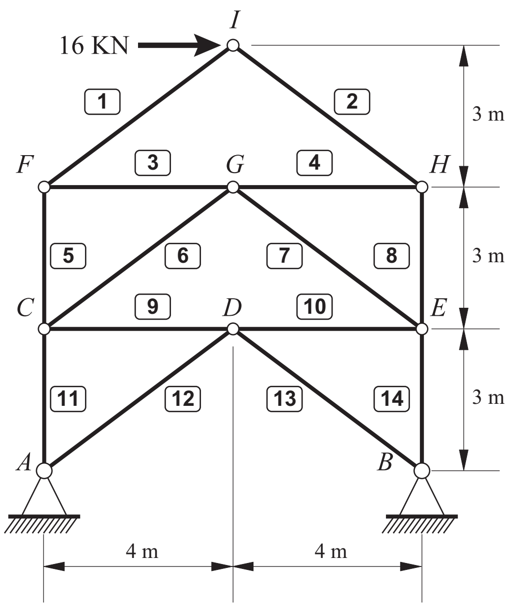
2- Using the method of joints, determine the internal forces in the truss shown in the figure below.



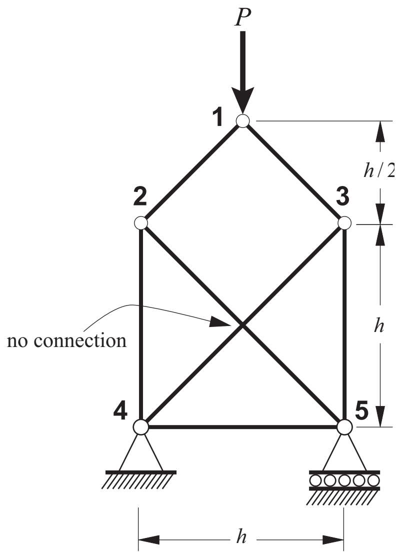
3- Using the method of joints, determine the internal forces in the truss shown in the figure below.



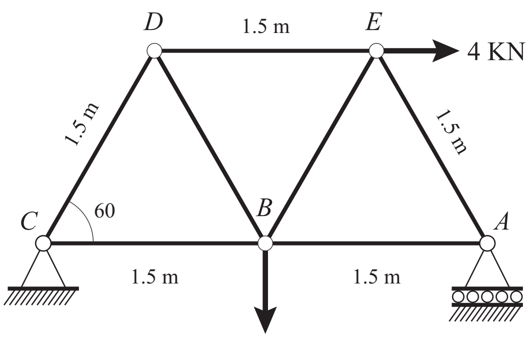
4- Using the method of joints, determine the internal forces in the truss shown in the figure below.



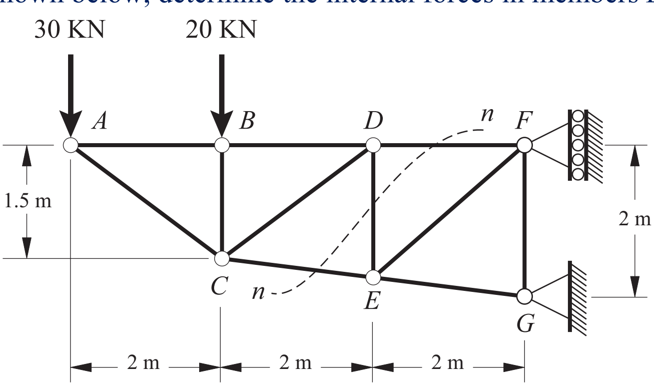
5- Using the method of joints, determine the internal forces in the truss shown in the figure below.



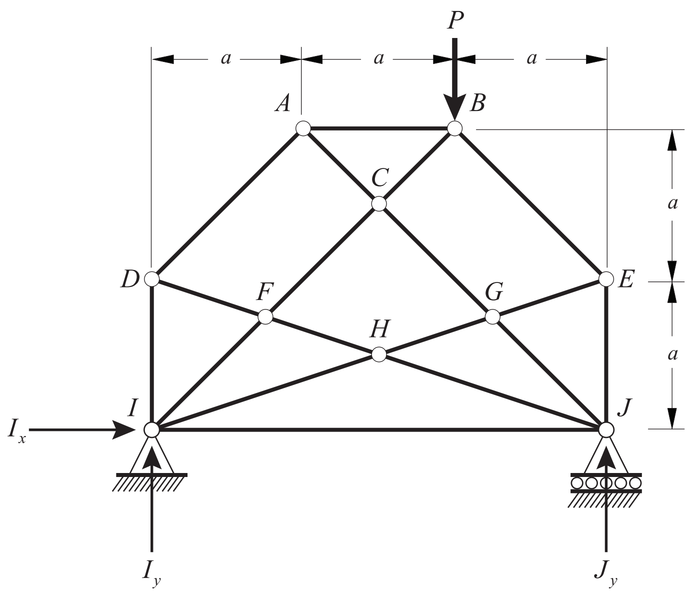
6- Using the method of joints, determine the internal forces in the truss shown in the figure below.



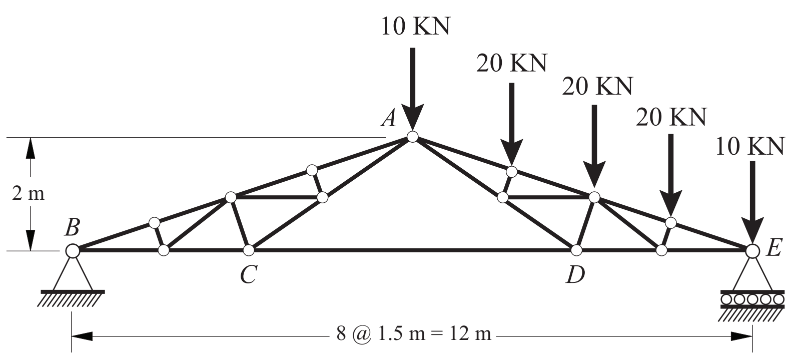
7- In the truss shown below, determine the internal forces in members *DE* and *DF*.



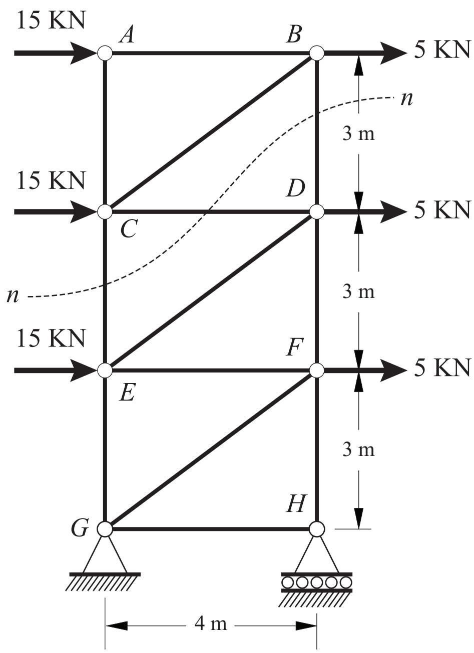
8- In the truss shown below, determine the internal forces in members *AB* and *EJ*.



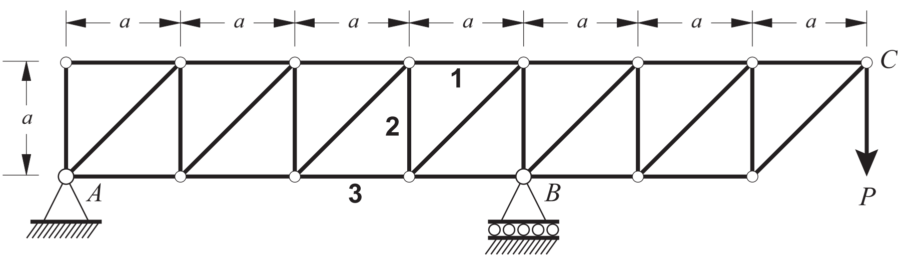
9- In the truss shown below, determine the internal force in member *CD*.



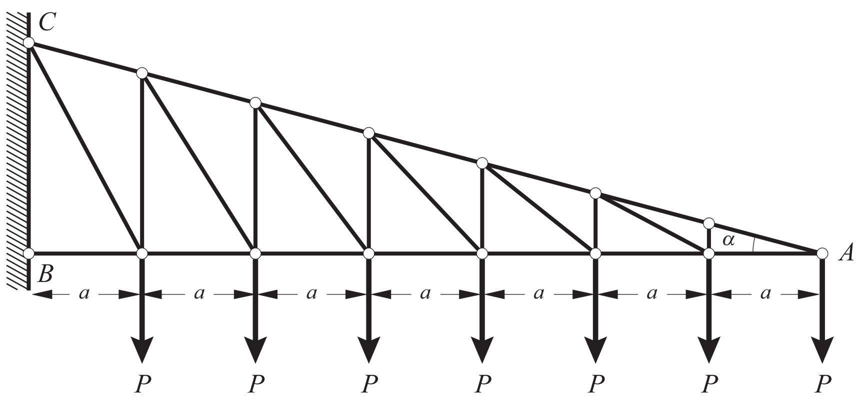
10- In the truss shown below, determine the internal forces in members *BD*, *CD* and *CE*.



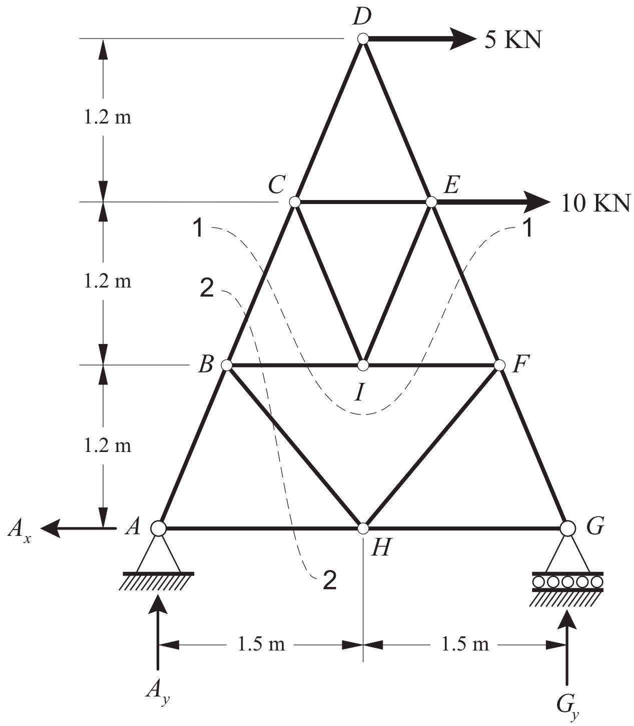
11- In the truss shown below, determine the internal forces in members 1, 2 and 3.



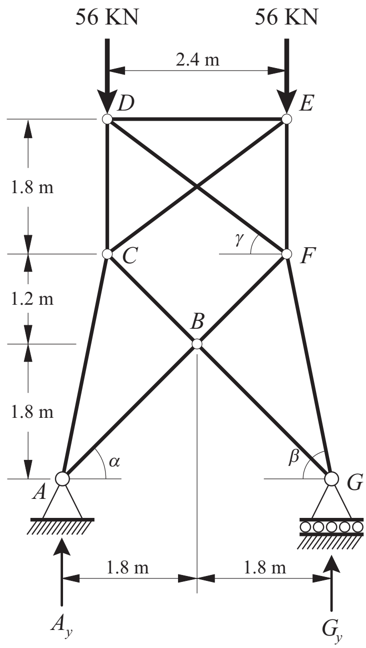
12- In the truss shown below, prove that the axial force in the nth vertical member measured from the free end of the truss is equal to $F_n = -\dfrac{(n-1)}{2}P$.



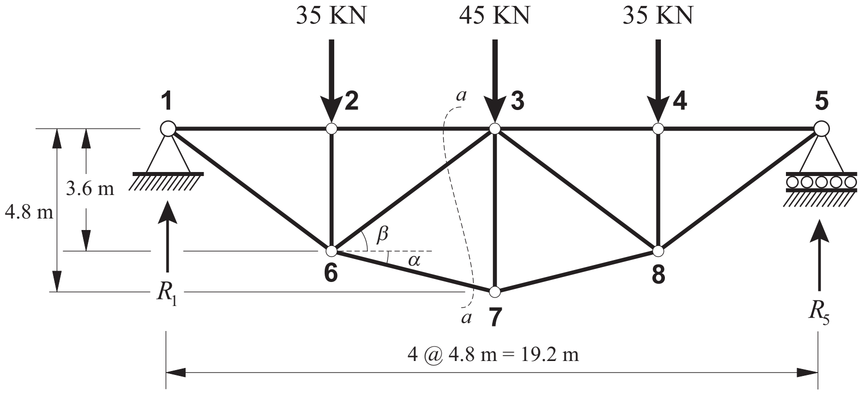
13- In the truss shown below, determine the internal forces in members *BI* and *BC*.



14- In the truss shown below, determine the internal forces in members *AC*, *BC* and *DF*.

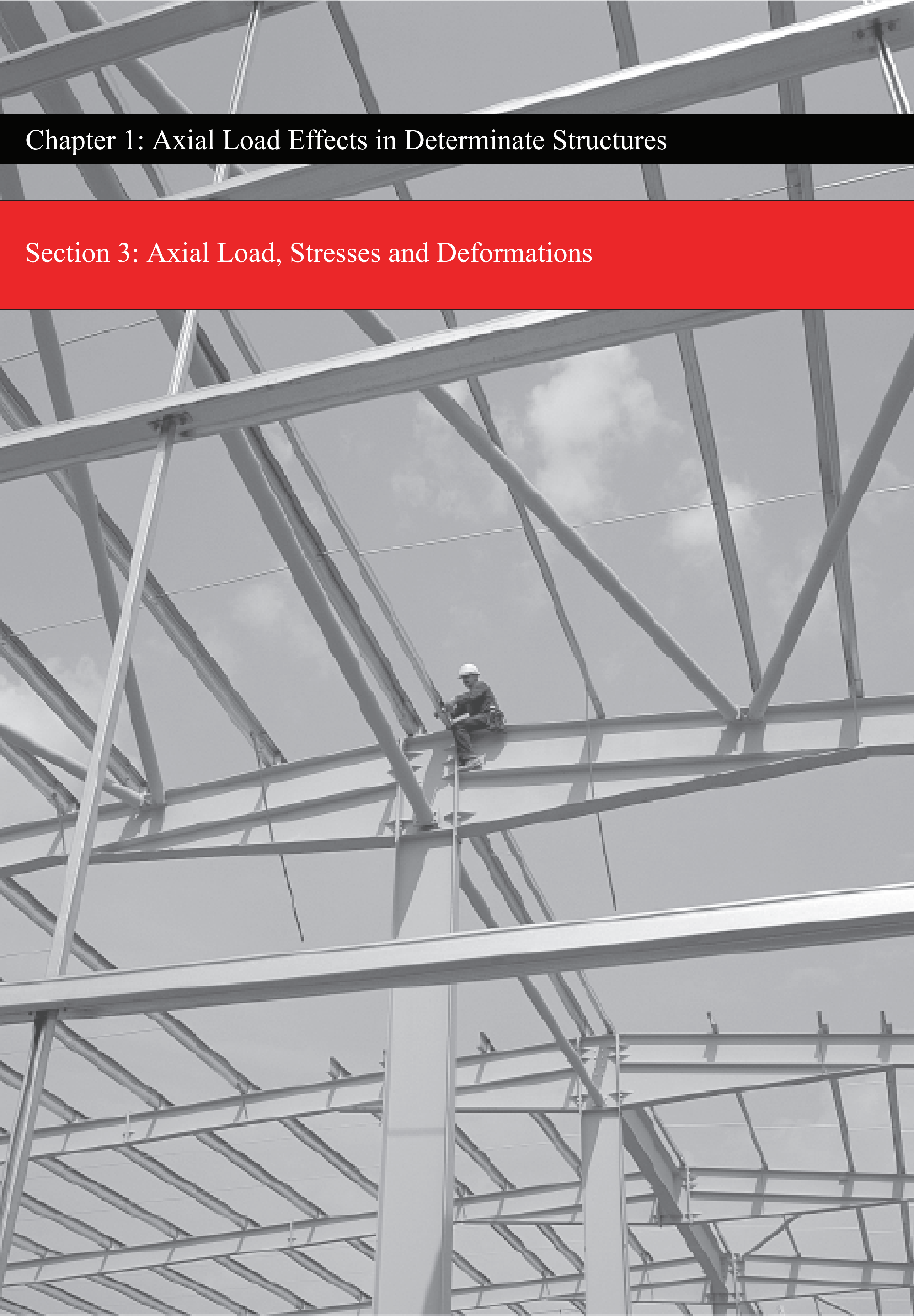


15- In the truss shown below, determine the internal forces in members 2-3, 2-6, 6-3, 6-7 and 3-7.



Chapter 1: Axial Load Effects in Determinate Structures

Section 3: Axial Load, Stresses and Deformations



1- If a wall is made of concrete with a compressive strength of 160 kg/cm^2 and a unit weight of $\gamma = 2000\ kg/m^3$, determine the maximum height of the wall assuming a safety factor of 4.

$\gamma = 2000\ kg/m^3 = (2000)(10^{-6})\ kg/cm^3 = 2\times10^{-3}\ kg/cm^3$

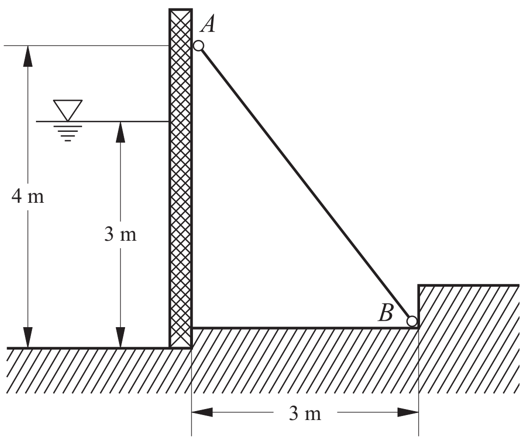
$(S.F = \dfrac{\sigma_c}{\sigma_W} = 4)$, $(\sigma_c = 160\ kg/cm^2) \rightarrow 4 = \dfrac{160}{\sigma_W} \rightarrow \sigma_W = 40\ kg/cm^3$ (I)

$\sigma_W = \dfrac{W}{A} = \dfrac{\gamma V}{A} = \dfrac{\gamma A h}{A} = \gamma h$ (II)

From the two relations (I) and (II), it is obtained that:

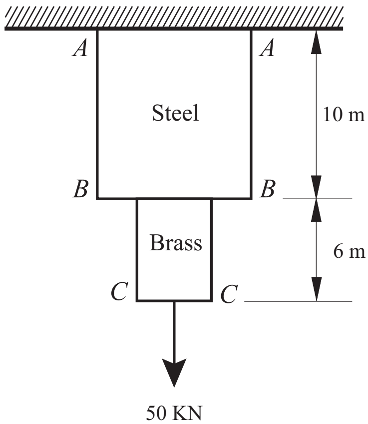
$40 = (2\times10^{-3})h \rightarrow h = 20000\ cm = 200\ m$

2- An impermeable water shield is held in place by wooden posts AB with a circular cross-section and a diameter of 15 cm to prevent overturning. The allowable stress of the wood is $20\ kg/cm^2$. Determine the maximum allowable spacing between the posts.



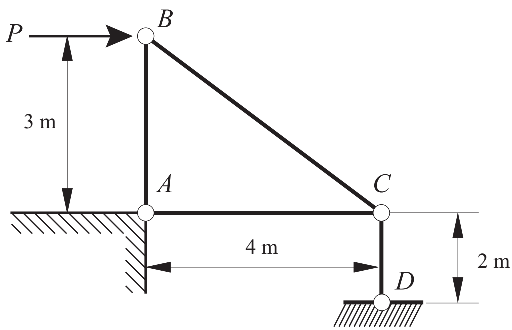
$x = 1.88\ m$

3- As shown below, two connected prismatic bars carry a vertical load of 50 KN. The upper rod is made of steel with a cross-sectional area of 60 cm^2 and a unit weight of $\gamma_{st} = 7.7\times10^4\ N/m^3$, while the lower bar is made of brass with a cross-sectional area of 50 cm^2 and a unit weight of $\gamma_B = 8.25\times10^4\ N/m^3$. The modulus of elasticity of steel is $E_{st} = 2\times10^{11}\ N/m^2$ and that of brass is $E_B = 9\times10^{10}\ N/m^2$. Determine the maximum stress developed in each bar.



$\sigma_{st,\max} = 9.5\ N/mm^2$ $\sigma_{B,\max} = 10.5\ N/mm^2$

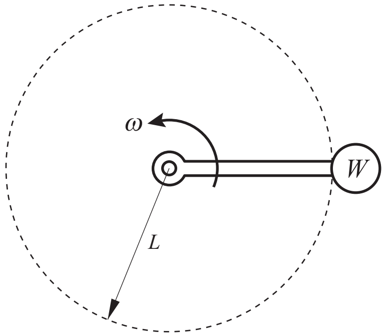
4- In the structure shown below, all bars are made of steel and have an equal cross-sectional area of 30 cm^2. The applied load P is 10 t. Determine the stresses developed in each bar.



$\sigma_{AB} = 250\ kg/cm^2\ (T)$ $\sigma_{AC} = 1333\ kg/cm^2\ (T)$

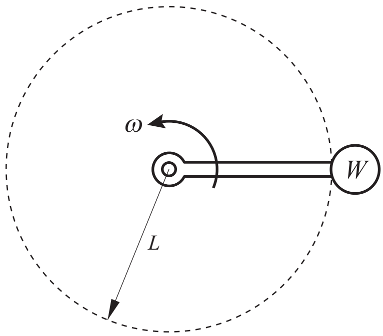
$\sigma_{BC} = 417\ kg/cm^2\ (C)$ $\sigma_{CD} = 250\ kg/cm^2\ (C)$

5- A bar of length L rotates in a horizontal plane about a vertical axis with an angular velocity ω. A weight W is attached to the end of the bar. If the allowable stress is σ_W and the weight of the bar is neglected, derive an expression for the required cross-sectional area (A) of the bar.



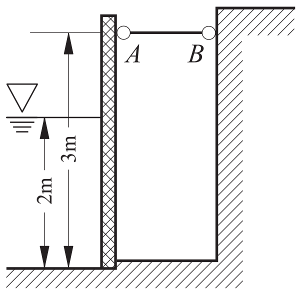
$$A = \frac{W L \omega^2}{g \sigma_W}$$

6- If the unit weight of the bar material in the previous example is γ, derive an expression for the required cross-sectional area (A) of the bar accounting for its weight.



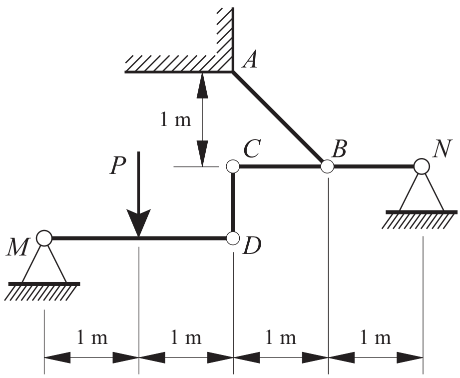
$$A = \frac{2 W L \omega^2}{2\sigma_W g - \gamma L^2 \omega^2}$$

7- As shown below, a rigid panel, supported by wooden compression bars AB, resists water pressure. The spacing between the wooden compression bars is 3 m. If the allowable compressive stress of the wood is $\sigma = 30\ kg / cm^2$, determine the appropriate circular cross-sectional dimensions for the wooden compression bars.



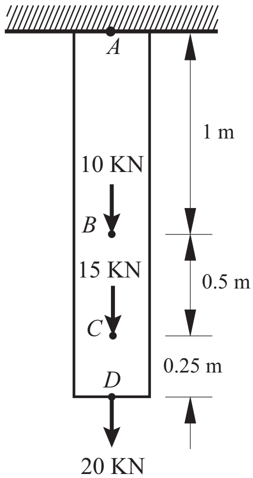
$d = 7.5\ cm$

8- In the figure below, the load P is 10 t, and the allowable stress of the material in members AB and CD is 100 kg / cm^2. Determine the diameters of these members.



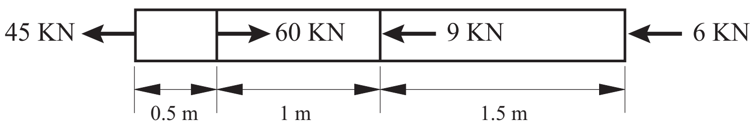
$D_{CD} = 25.2\ mm$
$D_{AB} = 42.4\ mm$

9- As shown below, a steel bar with a cross-sectional area of 5 cm^2 and a modulus of elasticity of $2 \times 10^{11}\ N / m^2$ is subjected to the loads indicated. Determine the elongation of the bar.



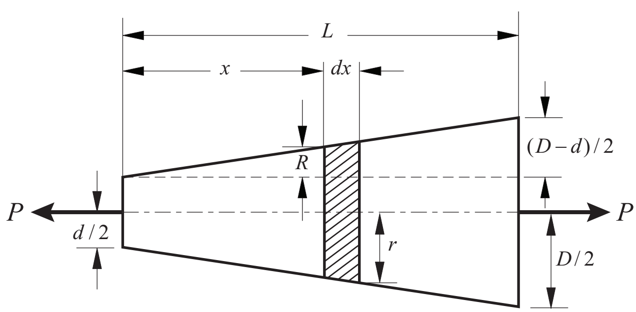
$\Delta = 0.675\ m$

10- As shown below, a brass bar with a cross-sectional area of 10 cm^2 and a modulus of elasticity of $E = 9 \times 10^{10}\ N/m^2$ is subjected to the loads indicated. Determine its elongation or shortening.



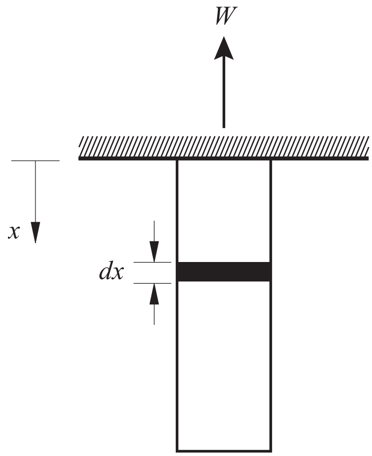
$\delta_{total} = 0.0017\ cm$

11- As shown below, a truncated conical bar is subjected to two axial tensile forces P at its ends. Determine the elongation of the bar.



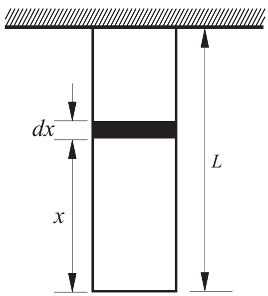
$$\delta = \frac{4PL}{\pi EDd}$$

12- A prismatic bar of length L, cross-sectional area A, and weight W is suspended under its own weight. Derive an expression for its elongation.



$$\delta = \frac{WL}{2AE}$$

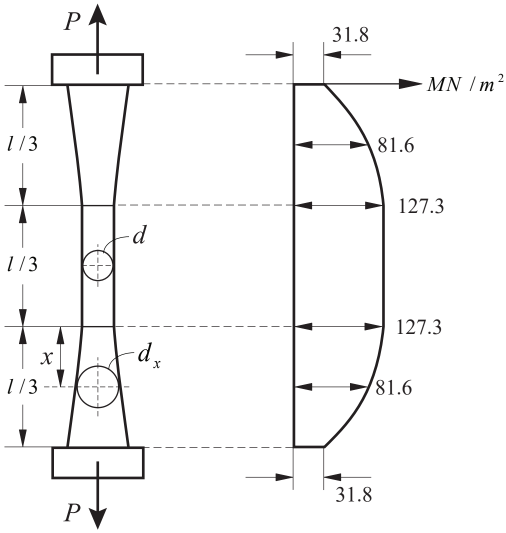
13- Compute the change in volume (ΔV) of a prismatic bar suspended under its own weight. Let W be the total weight of the bar, L its length, v the Poisson's ratio, and E the elastic modulus of the bar's material.



$$\Delta V = \frac{WL(1-2v)}{2E}$$

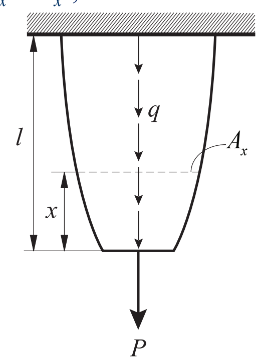
14- Consider the bar shown in the figure below. Plot the vertical stress distribution σ_x along the length of the bar and compute the resulting axial deformation Δl.

$P = 10\ KN \quad l = 0.3\ m \quad d = 0.01\ m \quad d_x = (0.01 + x^2)\ m \quad E = 2\times 10^5\ MN/m^2$



$\Delta l \simeq 0.0146\ cm$

15- Assuming the values of P, q, l, A_x, E and μ are known in the figure below, determine the values of ε_x, $\Delta A_x / A_x$, and ΔV.



$$\varepsilon_x = \frac{P + qx}{EA_x}$$

$$\frac{\Delta A_x}{A_x} = -2\frac{P + qx}{EA_x}$$

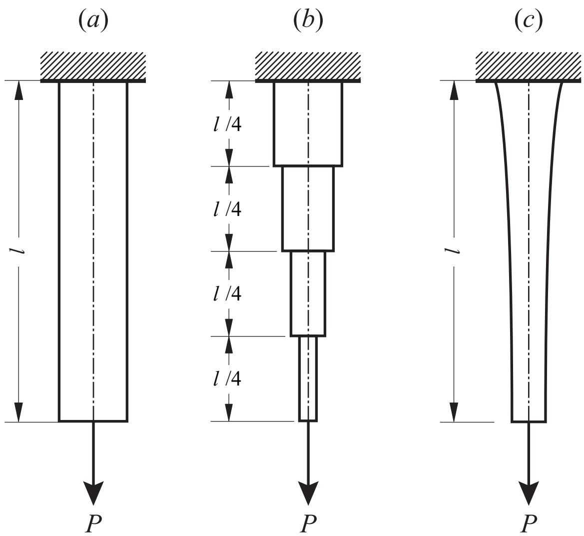
$$\Delta V = \frac{1 - 2\mu}{E}(P + \frac{ql}{2})l$$

16- Assuming $P = 16\ t$, $\gamma = 8\ gr / cm^3$, $\sigma = 1600\ kg / cm^2$, $E = 2 \times 10^6\ kg / cm^2$ and $l = 40\ m$, calculate:

a) The cross-sectional area A_{pr}, weight Q_{pr}, and elongation Δl_{pr} of a prismatic bar.

b) The maximum cross-sectional area A_{st}, weight Q_{st}, and elongation Δl_{st} of the stepped bar.

c) The maximum cross-sectional area A_{us}, weight Q_{us}, and elongation Δl_{us} of a bar with a varying cross-section.



(a)

$A_{pr} \simeq 10.204\ cm^2$

$Q_{pr} \simeq 326.53\ kg$

$\Delta l \simeq 3.168\ cm$.

(b)

$A_{st} \simeq 10.203\ cm^2$

$Q_{st} \simeq 324.8\ kg$

$\Delta l_{st} \simeq 3.192\ cm$

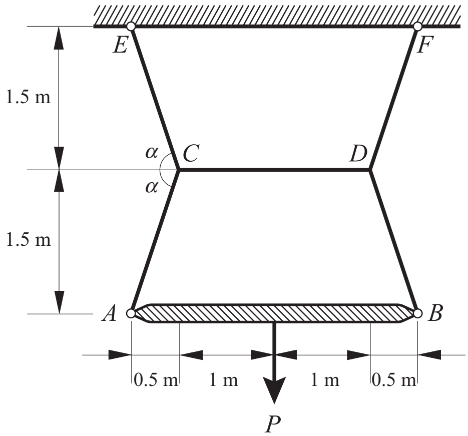
(c)

$A_{us} \simeq 10.202\ cm^2$

$Q_{us} \simeq 323.3\ kg$

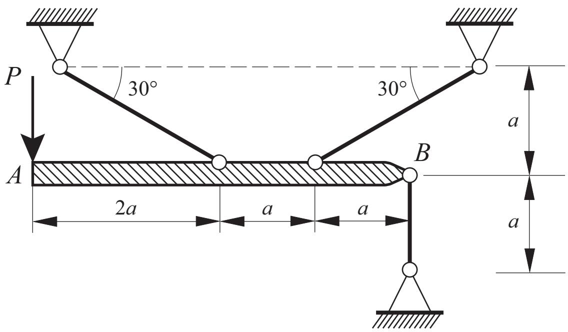
$\Delta l_{us} = 3.2\ cm$

17- As shown in the figure below, the rigid bar AB is suspended by cables. If the allowable stress of the cable material is $100\ kg/cm^2$ and the diameter of all cables is identical and equal to $d = 2.5\ cm$, calculate the maximum permissible load P. The effective cross-sectional area of each cable is 75% of its total area.



$P = 698.5\ kg$

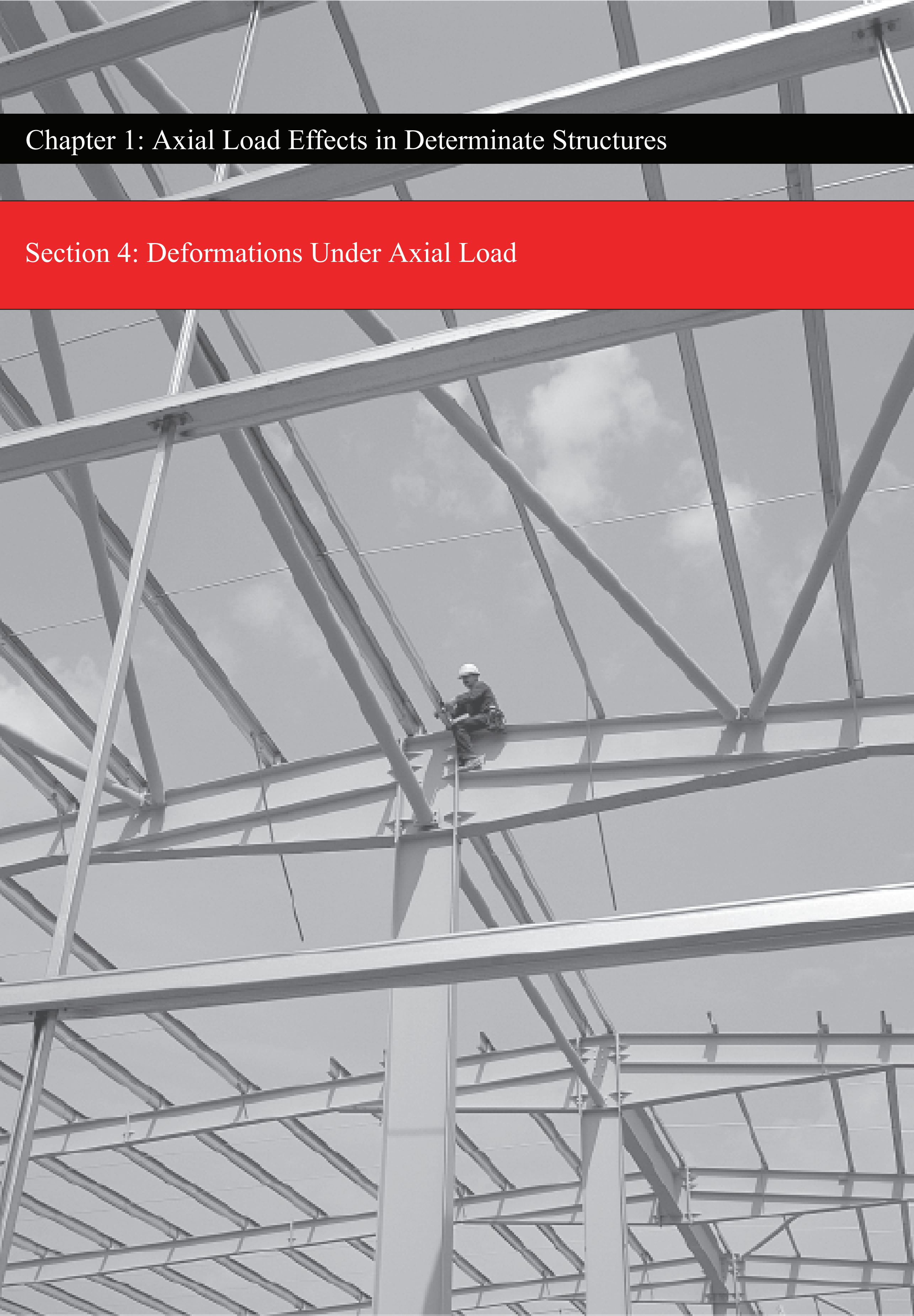
18- A rigid body AB is supported by three cables, each with a cross-sectional area of $A = 2\ cm^2$ and an allowable stress of $\sigma_W = 12 \times 10^7\ N/m^2$. This body is subjected to a load P at point A. Calculate the maximum permissible load P.



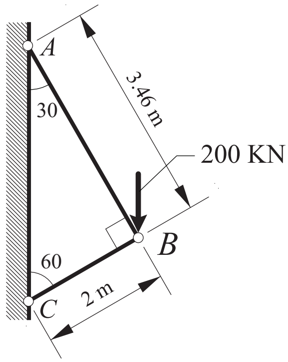
$P_{\max} = 9000\ N$

Chapter 1: Axial Load Effects in Determinate Structures

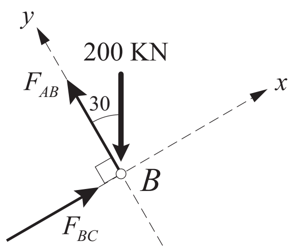
Section 4: Deformations Under Axial Load



1- The truss members shown in the figure below are made of steel, with a yield stress of 200 MPa and a modulus of elasticity of $E = 2\times10^{11}\ N/m^2$. Assuming safety factors in tension and compression of 2 and 2.5, respectively, determine the cross-sectional area of each member and also calculate the horizontal and vertical components of the displacement at hinge B.



Equilibrium equation at joint B:



$$\begin{cases}\sum F_x = 0 \\ \sum F_y = 0\end{cases} \rightarrow \begin{cases}F_{BC} - 200\sin 30 = 0 \\ F_{AB} - 200\cos 30 = 0\end{cases} \rightarrow \begin{cases}F_{BC} = 100\ KN\ (C) \\ F_{AB} = 100\sqrt{3}\ KN\ \ (T)\end{cases}$$

Since, according to the problem assumptions, the safety factors in tension and compression differ, the design and determination of the allowable cross-sectional area must account for the type of force and the corresponding safety factor. Bar AB is in tension; therefore:

$(\sigma_y = 200\ MPa)$, $(S.F_{(T)} = 2.0)$

$$\rightarrow\ \sigma_{all} = \frac{\sigma_y}{S.F_{(T)}} = \frac{200}{2} = 100\ MPa$$

$$S_{AB} = \frac{F_{AB}}{\sigma_{all}} = \frac{100\sqrt{3}\times10^3}{100\times10^6} = 1.73\times10^{-3}\ m^2 = 17.32\ cm^2$$

Bar BC is in compression; therefore:

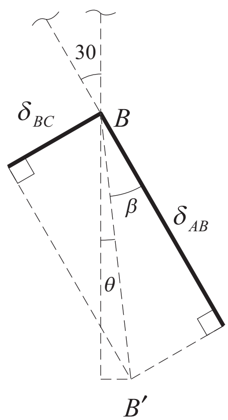
$(\sigma_y = 200\ MPa)$, $(S.F_{(C)} = 2.5)$

$$\rightarrow\ \sigma_{all} = \frac{\sigma_y}{S.F_{(C)}} = \frac{200}{2.5} = 80\ MPa$$

$$S_{BC} = \frac{F_{BC}}{\sigma_{all}} = \frac{100\times10^3}{80\times10^6} = 1.25\times10^{-3}\ m^2 = 12.5\ cm^2$$

To determine the final position of point B, the elongation of bar AB (in tension) and the shortening of bar BC (in compression) must be obtained. The point B must be placed in a position where δ_{AB} and δ_{BC} are created.

Diagram of member deformations and the position of point B:



$$\delta_{AB} = \frac{(100\times\sqrt{3}\times10^3)(3.46)}{(1.73\times10^{-3})(2\times10^{11})}$$

$$\delta_{AB} == 1.73\times10^{-3}\ m\ = 1.73\ mm$$

$$\delta_{BC} = \frac{(100\times10^3)(2)}{(1.25\times10^{-3})(2\times10^{11})}$$

$$\delta_{BC} = 8\times10^{-4}\ m\ = 0.8\ mm$$

$$BB' = \sqrt{(\delta_{BC})^2 + (\delta_{AB})^2} = 1.906\ \ mm$$

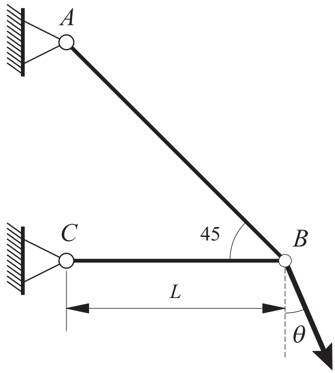
$$\beta = \tan^{-1}(\frac{\delta_{BC}}{\delta_{AB}}) = \tan^{-1}(\frac{0.8}{1.73}) = 24.8^\circ$$

$$\theta = 30 - \beta = 30 - 24.8 = 5.2^\circ$$

Vertical displacement downward $\quad \Delta B_V = BB'\cos\theta = 1.906\cos 5.2 = 1.898\ mm\ \downarrow$

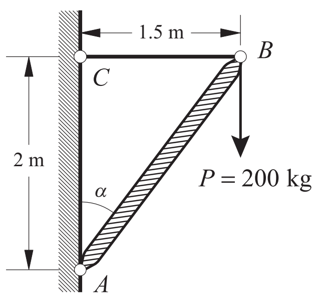
Horizontal displacement to the right $\quad \Delta B_H = BB'\sin\theta = 1.906\sin 5.2 = 0.17\ mm\ \leftarrow$

2- In the truss shown below, the cross-sectional areas of members AB and BC are A_1 and A_2, respectively. Determine the angle θ such that the displacement of hinge B occurs in the direction of the applied load P.



$$\cot 2\theta = -\sqrt{2}\,\frac{A_2}{A_1}$$

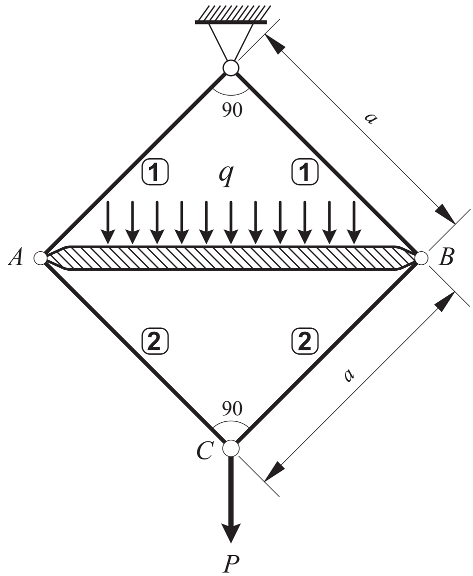
3- In the structure shown below, member BC is a steel wire with a diameter of $d_{st} = 0.3\ cm$ and an elastic modulus of $E_{st} = 2\times 10^6\ kg/cm^2$, while member AB is a wooden member with a square cross-section of side 2.5 cm and an elastic modulus of $E_W = 10^5\ kg/cm^2$. Compute the horizontal and vertical components of the displacement of point B under the vertical load $P = 200\ kg$.



$\Delta B_H = 1.59\ mm$

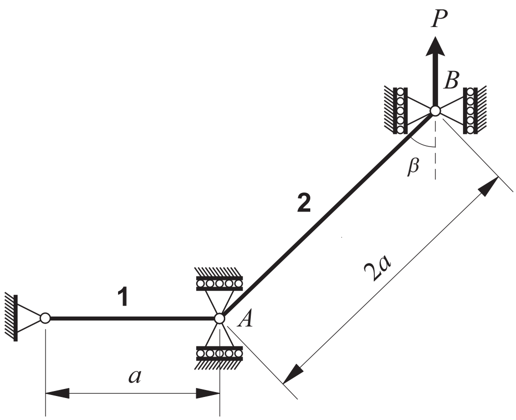
$\Delta B_V = 2.44\ mm \downarrow$

4- Compute the displacements produced by the external load P as well as the stresses developed in each of the members. The cross-sectional area of the members is A, and the elastic modulus is E.



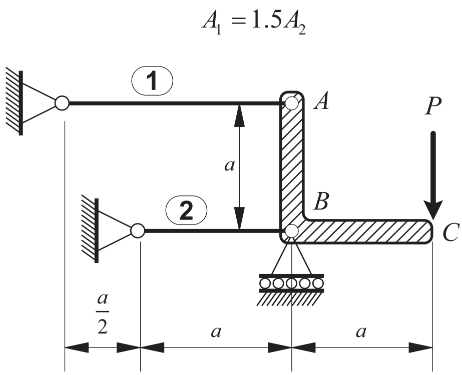
$$\Delta = \frac{a}{EA}(2P + \sqrt{2}qa)$$

5- Compute the displacements produced by the external load P as well as the stresses developed in each of the members. The cross-sectional area of the members is A, and the elastic modulus is E.



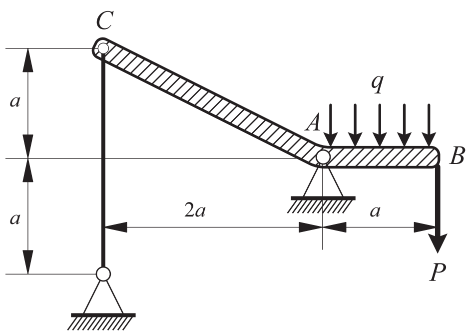
$$\Delta B_V = \frac{Pa}{EA\cos^2\beta}(2 + \sin^2\beta)$$

6- Compute the displacements produced by the external load P as well as the stresses developed in each of the members. The elastic modulus is E.



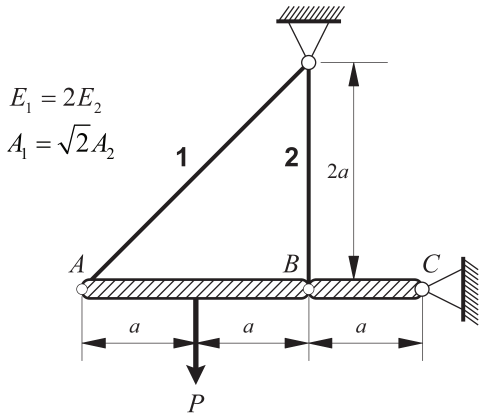
$$\Delta C_V = \frac{2Pa}{EA_2}$$

7- Compute the displacements produced by the external load P as well as the stresses developed in each of the members. The cross-sectional area of the members is A, and the elastic modulus is E.



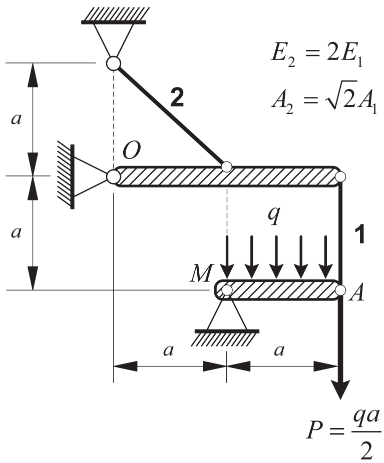
$$\delta_B = \frac{(qa + 2P)a}{4AE}$$

8- Compute the displacements produced by the external load P as well as the stresses developed in each of the members.



$$\delta_P = \frac{Pa}{E_2 A_2}$$

9- Compute the displacements produced by the external load P as well as the stresses developed in each of the bar.

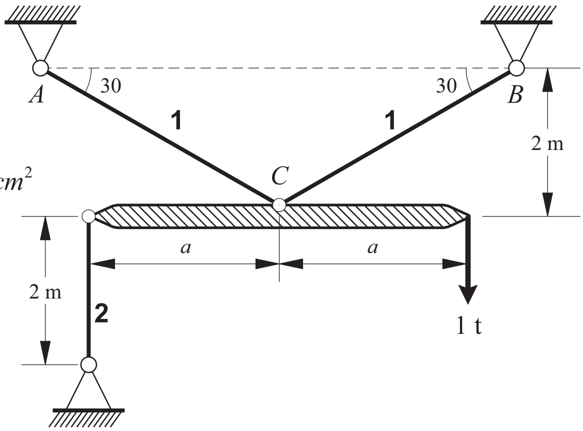


$$\Delta_A = \frac{10Pa}{A_1 E_1}$$

10- Compute the displacements produced by the external load P as well as the stresses developed in each of the bar.

$A_1 = A_2 = 2\ cm^2$

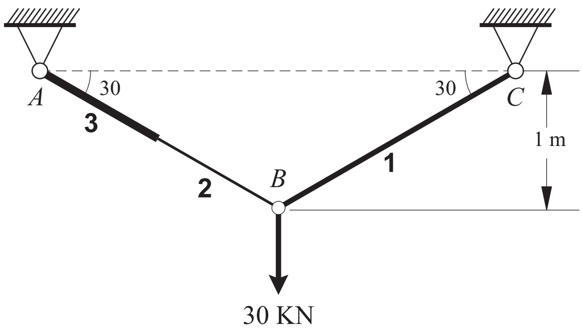
$E = 2\times10^6\ kg/cm^2$



$\Delta = 0.085\ mm$

11- Compute the displacements produced by the external load P as well as the stresses developed in each of the bar.

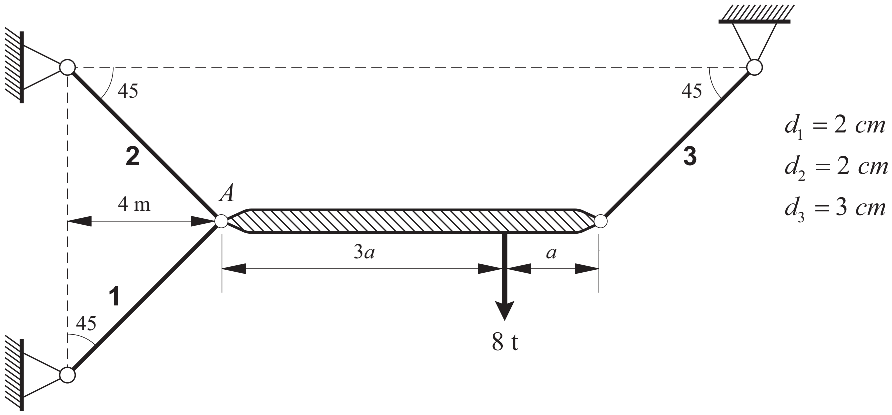
($A_1 = 3\ cm^2$, $A_3 = 2A_2 = 4\ cm^2$, $l_2 = l_3$, $E = 2\times10^5\ MN/m^2$)



$\Delta B_H = 0.072\ mm$

$\Delta B_V = 2.125\ mm$

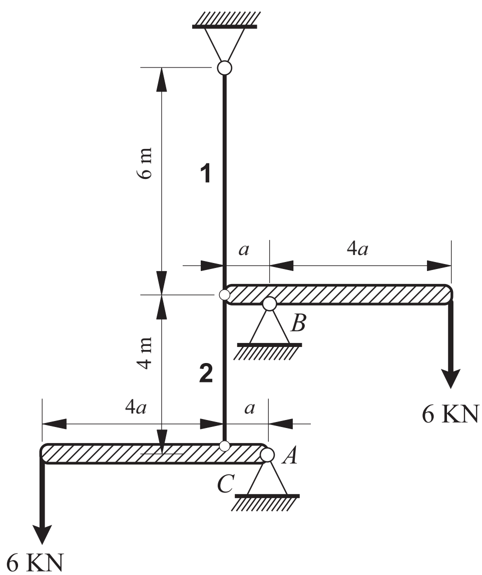
12- In the structure shown below, calculate the displacement of joint A and the stresses developed in the bars. The elastic modulus is $E = 2\times10^6\ kg/cm^2$.



$\Delta A_H \ = 5.4\ mm$

$\Delta A_V \ = 1.8\ mm$

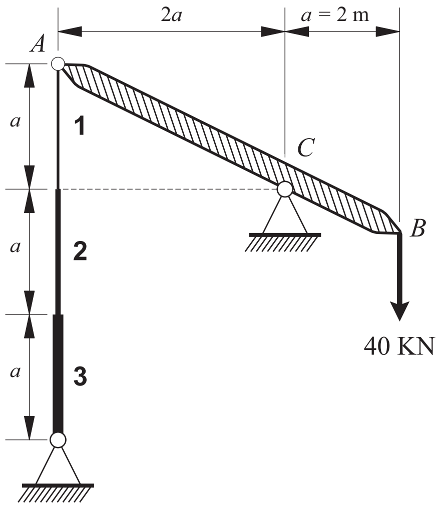
13- In the structure shown below, calculate the displacement of joint C and the stresses developed in the bars. The elastic modulus is $E = 2\times10^6\ MN/m^2$ and $A_1 = A_2 = 2\ cm^2$.



$\Delta_C = 0.39\ mm \downarrow$

14- In the structure shown below, calculate the displacements produced under the external load P as well as the stresses developed in the bars. The values are

$A_3 = 2A_2 = 10$, $A_1 = 2.5\ cm^2$ and $E = 2\times10^6\ kg/cm^2$.



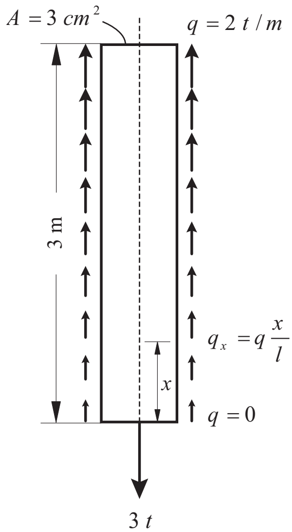
$\Delta B_V = 0.7\ mm$

$\Delta B_H = 0.35\ mm$

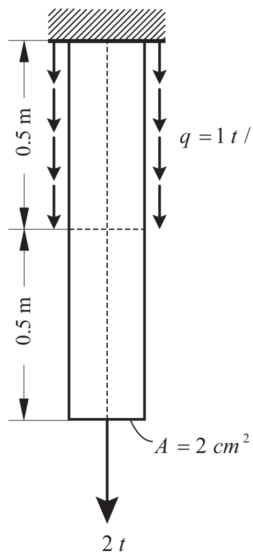
Chapter 1: Axial Load Effects in Determinate Structures

Section 5: Unsolved Problems

1- In the figures below, first draw the axial stress diagram of the bars and then calculate the resulting change in length created. The value of $E = 2\times10^6\ MN/m^2$.

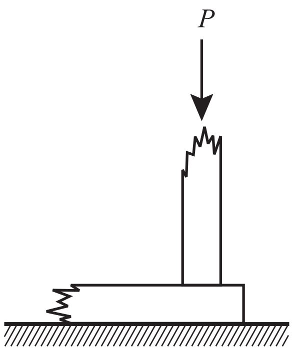


$\Delta = 1\ mm$



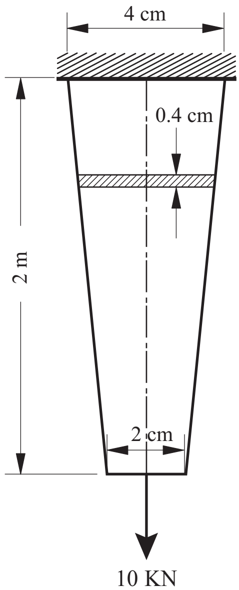
$\Delta = 0.531\ mm$

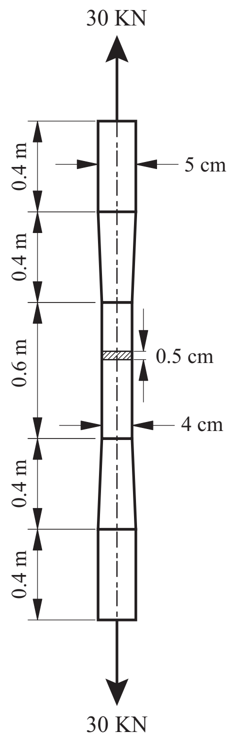
2- As shown in the figure below, a wooden column with dimensions $20 \times 20\ cm^2$ is supported by a strong wooden plate. The allowable compressive stress for the wooden column is $\sigma = 100\ kg/cm^2$, and the allowable compressive stress for the strong wooden plate is $\sigma = 30\ kg/cm^2$. Calculate the maximum load applied to the wooden column.



$P = 12\ t$

3- In the figures below, first draw the axial stress diagram of the bars and then calculate the resulting change in length created. The value of $E = 2 \times 10^5\ MN/m^2$.

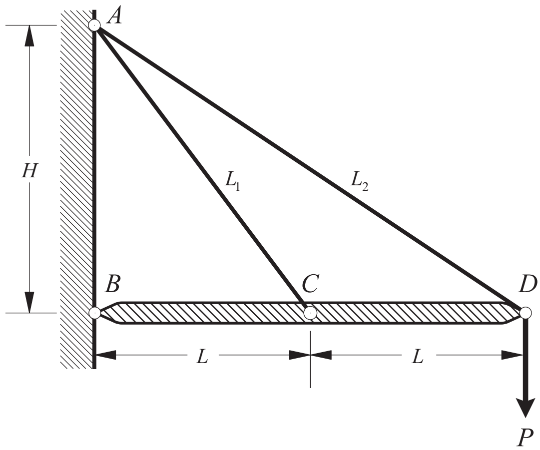



30 KN
0.4 m
5 cm
0.4 m
0.6 m
0.5 cm
4 cm
0.4 m
0.4 m
30 KN

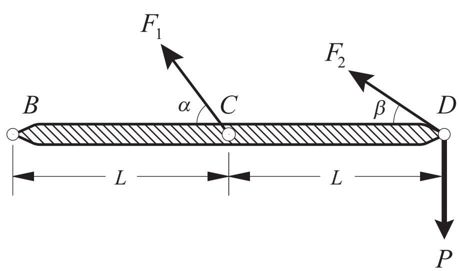
Chapter 2: Axial Load Effects in Indeterminate Structures

Section 1: Axial Load and Stress

1- As shown in the figure, a rigid bar *BD* is suspended by two barss, *AC* and *AD*. Initially, the bars are unstressed, and both have the same modulus of elasticity. If a load *P* is applied at point *D*, determine the tensile forces developed in each bar. Neglect the self-weight of all elements.



The free-body diagram of the rigid bar *BD* is shown as follows:

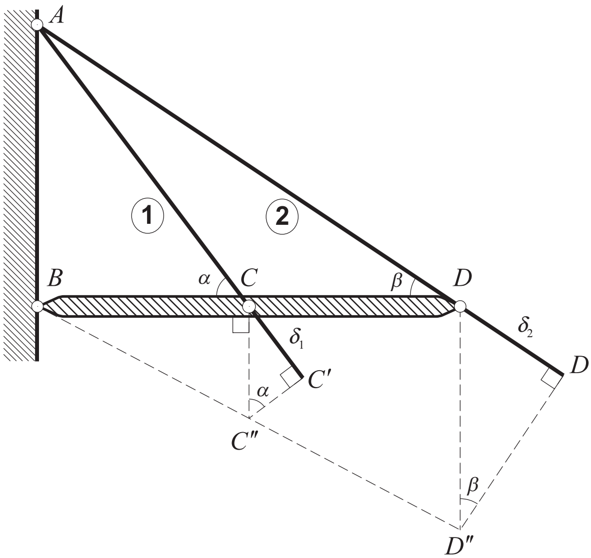


$$\sum M_B = 0$$

$$\rightarrow \ (F_1 \sin\alpha)(L) + (F_2 \sin\beta)(2L) = (P)(2L)$$

$$\rightarrow \ F_1(\frac{H}{L_1})(L) + F_2(\frac{H}{L_2})(2L) = 2PL \quad \text{(I)}$$

Two bars, 1 and 2, elongate under the tensile forces F_1 and F_2, causing point C to move to C' and point D to move to D'. Also, the rigid bar *BD* undergoes no deformation and rotates about point B. Consequently, an arc with center at B and radius *BC* (perpendicular to *BC*) intersects an arc with center at *A* and radius AC' (perpendicular to AC') at point C''. A similar condition occurs for point *D*, resulting in the rigid bar *BCD* moving to its new position $BC''D''$.



$$CC' = \delta_1 = \frac{F_1 L_1}{EA_1} \quad , \quad DD' = \delta_2 = \frac{F_2 L_2}{EA_2}$$

$$CC'' = \frac{CC'}{\sin\alpha} = \frac{\delta_1}{\sin\alpha} \quad , \quad DD'' = \frac{DD'}{\sin\beta} = \frac{\delta_2}{\sin\beta}$$

$$\Delta BCC'' \sim \Delta BDD'' \rightarrow \frac{CC''}{DD''} = \frac{L}{2L} \rightarrow DD'' = 2CC''$$

$$\rightarrow \frac{\delta_2}{\sin\beta} = 2\frac{\delta_1}{\sin\alpha} \rightarrow \delta_2 \sin\alpha - 2\delta_1 \sin\beta = 0 \rightarrow \delta_2 \frac{H}{L_1} - 2\delta_1 \frac{H}{L_2} = 0$$

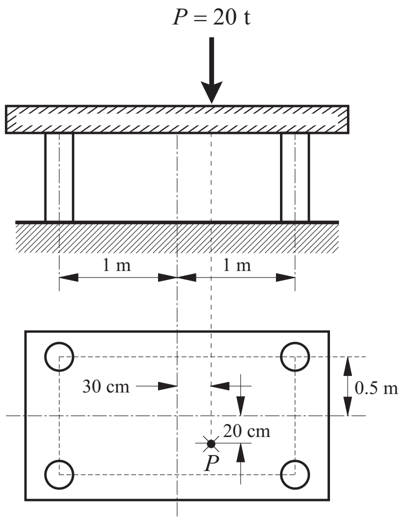
$$\rightarrow \left(\frac{F_2 L_2}{EA_2}\right)\left(\frac{H}{L_1}\right) - 2\left(\frac{F_1 L_1}{EA_2}\right)\left(\frac{H}{L_2}\right) = 0 \rightarrow F_2 \frac{L_2}{A_2 L_1} - 2F_1 \frac{L_1}{A_1 L_2} = 0 \quad \text{(II)}$$

Solving the two equations (I) and (II) yields:

$$\rightarrow \begin{cases} F_1 \dfrac{H}{L_1} + 2F_2 \dfrac{H}{L_2} = 2P \\[2ex] 2F_1 \dfrac{L_1}{A_1 L_2} - F_2 \dfrac{L_2}{A_2 L_1} = 0 \end{cases}$$

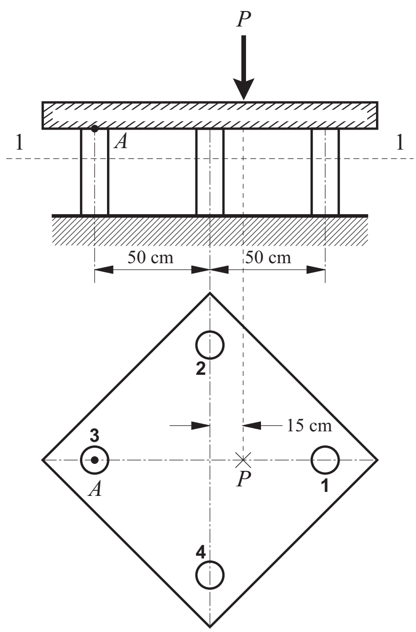
$$\rightarrow \begin{cases} F_{AD} = F_2 = \dfrac{2P}{\dfrac{A_1 L_2^3 H + 4A_2 L_1^3 H}{2A_2 L_2 L_1^3}} = \dfrac{2P}{\dfrac{A_1 L_2^2 H}{2A_2 L_1^3} + \dfrac{2H}{L_2}} \\[4ex] F_{AC} = F_1 = \dfrac{2P}{\dfrac{A_1 L_2^3 H + 4A_2 L_1^3 H}{A_1 L_2^3 L_1}} = \dfrac{2P}{\dfrac{4A_2 L_1^2 H}{A_1 L_2^3} + \dfrac{H}{L_1}} \end{cases}$$

2- A rectangular rigid slab is supported by four columns. If the columns have identical cross-sectional areas, lengths, and material, determine the forces developed in each of them.



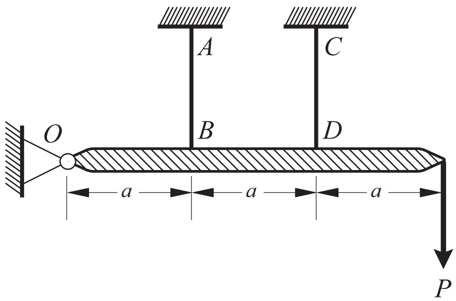
$F_1 = -4.5\ t \quad F_2 = -8.5\ t \quad F_3 = -5.5\ t \quad F_4 = -1.5\ t$

3- A square slab is supported symmetrically by four columns. All columns have the same length, cross-sectional area and material. Neglecting the deformation of the slab, determine the forces acting on the columns.



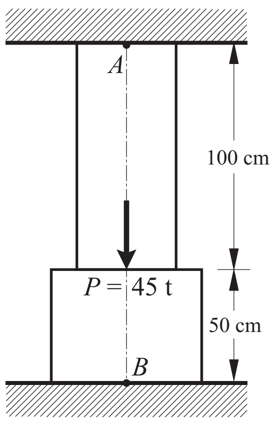
$F_1 = 0.4P$
$F_2 = 0.25P$
$F_3 = 0.1P$

4- A rigid bar of length $3a$, subjected to load $P = 25000\ kg$ at its free end, is supported by two identical cables AB and CD. The cross-sectional area of each cable is $A = 20\ cm^2$. Determine the stresses developed in each of the cables.



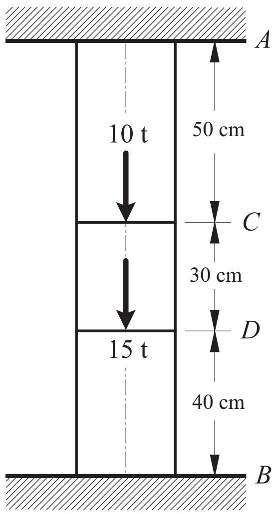
$\sigma_{AB} = 750\ kg/cm^2$
$\sigma_{CD} = 1500\ kg/cm^2$

5- The column shown in the figure is fixed at both its upper and lower ends. The cross-sectional area of the column is $10\ cm^2$ at the top and $40\ cm^2$ at the bottom. Determine the stresses developed in the upper and lower sections of the column.



$\sigma = 500\ kg/cm^2$ (Tension in upper part)
$\sigma = 1000\ kg/cm^2$ (Compression in lower part)

6- The bar shown in the figure, with a cross-sectional area of $10\ cm^2$,is fixed at both ends. Calculate the stresses developed in the different sections of the bar.

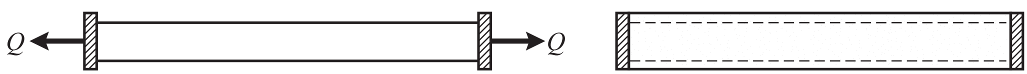


$\sigma_{AC} = 1083.3\ kg / cm^2$

$\sigma_{CD} = 83.3\ kg / cm^2$

$\sigma_{BD} = -1416.6\ kg / cm^2$

7- For the construction of prestressed concrete beams, first, as shown in Figure (a), steel cables are stretched under tensile stress σ_0 between two rigid end plates. Then, as shown in Figure (b), concrete is poured around the cables. After the concrete hardens, the external tensile forces Q are removed, resulting in a prestressed beam. If the modulus of elasticity of steel and concrete are in the ratio 12:1 and their cross-sectional areas are in the ratio 1:15, determine the final residual stresses developed in both materials.

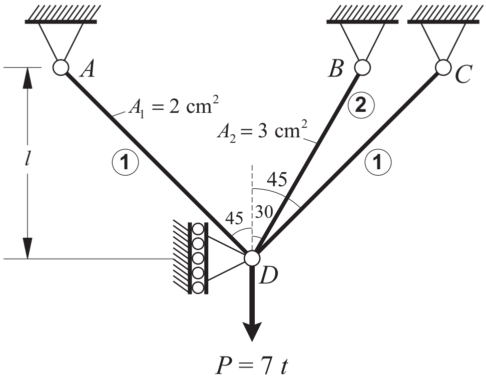


(a) (b)

$$\sigma_s = \frac{5}{9}\sigma_0$$

$$\sigma_c = \frac{1}{27}\sigma_0$$

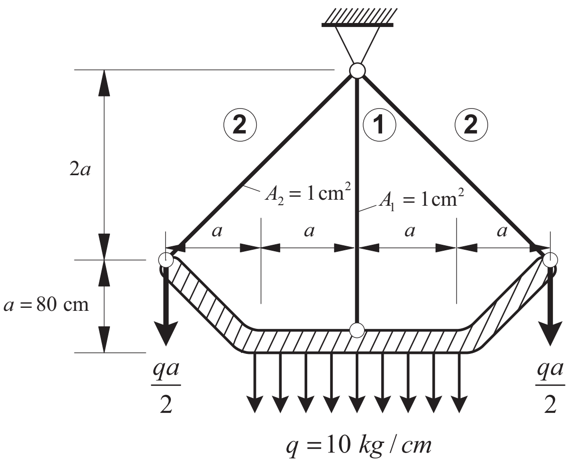
8- Assuming the modulus of elasticity is the same for all bars, determine the stresses in the bars.



$\sigma_1 = 1040.81\ kg/cm^2$

$\sigma_2 = 1561.21\ kg/cm^2$

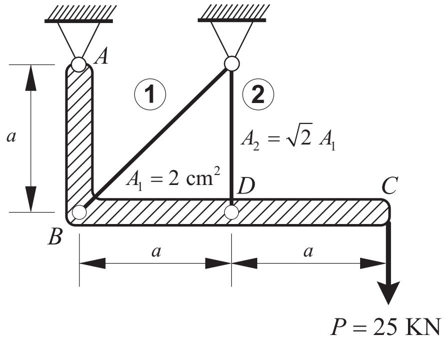
9- Assuming the modulus of elasticity is the same for all bars, determine the stresses in the bars.



$\sigma_1 = 1167.7\ kg/cm^2$

$\sigma_2 = 873.5\ kg/cm^2$

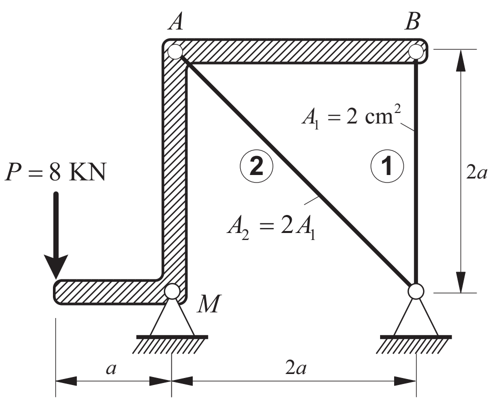
10- Assuming the modulus of elasticity is the same for all bars, determine the stresses in the bars.



$\sigma_1 = 70.7\ MN\ /\ m^2$

$\sigma_2 = 141.4\ MN\ /\ m^2$

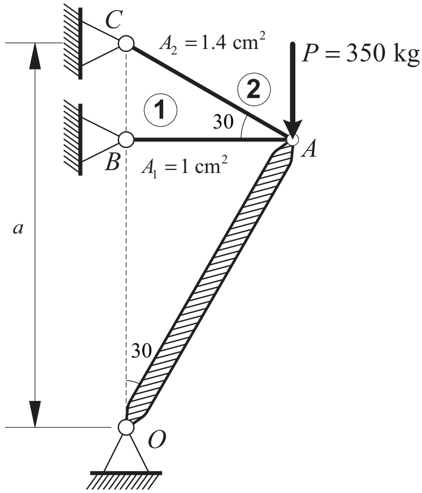
11- In the structure shown below, determine the stresses developed in the bars. Also, $E_1 = 0.7E_2$.



$\sigma_1 = 9.95\ MN\ /\ m^2$

$\sigma_2 = 7.1\ MN\ /\ m^2$

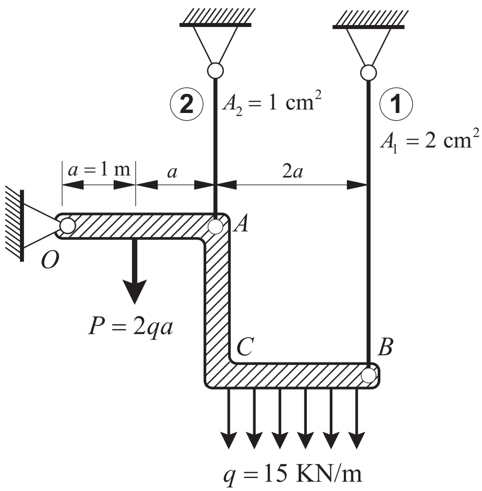
12- In the structure shown below, determine the stresses developed in the bars. Also, $E_1 = 0.7E_2$.



$\sigma_1 = 61.05\ kg/cm^2$

$\sigma_2 = 87.2\ kg/cm^2$

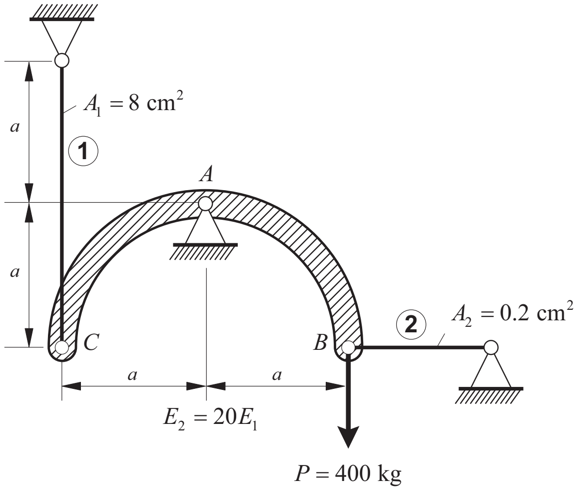
13- Assuming the modulus of elasticity is the same for all bars, determine the stresses in the bars.



$\sigma_1 = 120\ MN/m^2$

$\sigma_2 = 120\ MN/m^2$

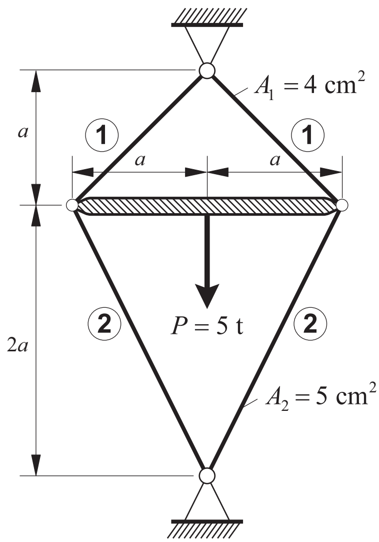
14- Assuming the modulus of elasticity is the same for all bars, determine the stresses in the bars.



$\sigma_1 = 25\ kg/cm^2$

$\sigma_2 = 1000\ kg/cm^2$

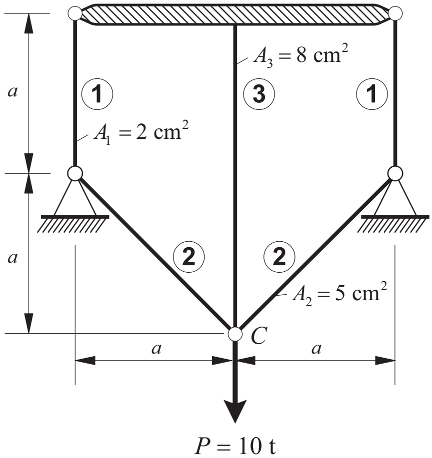
15- Assuming the modulus of elasticity is the same for all bars, determine the stresses in the bars.



$\sigma_1 = 390\ kg/cm^2$

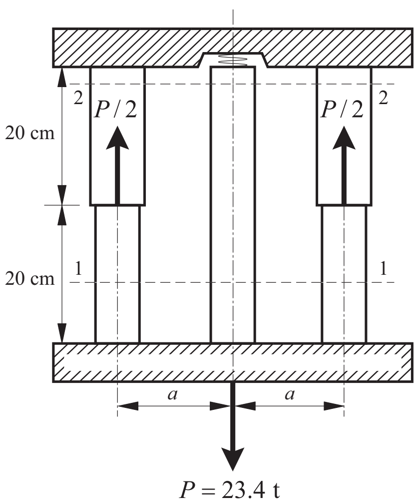
$\sigma_2 = 312\ kg/cm^2$

16- Assuming the modulus of elasticity is the same for all bars, determine the stresses in the bars.



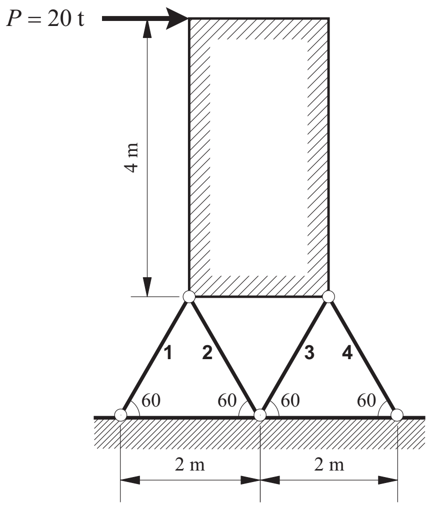
$\sigma_1 = 903\ kg/cm^2$ $\sigma_2 = 903\ kg/cm^2$ $\sigma_3 = 451.6\ kg/cm^2$

17- As shown in the figure below, two rigid bars are connected by three vertical bars. The two outer bars are made of steel, with cross-sectional areas of 16 cm^2 for the upper portions and 10 cm^2 for the lower portions. The middle bar is made of copper and has a cross-sectional area of 20 cm^2. A spring with stiffness $K = 0.8\times10^6\ kg/cm$ is placed between the top end of the copper bar and the upper rigid bar. The modulus of elasticity for steel and copper are $2\times10^6\ kg/cm^2$ and $10^6\ kg/cm^2$, respectively. Determine the stresses developed in the bars under the applied loading shown.



$\sigma_{cu} = 144\ kg/cm^2\ \ (T)\qquad \sigma_{st1} = 1026\ kg/cm^2\ \ (T)\qquad \sigma_{st2} = -90\ kg/cm^2\ \ (P)$

18- The rigid structure shown below is supported by four bars. All bars are made of the same material and have identical cross-sectional areas $25\ cm^2$. Determine the stresses developed in the bars.



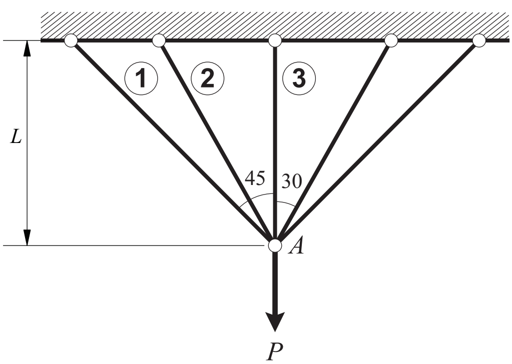
$\sigma_1 \simeq 1325\ kg/cm^2$

$\sigma_2 \simeq 525\ kg/cm^2$

$\sigma_3 \simeq -525\ kg/cm^2$

$\sigma_4 \simeq -1325\ kg/cm^2$

19- As shown in the figure below, all the truss bars are made of the same material and have a diameter of 3 *cm*. Determine the stresses developed in the bars, assuming a load *P* equal to 30 *t*.

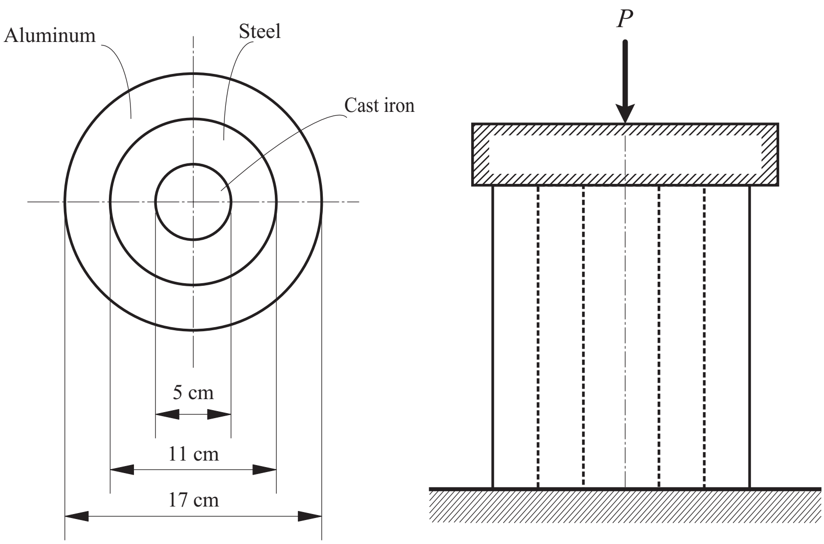


$\sigma_1 = 706 \; kg/cm^2$

$\sigma_2 = 1059 \; kg/cm^2$

$\sigma_3 = 1412 \; kg/cm^2$

20- The short cast iron bar is easily placed inside the steel tube, and both are housed inside an aluminum tube. Determine the stresses induced in the tubes and the cast iron bar due to the application of a load P through a rigid plate. The modulus of elasticity for cast iron, steel, and aluminum are $1.2\times10^6 \; kg/cm^2$, $2\times10^6 \; kg/cm^2$ and $0.7\times10^6 \; kg/cm^2$, respectively.

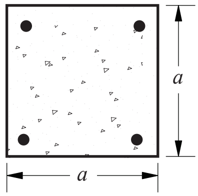


$\sigma_c = 180 \; kg/cm^2$

$\sigma_{st} = 300 \; kg/cm^2$

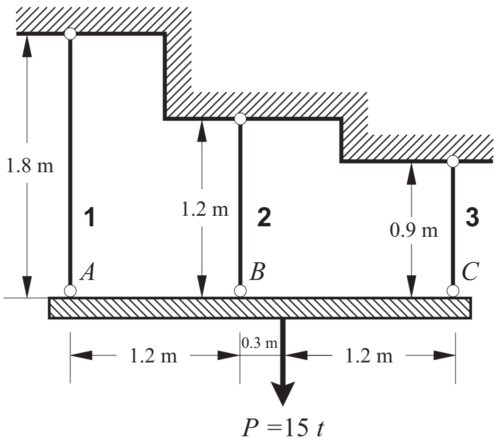
$\sigma_{al} = 105 \; kg/cm^2$

21- A square concrete column is reinforced with four steel bars. The total cross-sectional area of the steel bars is 1% of the column's cross-sectional area. The allowable stress of concrete is $60 \; kg/cm^2$ and the allowable stress of steel is $1200 \; kg/cm^2$. The ratio of the modulus of elasticity of steel to that of concrete is 10. If a load 100 t is applied to the column, determine the dimensions of the column's cross-section and the diameter of the steel bars.



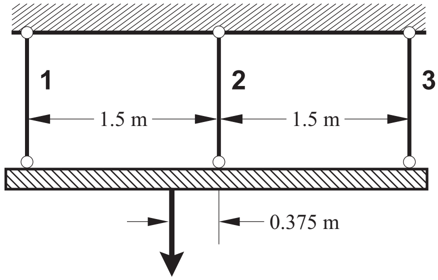
$a = 39\ cm$
$D = 22\ mm$

22- In the figure below, a rigid body is supported by several steel bars with equal cross-sectional areas. If the modulus of elasticity of the steel is $E = 2\times10^6\ kg/cm^2$ and its allowable stress is $1600\ kg/cm^2$, determine the required cross-sectional area for each bar.



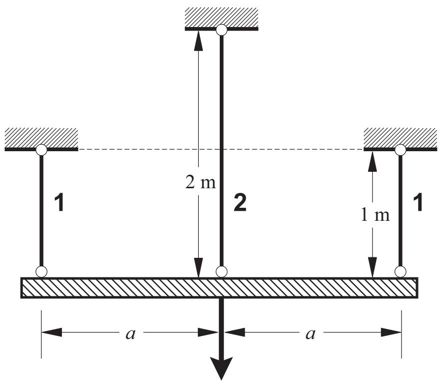
$A = 3.77\ cm^2$

23- In the figure below, a rigid body is supported by several steel bars with identical cross-sectional areas. If the modulus of elasticity of the steel is $E = 2\times10^6\ kg/cm^2$ and its allowable stress is $1600\ kg/cm^2$, determine the required cross-sectional area for each bar.



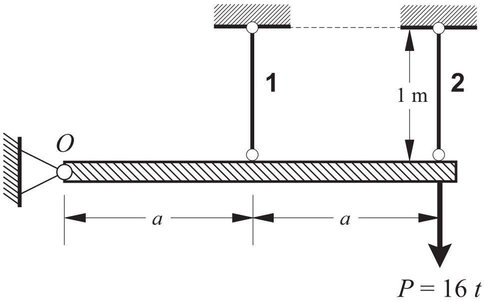
$A = 4.3\ cm^2$

24- In the figure below, a rigid body is supported by several steel bars with identical cross-sectional areas. If the modulus of elasticity of the steel is $E = 2\times10^6\ kg/cm^2$ and its allowable stress is $1600\ kg/cm^2$ calculate the required cross-sectional area for each bar.



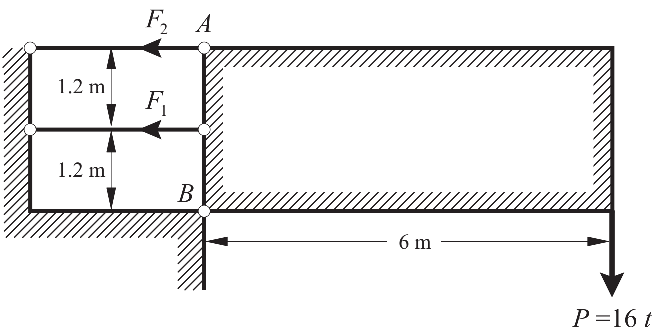
$A = 5\ cm^2$

25- In the figure below, a rigid body is supported by a number of steel bars of equal cross-section. If the modulus of elasticity of the steel is $E = 2\times10^6\ kg/cm^2$ and its allowable stress is $1600\ kg/cm^2$, determine the required cross-sectional area of each bar.



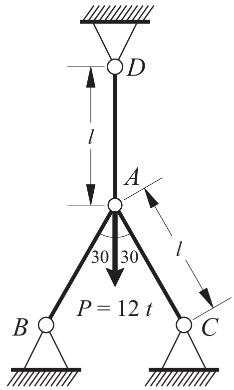
$A = 8\ cm^2$

26- In the figure below, a rigid body is supported by a number of steel bars of equal cross-section. If the modulus of elasticity of the steel is $E = 2\times10^6\ kg/cm^2$ and its allowable stress is $1600\ kg/cm^2$, determine the required cross-sectional area of the bars.



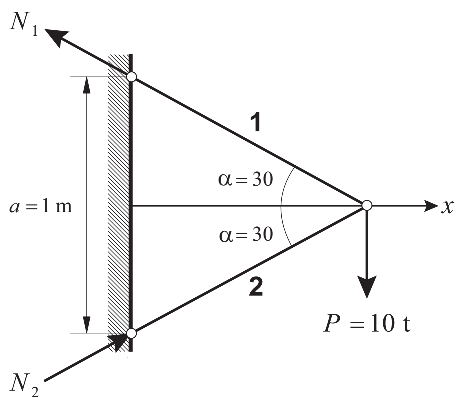
$A = 20\ cm^2$

27- As shown in the figure below, the three truss members have identical cross-sectional areas. Their modulus of elasticity and allowable stress are $2\times10^6\ kg/cm^2$ and $1600\ kg/cm^2$ respectively. Determine the required cross-sectional area of the members.



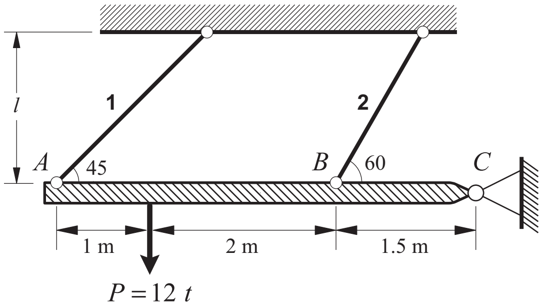
$A = 3\ cm^2$

28- In the structure shown in the figure, calculate the values of A_1 and A_2. Assume that $\sigma_{c,2} = 100\ kg/cm^2$, $\sigma_{t,1} = 1000\ kg/cm^2$, $E_1 = 2\times10^6\ kg/cm^2$ and $E_2 = 0.1\times10^6\ kgf/cm^2$ are given. The allowable displacements under the load P in the x and y directions are 1.3 mm.



$A_1 = 1\ cm^2 \quad A_2 = 12.5\ cm^2$

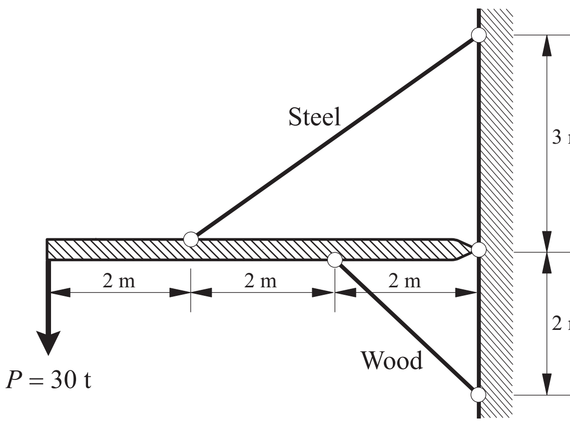
29- A rigid body is suspended by two steel cables with a modulus of elasticity $2\times10^6\ kg/cm^2$ and an allowable stress $1600\ kg/cm^2$. Cable 1 has a cross-sectional area twice that of cable 2. Determine the cross-sectional area of each cable.



$A_1 = 7.5\ cm^2$

$A_2 = 3.75\ cm^2$

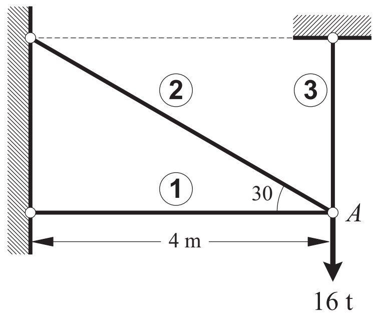
30- As shown below, the rigid bar is connected to the wall through a pin joint, a steel tie, and a wooden column. If the cross-sectional area of the column is 10 times that of the steel tie, and the allowable stresses for steel and wood are $1600\ kg/cm^2$ and $60\ kg/cm^2$, respectively, with the modulus of elasticity of steel $2\times10^6\ kg/cm^2$ and that of wood $10^5\ kg/cm^2$, calculate their cross-sectional areas.



$A_W \simeq 500\ cm^2$

$A_s = 50\ cm^2$

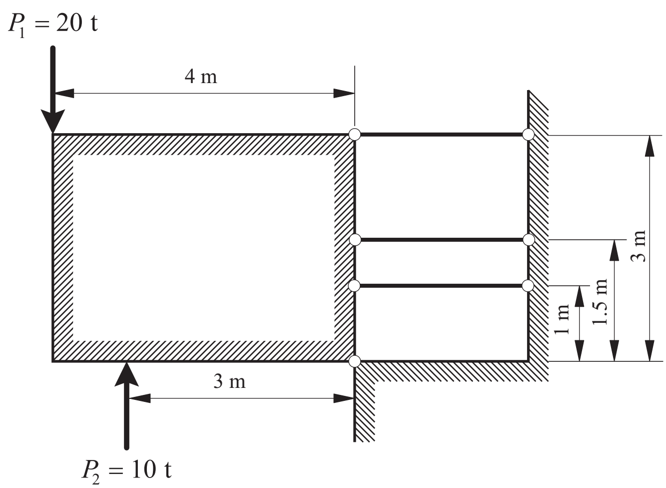
31- In the structure shown below, bar 1 is made of cast iron with allowable stress $800\ kg/cm^2$ and modulus of elasticity $1.2\times10^6\ kg/cm^2$, bar 2 is made of copper with allowable stress $600\ kg/cm^2$ and modulus of elasticity $10^6\ kg/cm^2$, and bar 3 is made of steel with allowable stress $1200\ kg/cm^2$ and modulus of elasticity $2\times10^6\ kg/cm^2$. The cross-sectional areas of bars 1 and 2 are equal, while the cross-sectional area of bar 3 is half that of bars 1 and 2. Calculate the cross-sectional areas of the bars.



$A_1 = A_2 = 24.6\ cm^2$

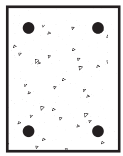
$A_3 = 12.3\ cm^2$

32- As shown in the figure below, a rigid structure is connected to the foundation by a hinge and three steel bars, all having the same cross-sectional area and length. If the allowable stress of the steel is $\sigma = 1600\ kg/cm^2$ and its modulus of elasticity is $2\times10^6\ kg/cm^2$, determine the required cross-sectional area for the bars.



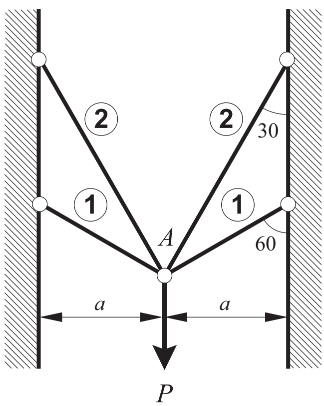
$A = 7.65\ cm^2$

33- The cross-sectional area of a short reinforced concrete column is $A_c = 645\ cm^2$, and four longitudinal steel bars, each with a cross-sectional area of $A_s = 10\ cm^2$, are symmetrically embedded in it. If the allowable stress of concrete $f'_c = 80\ kg/cm^2$ and the allowable stress of steel is $F_s = 1400\ kg/cm^2$, determine the allowable load of the column. The modulus of elasticity of steel is $E_s = 2\times10^6\ kg/cm^2$ and that of concrete is $E_c = 2\times10^5\ kg/cm^2$.



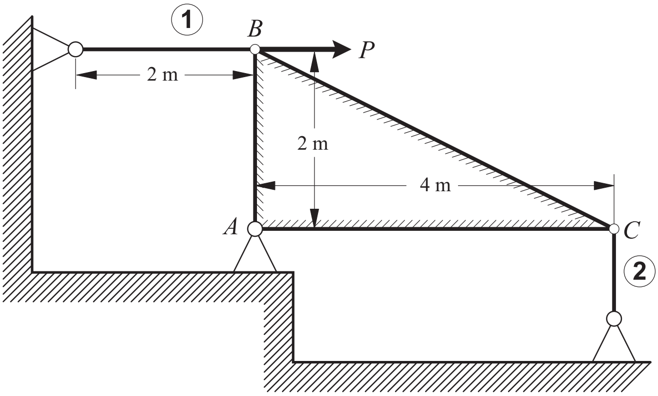
$P_{all} = 83.6\ t$

34- In the structure shown below, the bar 1 is made of steel with allowable stress $1600\ kg/cm^2$, cross-sectional area $10\ cm^2$ and modulus of elasticity $2\times10^6\ kg/cm^2$, while the bar 2 is made of copper with allowable stress $600\ kg/cm^2$, cross-sectional area $20\ cm^2$, and modulus of elasticity $10^6\ kg/cm^2$. Calculate the allowable load P_{all}.



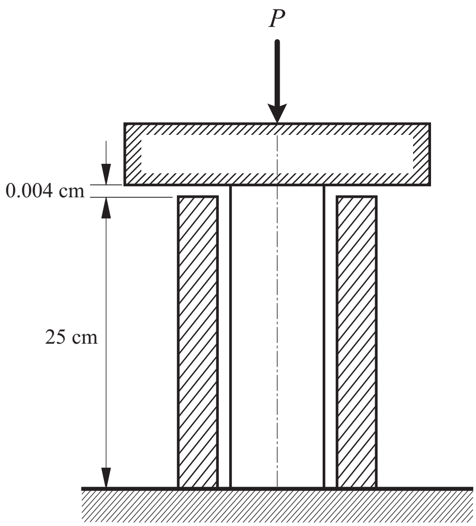
$P_{all} \simeq 32.8\ ton$

35- The rigid frame shown below is connected to the foundation through one hinge and two bars. Bar 1 is made of steel with an allowable stress of $1600\ kg/cm^2$ and a modulus of elasticity $2\times10^6\ kg/cm^2$, while Bar 2 is made of cast iron with an allowable stress of 1000 kg/cm^2 and a modulus of elasticity 1.2×10^6 kg/cm^2. If the cross-sectional areas of the steel and cast iron bars are 30 cm^2 and 50 cm^2 respectively, determine the maximum allowable load P.



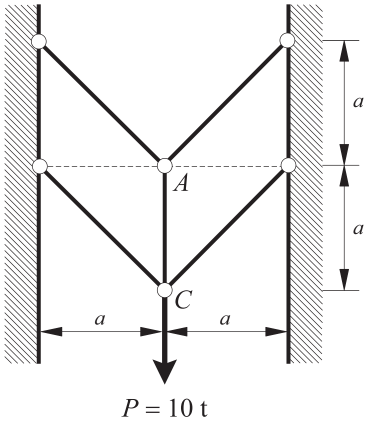
$P_{all} = 112.5\ t$

36- As shown in the figure below, an aluminum bar with a cross-sectional area 20 cm^2 has a stress-free length of 25.004 cm. Another steel bar with the same cross-sectional area, under the same conditions, has a length of 25 cm. Determine the load P required so that the stresses developed in the steel and aluminum bars are equal. The modulus of elasticity for steel and aluminum are 2×10^6 kg/cm^2 and 0.7×10^6 kg/cm^2, respectively.



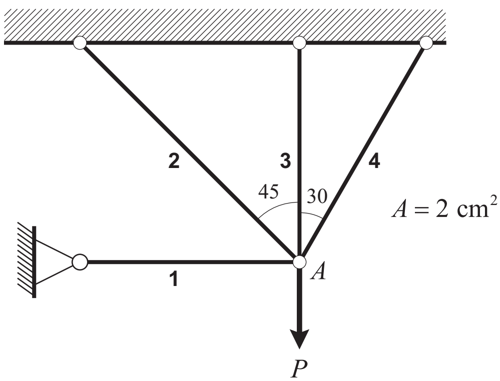
$P = 6.9\ t$

37- If the cross-sectional area of each bar in the structure shown below is $5\ cm^2$, calculate the vertical displacement of joint C. The values of $a = 1\ m$ and $E = 2\times10^6\ kg/cm^2$ are also assumed.



$\Delta_{yc} = 0.892\ mm$

38- Assuming the values $E = 2\times10^5\ MN/m^2$ and $\sigma_w = 160\ MN/m^2$ for the bars, determine the maximum allowable load based on the permissible stress.

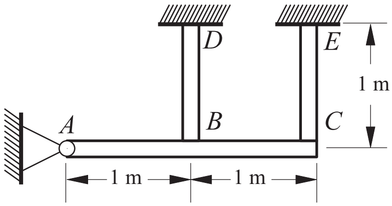


$P_{all} = 64\ KN$

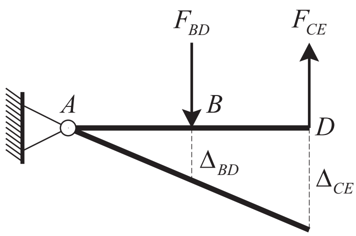
Chapter 2: Axial Load Effects in Indeterminate Structures

Section 2: Temperature Change Effects

1- The bar *AC* shown in the figure is completely rigid, with a hinge at point *A* and suspended from bars *DB* and *EC* at points *B* and *C*. The weight of bar *AC* is $50\ KN$, while the weights of the other two bars can be neglected. If the temperature of bars *DB* and *EC* increases by $35^\circ\ C$, determine the stresses developed in both bars. Bar *DB* is made of copper with a modulus of elasticity $E = 90000\ MN/m^2$, coefficient of thermal expansion $\alpha = 18\times10^{-6}\ 1/C^\circ$ and cross-sectional area $1000\ mm^2$, while bar *EC* is made of steel with $E = 200000\ MN/m^2$, $\alpha = 12\times10^{-6}\ 1/C^\circ$ and cross-sectional area $500\ mm^2$. Lateral buckling of the bars is neglected.



Since $\alpha_{DB} > \alpha_{EC}$, due to the temperature increase, bar *DB* is subjected to compression, while bar *EC* is subjected to tension.



$$\sum M_A = 0$$

$$\rightarrow\ (F_{CE})(2) - (F_{BD})(1) = (0.05)(1) \quad \text{(I)}$$

From the similarity of the triangles, the following is obtained:

$$\rightarrow\ \Delta_{CE} = 2\Delta_{BD}$$

$$\Delta_{CE} = \delta_T + \delta_P = \alpha l \Delta T + \frac{Pl}{EA} = (12\times10^{-6})(1)(35) + \frac{(F_{CE})(1)}{(200000)(500\times10^{-6})}$$

$$\Delta_{BD} = \delta_T - \delta_P = \alpha l \Delta T - \frac{Pl}{EA} = (18\times10^{-6})(1)(35) - \frac{(F_{BD})(1)}{(90000)(1000\times10^{-6})}$$

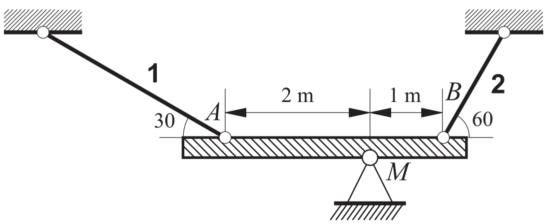
$$\Delta_{CE} = 2\Delta_{BD}\ \rightarrow\ 420\times10^{-6} + \frac{F_{CE}}{100} = 1260\times10^{-6} - \frac{2F_{BD}}{90} \quad \text{(II)}$$

From the solution of equations (I) and (II), the following is obtained:

Tension $F_{CE} = 0.0358\ MN\ \rightarrow\ \sigma_{CE} = \dfrac{F_{CE}}{A_{CE}} = \dfrac{0.0358}{500\times10^{-6}} = 71.7\ MPa$

Compression $F_{BD} = 0.0217\ MN \quad \rightarrow \sigma_{BD} = \dfrac{F_{BD}}{A_{BD}} = \dfrac{0.0217}{1000 \times 10^{-6}} = 21.7\ MPa$

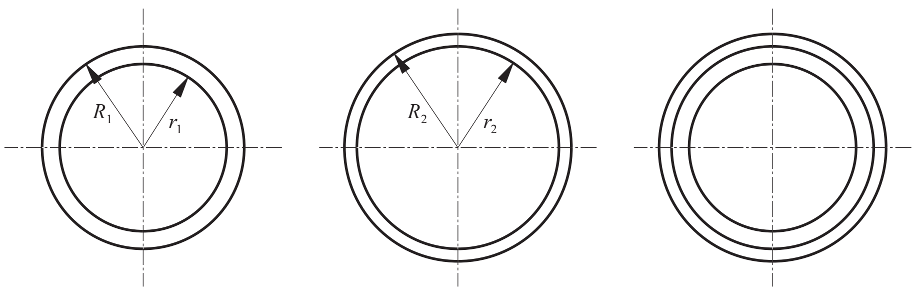
2- As shown in the figure below, a rigid bar is supported by a hinged support and suspended by two bars with identical cross-sectional areas $40\ cm^2$. After installation, the temperature increases by 20°. Determine the thermal stresses developed in the bars. The bars are made of steel with a modulus of elasticity $E = 2 \times 10^6\ kg/cm^2$ and a coefficient of thermal expansion $\alpha = 12 \times 10^{-6}\ 1/C^\circ$.



$\sigma_1 = 452.2\ kg/cm^2$

$\sigma_2 = 522.2\ kg/cm^2$

3- A cylindrical steel tube 2, with inner radius $r_2 = 41.96\ mm$ and outer radius $R_2 = 43\ mm$, as shown in the figure, is placed inside another cylindrical steel tube with inner radius $r_1 = 40\ mm$ and outer radius $R_1 = 42\ mm$. Determine the stresses induced in the walls of tubes 1 and 2 when the temperature of the outer tube is lowered to the temperature of the inner tube. The modulus of elasticity of steel is $E = 2 \times 10^6\ kg/cm^2$.



$\sigma_1 = 651\ kg/cm^2$

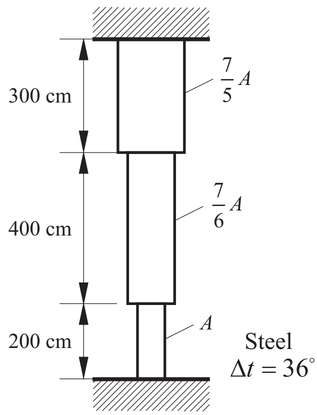
$\sigma_2 = 1250\ kg/cm^2$

4- Determine the necessary gap between railway rails so that they do not press against each other in summer. The rails are installed at a temperature of $10^\circ\ C$, and the maximum summer temperature is $60^\circ\ C$. Also, the length of each rail is 8 *m*, their modulus of elasticity is $E = 2\times10^6\ kg/cm^2$, and their coefficient of thermal expansion is $\alpha = 12.5\times10^{-6}\ 1^\circ C$. If no gap is provided, calculate the induced stresses.

$\Delta = 5\ mm$

$\sigma = 1250\ kg/cm^2$

5- If the modulus of elasticity and the coefficient of thermal expansion of steel are equal to $E = 2\times10^6\ kg/cm^2$ and $\alpha = 125\times10^{-7}\ 1/C^\circ$, respectively, determine the stresses induced in each bar due to the temperature change.

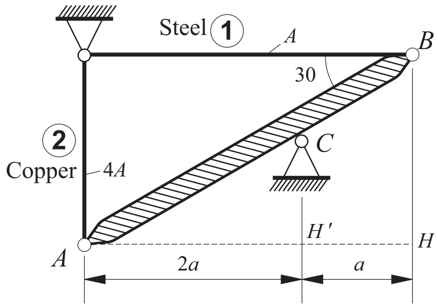


$\sigma_1 = 764\ kg/cm^2$

$\sigma_2 = 917\ kg/cm^2$

$\sigma_3 = 1069.8\ kg/cm^2$

6- If the modulus of elasticity and the coefficient of thermal expansion of steel are $E = 2\times10^6\ kg/cm^2$ and $\alpha = 125\times10^{-7}\ 1/C^\circ$ respectively, and for copper are $E = 10^6\ kg/cm^2$ and $\alpha = 165\times10^{-7}\ 1/C^\circ$, respectively, determine the stresses induced in each bar due to the temperature change.

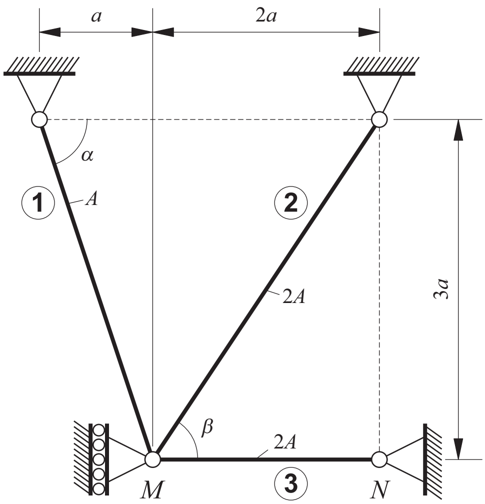


$\Delta t = 20^{\circ}$

$\sigma_1 = 596.8\ kg/cm^2$

$\sigma_2 = 43.1\ kg/cm^2$

7- If the modulus of elasticity and the coefficient of thermal expansion of steel are $E = 2\times10^6\ kg/cm^2$ and $\alpha = 125\times10^{-7}\ 1/C^{\circ}$, respectively, determine the stresses induced in each mbar due to the temperature change.

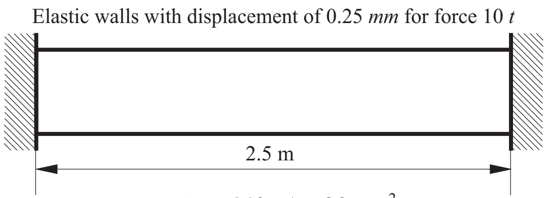


Steel $\Delta t = 40^{\circ}$

$\sigma_1 = -172.3\ kg/cm^2$

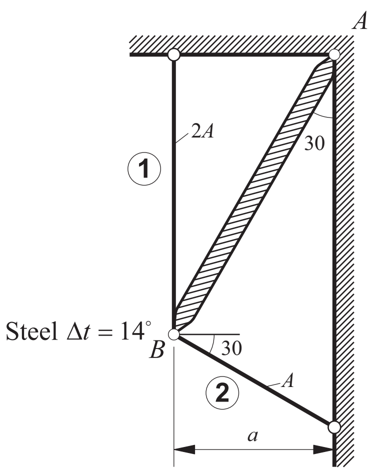
$\sigma_2 = 98.2\ kg/cm^2$

8- If the modulus of elasticity and the coefficient of thermal expansion of steel are $E = 2\times10^5\ MN/m^2$ and $\alpha = 125\times10^{-7}\ 1/C^\circ$ respectively, determine the stresses induced in bar due to the temperature change.



$\sigma = 105.4\ MPa$

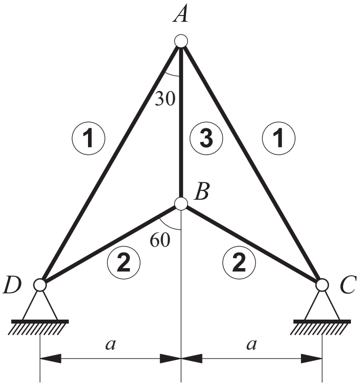
9- If the modulus of elasticity and the coefficient of thermal expansion of steel are $E = 2\times10^6\ kg/cm^2$ and $\alpha = 125\times10^{-7}\ 1/C^\circ$ respectively, determine the stresses induced in each bar due to the temperature change.



$\sigma_1 = 35\ MN/m^2$

$\sigma_2 = 35\ MN/m^2$

10- If the modulus of elasticity and the coefficient of thermal expansion of steel are $E = 2\times10^6\ kg/cm^2$ and $\alpha = 125\times10^{-7}\ 1/C^\circ$ respectively, determine the stresses induced in each bar due to the temperature change.



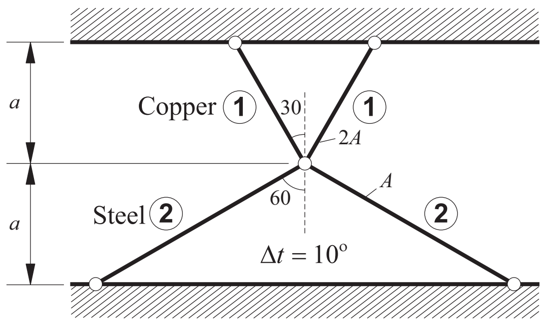
Steel $\Delta t_3 = 17.3^\circ$

$\sigma_1 = 6\ MN\ /m^2$

$\sigma_2 = 10.4\ MN\ /m^2$

$\sigma_3 = 10.4\ MN\ /m^2$

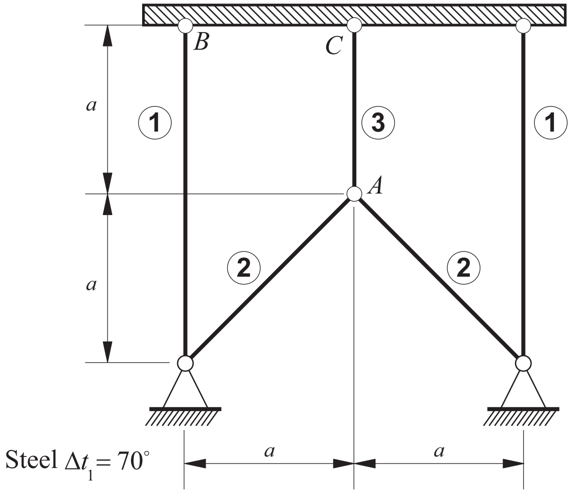
11- If the modulus of elasticity and the coefficient of thermal expansion of steel are $E = 2\times10^6\ kg/cm^2$ and $\alpha = 125\times10^{-7}\ 1/C^\circ$, respectively, and for copper are $E = 10^6\ kg/cm^2$ and $\alpha = 165\times10^{-7}\ 1/C^\circ$, respectively, determine the stresses induced in each bar due to the temperature change.



$\sigma_1 = 87.1\ kg\ /cm^2$

$\sigma_2 = 302\ kg\ /cm^2$

12- If the modulus of elasticity and the coefficient of thermal expansion of steel are $E = 2\times10^6\ kg/cm^2$ and $\alpha = 125\times10^{-7}\ 1/C^\circ$, respectively, determine the stresses induced in each bar due to the temperature change.

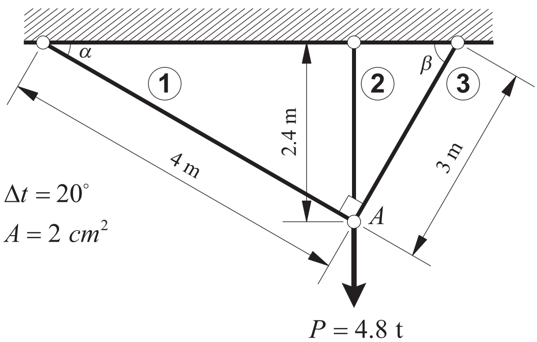


$\sigma_1 = 512.6\ kg/cm^2$

$\sigma_2 = 724.9\ kg/cm^2$

$\sigma_3 = 1025.2\ kg/cm^2$

13- If the modulus of elasticity and the coefficient of thermal expansion of steel are $E = 2\times10^6\ kg/cm^2$ and $\alpha = 12\times10^{-6}\ 1/C^\circ$ respectively, determine the stresses induced in each bar due to the applied force and temperature change.

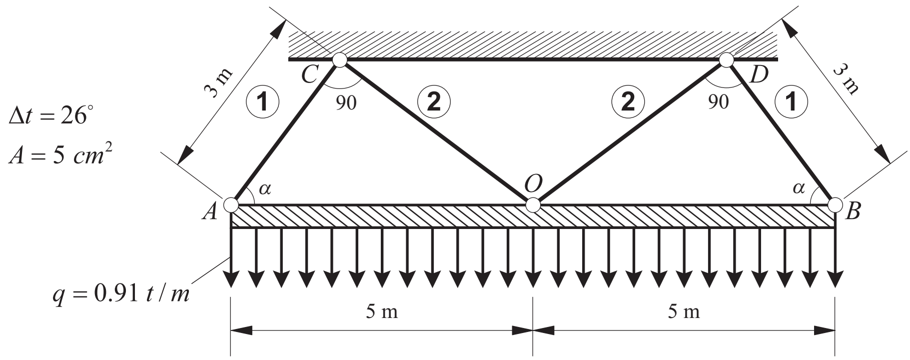


$\sigma_1 = 120\ kg/cm^2$

$\sigma_2 - 200\ kg/cm^2$

$\sigma_3 = 160\ kg/cm^2$

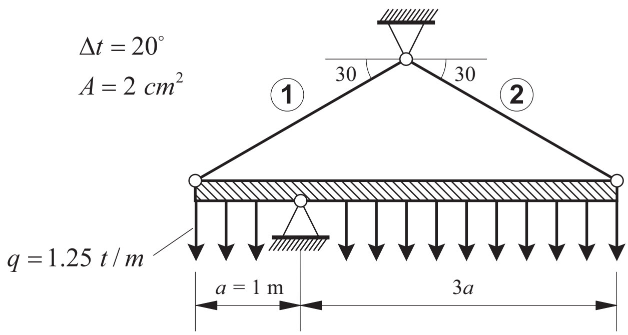
14- If the modulus of elasticity and the coefficient of thermal expansion of bars are $E = 2\times10^6\ kg/cm^2$ and $\alpha = 12\times10^{-6}\ 1/C^\circ$ respectively, determine the stresses induced in each bar due to the applied forces and temperature change.



$\sigma_1 = 144.6\ kg/cm^2$

$\sigma_2 = 192.9\ kg/cm^2$

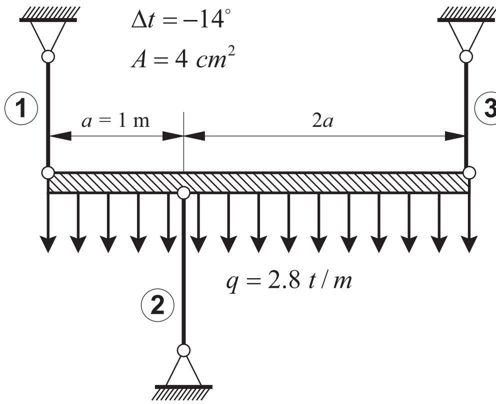
15- If the modulus of elasticity and the coefficient of thermal expansion of bars are $E = 2\times10^6\ kg/cm^2$ and $\alpha = 12\times10^{-6}\ 1/C^\circ$ respectively, determine the stresses induced in each bar due to the applied forces and temperature change.



$\sigma_1 = 576\ kg/cm^2$

$\sigma_2 = 192\ kg/cm^2$

16- If the modulus of elasticity and the coefficient of thermal expansion of bars are $E = 2\times10^6\ kg/cm^2$ and $\alpha = 12\times10^{-6}\ 1/C^\circ$ respectively, determine the stresses induced in each bar due to the applied forces and temperature change.

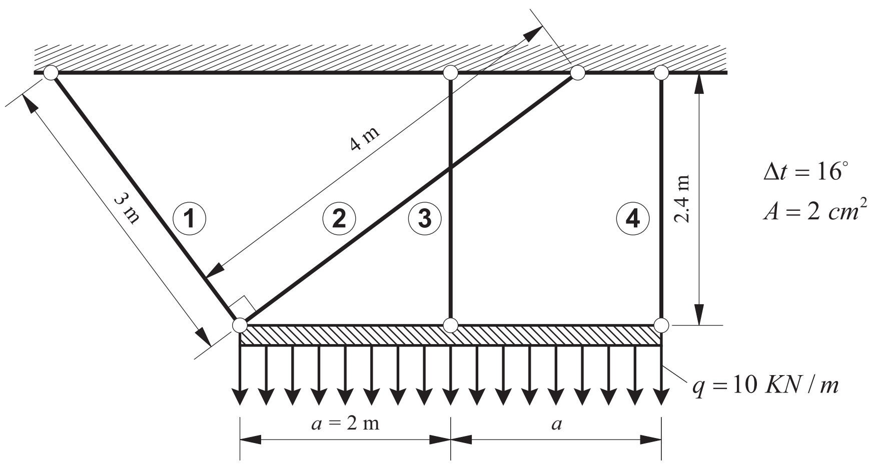


$\sigma_1 = 288\ kg/cm^2$

$\sigma_2 = 432\ kg/cm^2$

$\sigma_3 = 144\ kg/cm^2$

17- If the modulus of elasticity and the coefficient of thermal expansion of bars are $E = 2\times10^5\ MN/m^2$ and $\alpha = 12\times10^{-6}\ 1/C^\circ$ respectively, determine the stresses induced in each bar due to the applied forces and temperature change.



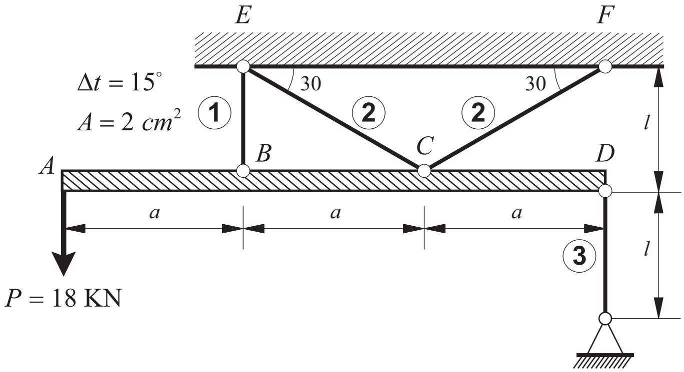
$\sigma_1 = 4.8\ MN/m^2$

$\sigma_2 = 3.6\ MN/m^2$

$\sigma_3 = 12\ MN/m^2$

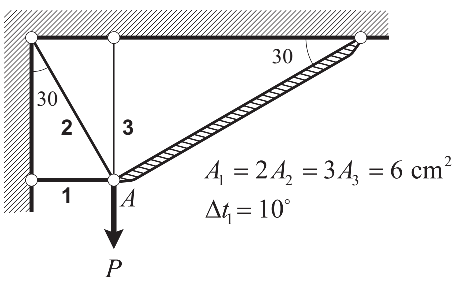
$\sigma_4 = 6\ MN/m^2$

18- If the modulus of elasticity and the coefficient of thermal expansion of bars are $E = 2\times10^5\ MN/m^2$ and $\alpha = 12\times10^{-6}\ 1/C^\circ$ respectively, determine the stresses induced in each bar due to the applied forces and temperature change.



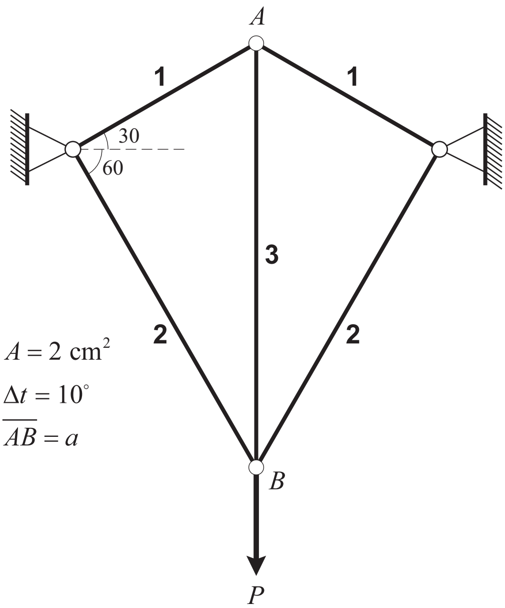
$\sigma_1 = 16\ MPa$
$\sigma_2 = 32\ MPa$
$\sigma_3 = 16\ MPa$

19- Assuming the values $E = 2\times10^6\ kg/cm^2$, $\sigma_W = 1600\ kg/cm^2$ and $\alpha = 12\times10^{-6}\ 1/C^\circ$ for the bars, determine the maximum allowable load based on the permissible stress. Also, calculate the thermal stresses induced by the temperature change.



$P_{all} = 14.26\ t$
$\sigma_1 = 146.8\ kg/cm^2$
$\sigma_2 = 93.2\ kg/cm^2$
$\sigma_3 = 92.9\ kg/cm^2$

20- Assuming the values $E = 2\times10^6\ kg/cm^2$, $\sigma_W = 1600\ kg/cm^2$ and $\alpha = 12\times10^{-6}\ 1/C^\circ$ for the bars, determine the maximum allowable load based on the permissible stress. Also, calculate the thermal stresses induced by the temperature change.



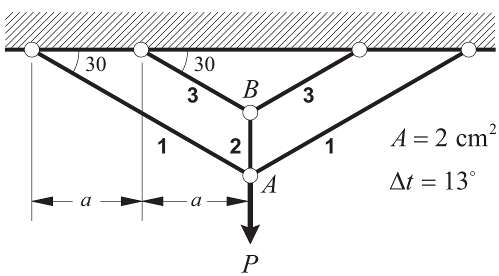
$P_{all} = 7143\ kg = 7.14\ t$

$\sigma_1 = 93.1\ kg\ /\ cm^2$

$\sigma_2 = 53.8\ kg\ /\ cm^2$

$\sigma_3 = 93.1\ kg\ /\ cm^2$

21- Assuming the values $E = 2\times10^6\ kg\ /\ cm^2$, $\sigma_W = 1600\ kg\ /\ cm^2$ and $\alpha = 12\times10^{-6}\ 1/C^\circ$ for the bars, determine the maximum allowable load based on the permissible stress. Also, calculate the thermal stresses induced by the temperature change.



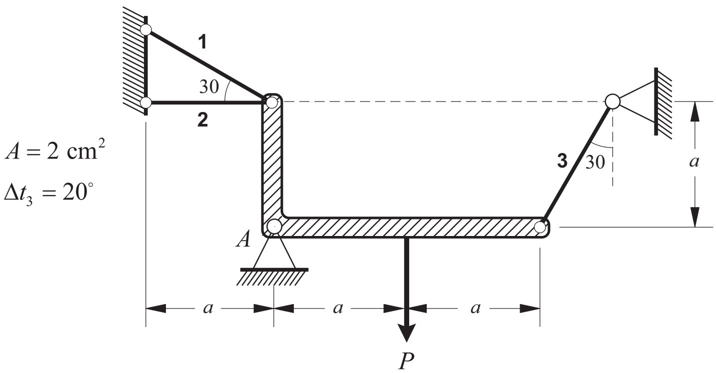
$P_{all} = 5.2\ t$

$\sigma_1 = 72\ kg/cm^2$

$\sigma_2 = -72\ kg/cm^2$

$\sigma_3 = -72\ kg/cm^2$

22- Assuming the values $E = 2\times10^6\ kg/cm^2$, $\sigma_W = 1600\ kg/cm^2$ and $\alpha = 12\times10^{-6}\ 1/C^\circ$ for the bars, determine the maximum allowable load based on the permissible stress. Also, calculate the thermal stresses induced by the temperature change.



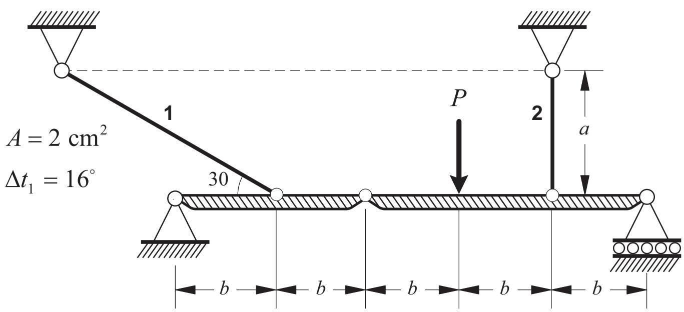
$P_{all} = 9.06\ t$

$\sigma_1 = 175\ kg/cm^2$

$\sigma_2 = 73.4\ kg/cm^2$

$\sigma_3 = -129.9\ kg/cm^2$

23- Assuming the values $E = 2\times10^6\ kg/cm^2$, $\sigma_W = 1600\ kg/cm^2$ and $\alpha = 12\times10^{-6}\ 1/C^\circ$ for the bars, determine the maximum allowable load based on the permissible stress. Also, calculate the thermal stresses induced by the temperature change.

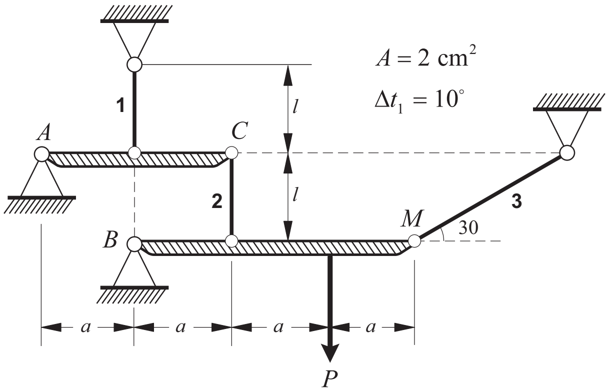


$P_{all} = 2.05\ t$

$\sigma_1 = 300\ kg\ /cm^2$

$\sigma_2 = 225\ kg\ /cm^2$

24- Assuming the values $E = 2\times10^5\ MN\ /\ m^2$, σ_W =160 $MN\ /\ m^2$ and $\alpha = 12\times10^{-6}\ 1/C^{\circ}$ for the bars, determine the maximum allowable load based on the permissible stress. Also, calculate the thermal stresses induced by the temperature change.



$P_{all} = 28.3\ KN$

$\sigma_1 = 16.3\ MPa$

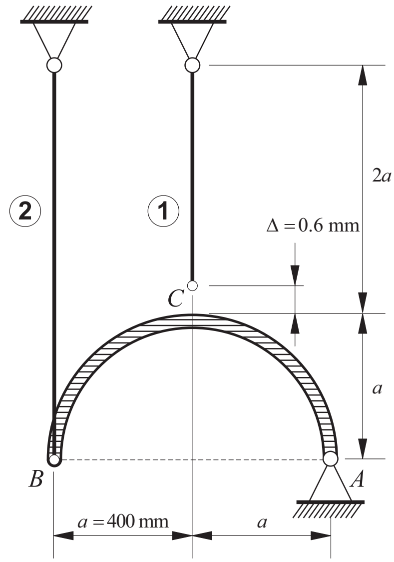
$\sigma_2 = 8.15\ MPa$

$\sigma_3 = 5.4\ MPa$

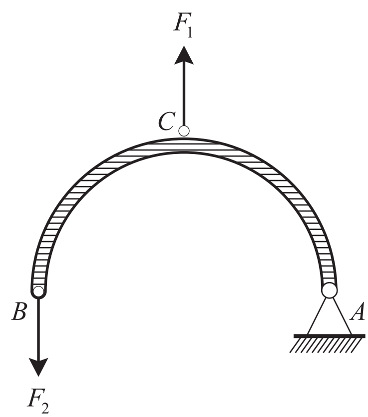
Chapter 2: Axial Load Effects in Indeterminate Structures

Section 3: Fabrication Error Effects

1- If Δ represents the fabrication error in the length of bar 1 as shown, determine the stresses induced in the structure below. The bars are assumed to have an elastic modulus of $E = 2\times10^6\ kg/cm^2$.



When bar 1 is connected to the rigid segment, due to the fabrication error ($\Delta = 0.6\ mm$), the structure rotates about point A. To compensate for the gap Δ, bar 1 must be stretched, resulting in bar 1 being in tension and bar 2 being in compression.

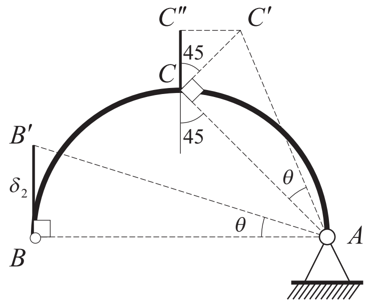


$\sum M_A = 0$

$\rightarrow F_1 a - F_2(2a) = 0$

$\rightarrow F_1 = 2F_2 \quad \text{(I)}$

CC'' is the distance by which point C rises due to the shortening of bar 2. To compensate for the gap Δ, the additional elongation of bar 1 (δ_1) must also be added. Due to the rigidity of the segment *ACB*, the rotation angles of radius *AB* and *AC* will be equal.



$$\Delta ABB' \sim \Delta ACC'$$

$$\to \frac{AB}{BB'} = \frac{AC}{CC'} \quad \left\{ AB = 2a \quad , \quad AC = a\sqrt{2} \quad , \quad BB' = \delta_2 \right\}$$

$$\to \frac{2a}{\delta_2} = \frac{a\sqrt{2}}{CC'} \quad \to CC' = \frac{\sqrt{2}}{2}\delta_2$$

$$CC' = \frac{CC''}{\cos 45} = \sqrt{2}CC''$$

$$\to \sqrt{2}CC'' = \frac{\sqrt{2}}{2}\delta_2 \quad \to CC'' = \frac{1}{2}\delta_2$$

$$CC'' + \delta_1 = \Delta$$

$$\to \frac{1}{2}\delta_2 + \delta_1 = \Delta$$

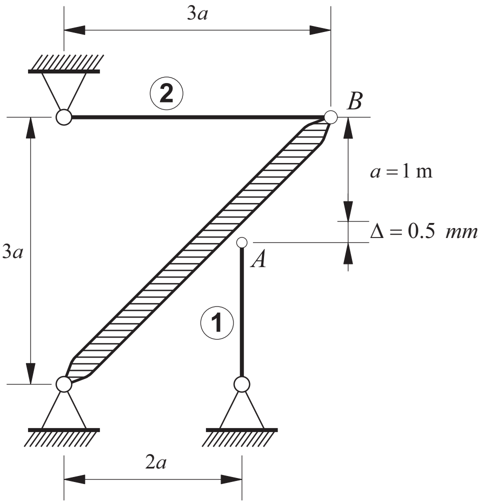
$$\to \frac{1}{2}\frac{(F_2)(3a)}{EA} + \frac{(F_1)(2a)}{EA} = \Delta \quad \text{(II)}$$

By solving equations (I) and (II), it follows that:

$$\begin{cases} F_1 = \dfrac{4EA\Delta}{11a} \\ F_2 = \dfrac{2EA\Delta}{11a} \end{cases}$$

$$\to \begin{cases} \sigma_1 = \dfrac{F_1}{A} = \dfrac{4E\Delta}{11a} = \dfrac{(4)(2\times10^6)(0.6\times10^{-1})}{(11)(400\times10^{-1})} = 1091\ kg/cm^2 \\ \sigma_2 = \dfrac{F_2}{A} = \dfrac{2E\Delta}{11a} = \dfrac{(2)(2\times10^6)(0.6\times10^{-1})}{11(400\times10^{-1})} = 545\ kg/cm^2 \end{cases}$$

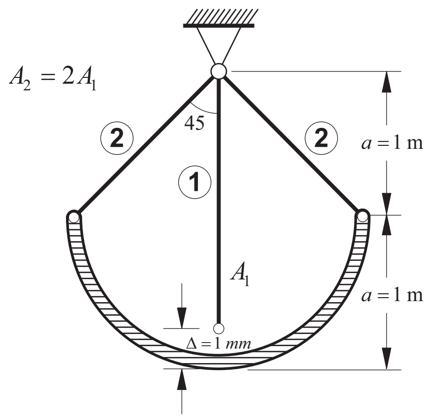
2- If Δ represents the fabrication error in the length of member 1 as shown, determine the stresses induced in the structure below. The bars are assumed to have an elastic modulus of $E = 2\times10^6\ kg/cm^2$.



$\sigma_1 = 300\ kg/cm^2$

$\sigma_2 = 200\ kg/cm^2$

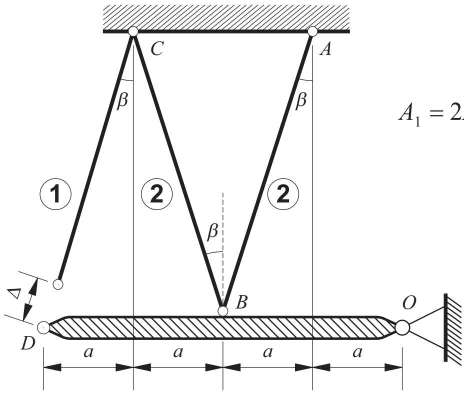
3- If Δ represents the fabrication error in the length of member 1 as shown, determine the stresses induced in the structure below. The bars are assumed to have a modulus of elasticity of $E = 2 \times 10^6\ kg/cm^2$.



$\sigma_1 \simeq 739\ kg/cm^2$

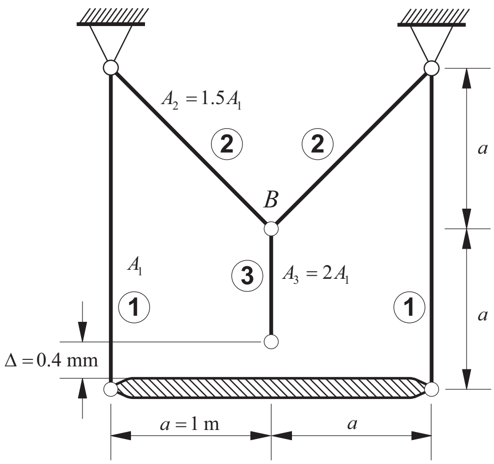
$\sigma_2 = 261.2\ kg/cm^2$

4- If Δ represents the fabrication error in the length of bar 1 as shown, determine the stresses induced in the structure below. The bars are assumed to have an elastic modulus of $E = 2\times10^6\ kg/cm^2$.



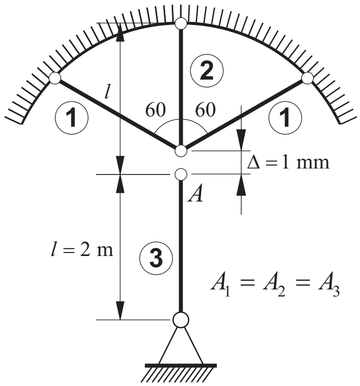
$$\sigma_1 = \frac{E\Delta\sin\beta}{5a}$$
$$\sigma_2 = \frac{2E\Delta\sin\beta}{5a}$$

5- If Δ represents the fabrication error in the length of bar 3 as shown, determine the stresses induced in the structure below. The bars are assumed to have an elastic modulus of $E = 2\times10^5\ MN/m^2$.



$$\sigma_1 = 16.4\ MN/m^2$$
$$\sigma_2 = 15.4\ MN/m^2$$
$$\sigma_3 = 16.4\ MN/m^2$$

6- If Δ represents the fabrication error in the length of bar 3 as shown, determine the stresses induced in the structure below. The bars are assumed to have an elastic modulus of $E = 2\times10^5\ MN/m^2$.

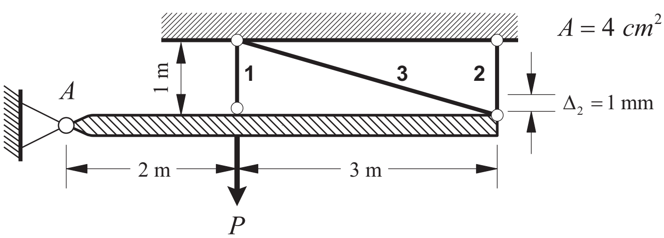


$\sigma_1 = 20\ MPa$

$\sigma_2 = 40\ MPa$

$\sigma_3 = 60\ MPa$

7- Assuming the values $E = 2\times10^6\ kg/cm^2$ and $\sigma_W = 1600\ kg/cm^2$ for the bars, determine the maximum allowable load based on the permissible stress. Also, calculate the stresses resulting from the fabrication error in member 2.



$P_{all} = 19.1\ t$

$\sigma_1 = 671\ kg/cm^2$

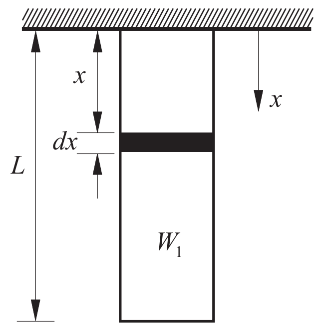
$\sigma_2 = 322\ kg/cm^2$

$\sigma_3 = 168\ kg/cm^2$

Chapter 2: Axial Load Effects in Indeterminate Structures

Section 4: Ultimate Load Determination

1- A vertical bar is suspended from its upper end. Determine its elongation due to its self-weight. Assuming that B and n are constant, L is the length of the bar, and γ is the unit weight of the bar material. The relation between stress and strain, expressed as $\sigma^n = B\varepsilon$.



In the considered element, the weight of the lower portion of the bar acts as a tensile force.

$$W_1 = \gamma A(L-x)$$

$$\sigma = \frac{W_1}{A} = \gamma(L-x)$$

$$\varepsilon_x = \frac{[\gamma(L-x)]^n}{B} = \frac{\gamma^n}{B}(L-x)^n$$

$$\varepsilon_x = \frac{d\delta}{dx} \rightarrow d\delta = \varepsilon_x dx$$

$$\rightarrow d\delta = \frac{\gamma^n}{B}(L-x)^n\, dx$$

$$\delta = \int_0^L d\delta$$

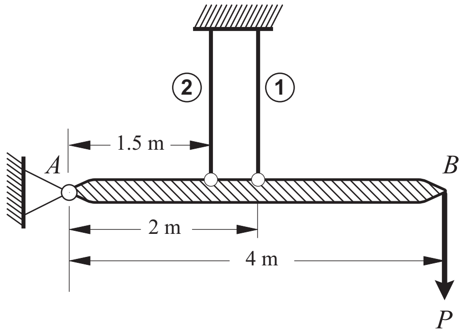
$$\rightarrow \delta = \int_0^L \frac{\gamma^n}{B}(L-x)^n dx = \frac{\gamma^n}{B}\int_0^L (L-x)^n dx$$

$$\rightarrow \delta = \frac{\gamma^n}{B}\left[-\frac{(L-x)^{n+1}}{n+1}\right]_0^L$$

$$\rightarrow \delta = \frac{\gamma^n}{B}\left[0-(-\frac{L^{n+1}}{n+1})\right] = (\frac{\gamma^n}{B})(\frac{L^{n+1}}{n+1})$$

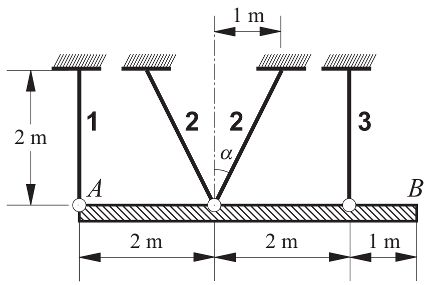
$$\rightarrow \delta = \frac{\gamma^n L^{n+1}}{B(n+1)}$$

2– The rigid bar shown is suspended from two vertical steel bars and is pin-supported at support A. If the yield stress of the steel is $400\ MPa$ and the cross-sectional area of each bar is $100\ mm^2$, determine the ultimate load P_u .



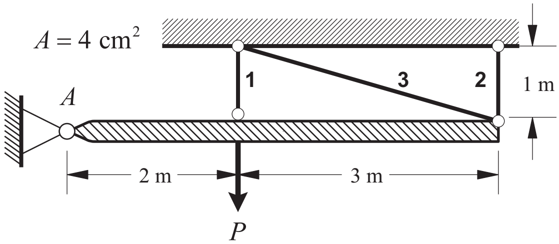
$P_u = 35\ KN$

3- As shown in the figure, the rigid bar *AB* is suspended from four circular bars, each with diameter 50 *mm*. The yield stress of the bars is 300 *MPa*. Assuming the weight of the rigid bar is uniformly distributed along its length, determine the maximum weight of the bar *AB* based on plastic analysis.



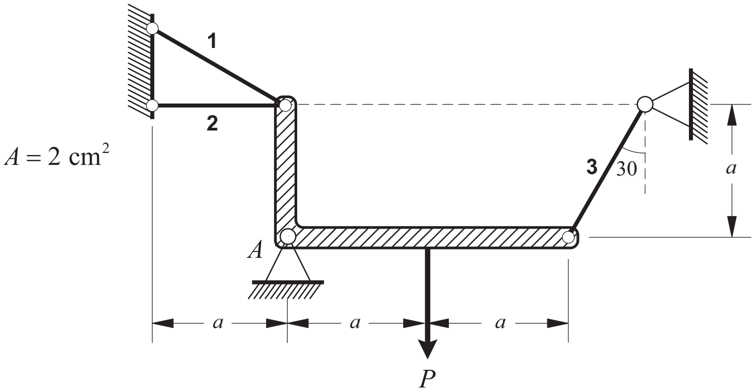
$W = 1.79\ MN$

4- As shown in the figure, if the yield stress of the bar material is $2400\ kg/cm^2$ and the safety factor is 1.5, determine the allowable load *P* based on plastic analysis.



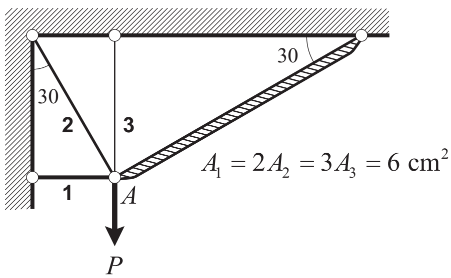
$P_{all} = 27.5\ t$

5– As shown in the figure, if the yield stress of the bar material is $2400\ kg/cm^2$ and the safety factor is 1.5, determine the allowable load P based on plastic analysis.



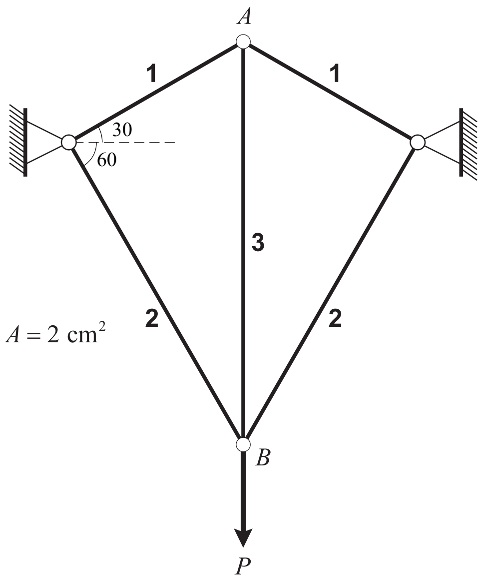
$P_{all} = 11.51\ t$

6– As shown in the figure, if the yield stress of the bar material is $2400\ kg/cm^2$ and the safety factor is 1.5, determine the allowable load P based on plastic analysis.



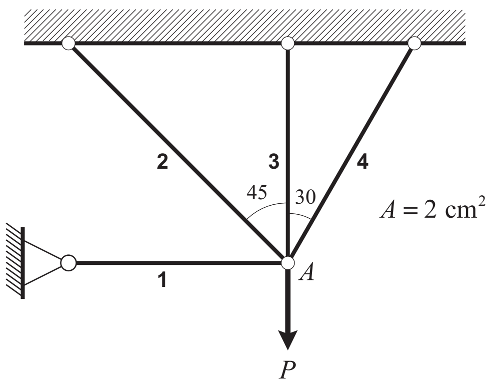
$P_{all} = 14.3\ t$

7– As shown in the figure, if the yield stress of the bar material is $2400\ kg/cm^2$ and the safety factor is 1.5, determine the allowable load P based on plastic analysis.



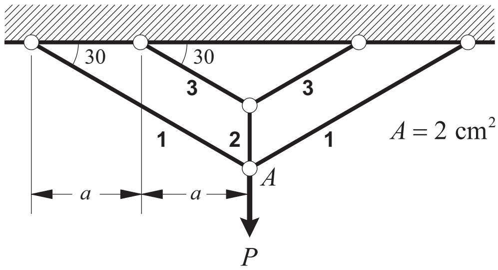
$P_{all} = 8.74\ t$

8– As shown in the figure, if the yield stress of the bar material is $2400\ kg/cm^2$ and the safety factor is 1.5, determine the allowable load P based on plastic analysis.



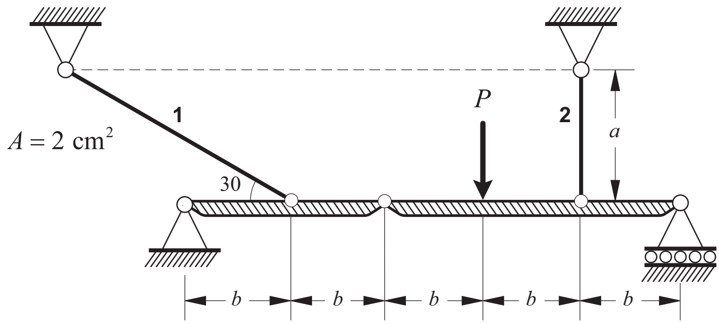
$P_{all} = 82\ KN$

9– As shown in the figure, if the yield stress of the bar material is $2400\ kg/cm^2$ and the safety factor is 1.5, determine the allowable load P based on plastic analysis.



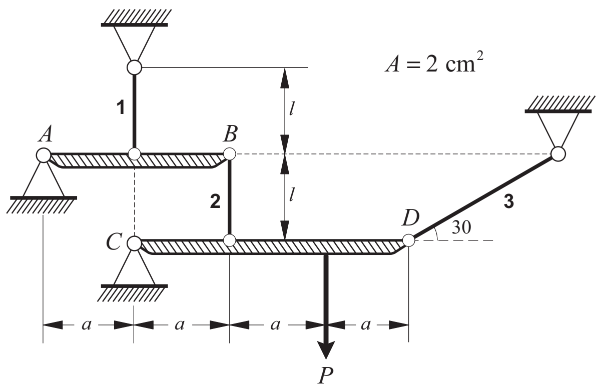
$P_{all} = 6.4\ t$

10– As shown in the figure, if the yield stress of the bar material is $2400\ kg/cm^2$ and the safety factor is 1.5, determine the allowable load P based on plastic analysis.



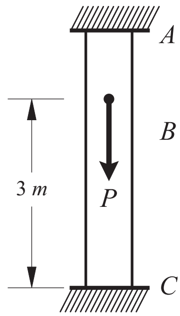
$P_{all} = 2.8\ t$

11– As shown in the figure, if the yield stress of the bar material is $2400\ kg/cm^2$ and the safety factor is 1.5, determine the allowable load P based on plastic analysis.



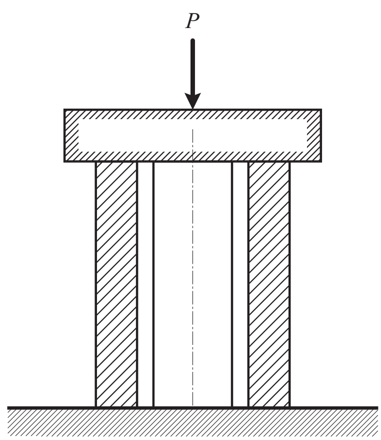
$P_{all} = 32\ KN$

12– As shown in the figure, a vertical steel bar is subjected to a downward concentrated axial load P. The cross-sectional area of the bar is $A = 200\ cm^2$, and its length is $L = 5\ m$. Assuming the yield stress of the material is $2400\ kg/cm^2$ and using a safety factor of 2, determine the allowable load P based on plastic analysis.



$P_{all} = 480\ t$

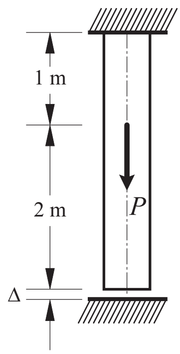
13- The load P is transferred through the rigid slab to a solid steel bar and a hollow copper cylinder. The cross-sectional area of the steel member is $A_{st} = 15\ cm^2$, and that of the hollow copper cylinder is $A_{cu} = 20\ cm^2$. If the yield stress of the steel is $\sigma_{y,st} = 2400\ kg/cm^2$ and the yield stress of the copper is $\sigma_{y,cu} = 1800\ kg/cm^2$, calculate the internal forces carried by the steel and copper members at the instant when both materials reach their respective yield stresses.



$P_{st} = 36\ t$

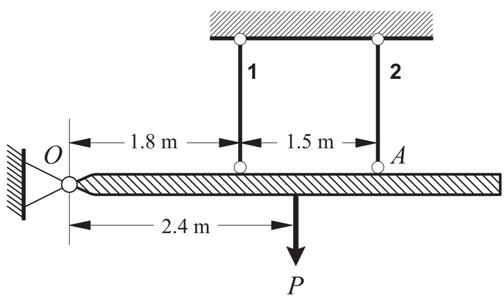
$P_{cu} = 36\ t$

14– As shown in the figure, a bar with cross-sectional area $A = 100\ cm^2$, fixed at its upper end, is subjected to loading. Before loading, a gap of size $\Delta = 0.02\ mm$ exists between the lower end of the bar and the rigid plate beneath it. If the yield stress of the bar material is $1500\ kg/cm^2$ and safety factor of 1.5 is specified, determine the allowable load P using plastic analysis.



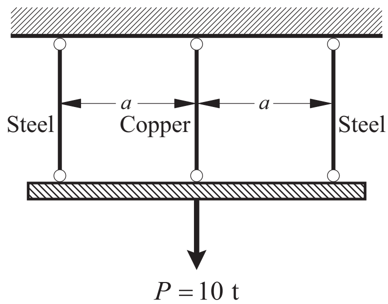
$P_{all} = 200\ t$

15- As shown in the figure, a rigid bar is suspended from two vertical bars. The cross-sectional area of bar 1 is $10\ cm^2$ with yield stress $2600\ kg/cm^2$, and the cross-sectional area of bar 2 is $15\ cm^2$ with yield stress $1500\ kg/cm^2$. Using a safety factor of 2, determine the allowable load P based on plastic analysis.



$P_{all} = 25.21\ t$

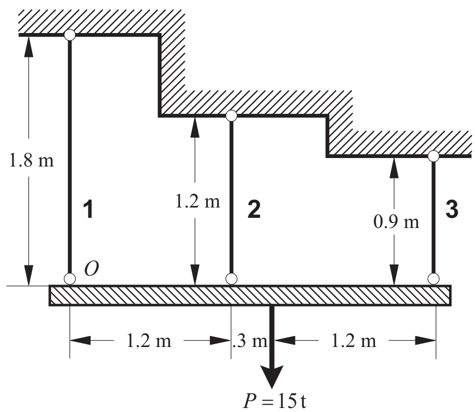
16- As shown in the figure, a rigid bar is suspended from three vertical bars. The two outer bars are made of steel with the same cross-sectional area, while the middle bar is made of copper with a cross-sectional area 1.5 times that of the steel bars. If the yield stresses of steel and copper are $2400\ kg/cm^2$ and $1800\ kg/cm^2$, respectively, and a safety factor of 1.5 is applied, determine the required cross-sectional area of each bar based on the ultimate load method.



$A_{st} = 2\ cm^2$

$A_{cu} = 3\ cm^2$

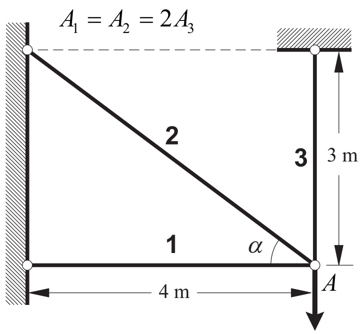
17- As shown in the figure, a rigid bar is suspended from three vertical steel bars. The yield stress of the bars is $2400\ kg/cm^2$, and their cross-sectional areas are identical. Using a safety factor of 1.5, determine the required cross-sectional area of the bars based on the ultimate load method.



$A = 3.61\ cm^2$

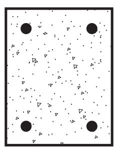
18– As shown in the figure, bar 1 is made of aluminum with yield stress $1600\ kg/cm^2$, bar 2 is made of copper with yield stress $1200\ kg/cm^2$, and bar 3 is made of steel with yield stress $2400\ kg/cm^2$. The cross-sectional areas of bars 1 and

2 are equal, while the cross-sectional area of bar 3 is half that of bar 1. Using a safety factor of 2, determine the required cross-sectional area of each bar based on the ultimate load method.



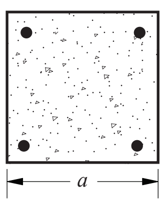
$A_3 = 8.33 \qquad A_1 = A_2 = 16.66\ cm^2$

19– The cross-sectional area of concrete in a short reinforced concrete column is $645\ cm^2$. The column contains four longitudinal reinforcing bars symmetrically arranged, each with a cross-sectional area $10\ cm^2$. The yield stresses of the concrete and steel reinforcement are $120\ kg/cm^2$ and $2100\ kg/cm^2$, respectively. Using a safety factor of 1.5, determine the allowable load of the column based on the ultimate load method.



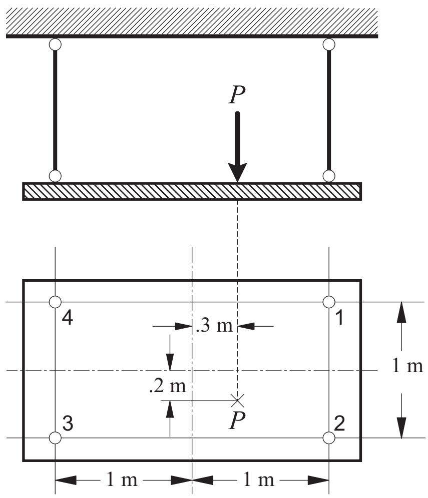
$P_{all} = 107.6\ t$

20– A square concrete column is to be designed for an axial load 100 t. The column contains four steel bars, whose total cross-sectional area is 1% of the column's cross-sectional area. The yield stresses of the concrete and steel are $120\ kg/cm^2$ and $2400\ kg/cm^2$, respectively. Using a safety factor of 2, determine the dimensions of the column's cross-section and the diameter of the reinforcing bars.



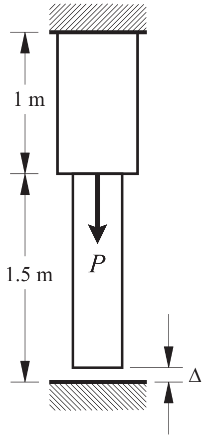
$a = 37.4\ cm$
$D = 21\ mm$

21- A rectangular rigid slab is suspended at its four corners by four bars, each with identical cross-sectional area, length, and material. The cross-sectional area of each bar is $4\ cm^2$, and the yield stress of the material is $2400\ kg/cm^2$. Using a safety factor of 2, determine the allowable load P based on the ultimate load method.



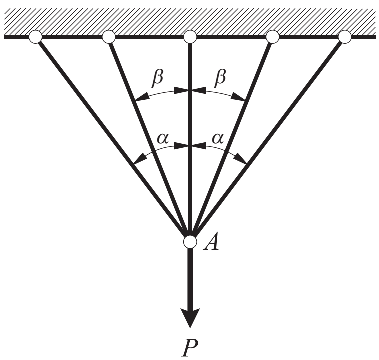
$F_1 = 0.225P$
$F_2 = 0.425P$
$F_3 = 0.275P$
$F_4 = 0.075P$

22- As shown in the figure, a bar with its upper end fixed is subjected to a load P. A gap of size $\Delta = 0.05\ mm$ exists between the lower end of the bar and the rigid support. The upper portion of the bar, made of copper, has a cross-sectional area of $150\ cm^2$, while the lower portion, made of steel, has a cross-sectional area of $50\ cm^2$. The yield stresses of copper and steel are $1500\ kg/cm^2$ and $2100\ kg/cm^2$, respectively. Using a safety factor of 1.5, determine the allowable load based on the ultimate load method.



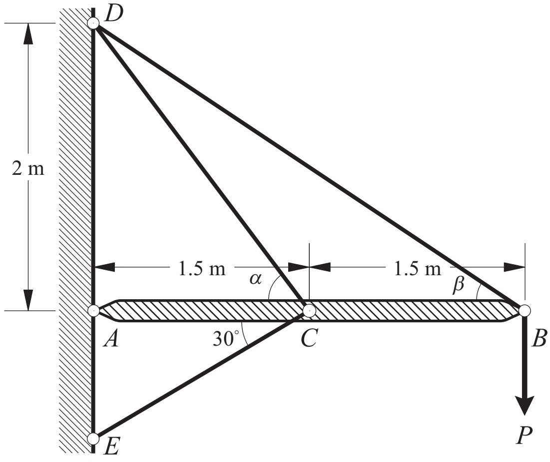
$P_{all} = 220\ t$

23- As shown in the figure below, a vertical load P is carried by five steel wires that are symmetrically connected at point A. The cross-sectional area of each wire is $A = 0.65\ cm^2$. If the yield stress of the wires is $3000\ kg / cm^2$, and the angles α and β are 45° and 30° respectively, determine the ultimate load P_u.



$P_u = 8085\ kg$

24- A rigid horizontal bar AB is hinged to a vertical wall at point A and is suspended by two steel cables BD and CD, while supported by column EC. The cross-sectional area of each cable is $A = 0.65\ cm^2$, and their yield stress is $3000\ kg / cm^2$. The compressive buckling resistance of the column EC is $Q = 1500\ kg$. Determine the ultimate load P_u.

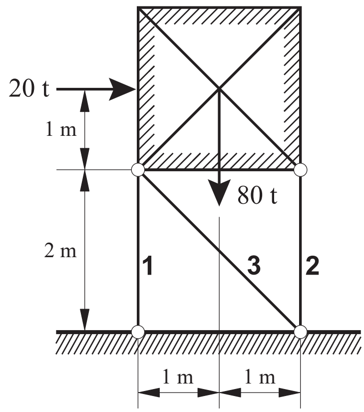


$P_u = 2237.25\ kg$

Chapter 2: Axial Load Effects in Indeterminate Structures

Section 5: Unsolved Problems

1- The rigid structure *ABCD* is supported by two vertical bars 1 and 2 and an inclined bar 3. The weight of the structure is 80 *t*, and a lateral force of 20 *t* is applied to it. Determine the appropriate cross-sectional dimensions for the supporting bars. The supporting bars are constructed from four identical angles. The allowable stress is $1000\ kg/cm^2$.

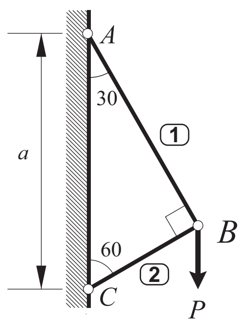


$L_1 = 35\times35\times4$ mm

$L_2 = 80\times80\times8$ mm

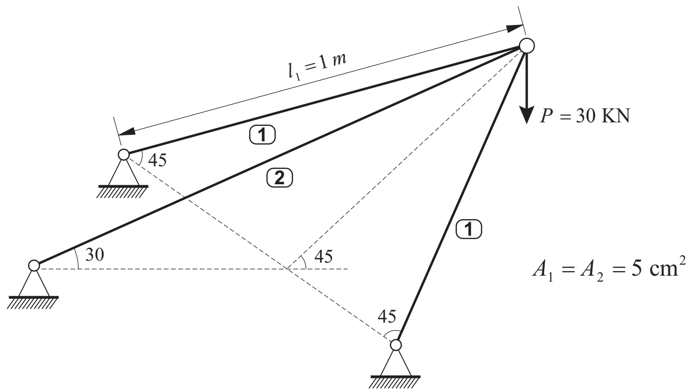
$L_3 = 65\times65\times6$ mm

2- Determine the displacement of the point under load *P* and the induced stresses in the bars.

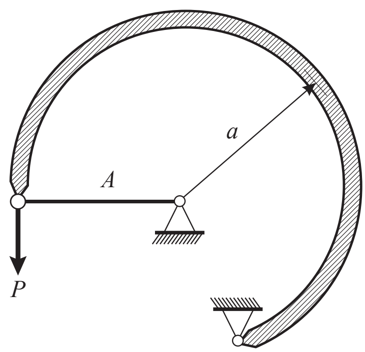


$$\delta_x = \frac{Pa}{8EA}(3-\sqrt{3}) \qquad \delta_y = \frac{Pa}{8EA}(1+3\sqrt{3})$$

$$\sigma_1 = (\frac{\sqrt{3}}{2})(\frac{P}{A}) \qquad \sigma_2 = \frac{P}{2A}$$

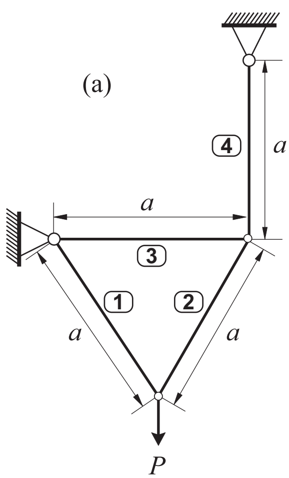


$\delta = 5.6\ mm$, $\sigma_1 = 142\ MN/m^2$, $\sigma_2 = 165\ MN/m^2$



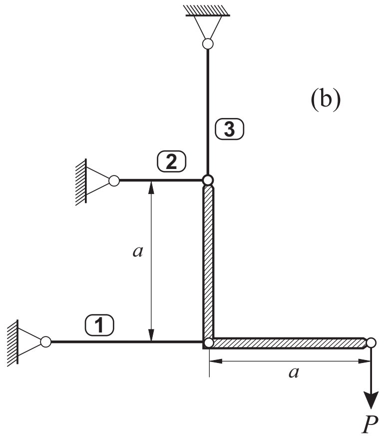
$$\delta_x = \delta_y = \frac{Pa}{EA} \quad , \ \sigma = \frac{P}{A}$$

3- In the following figures, determine the cross-sectional area (A) of the non-rigid bars. The modulus of elasticity for problems (c) and (f) is given as $2\times10^6\ kgf/cm^2$.

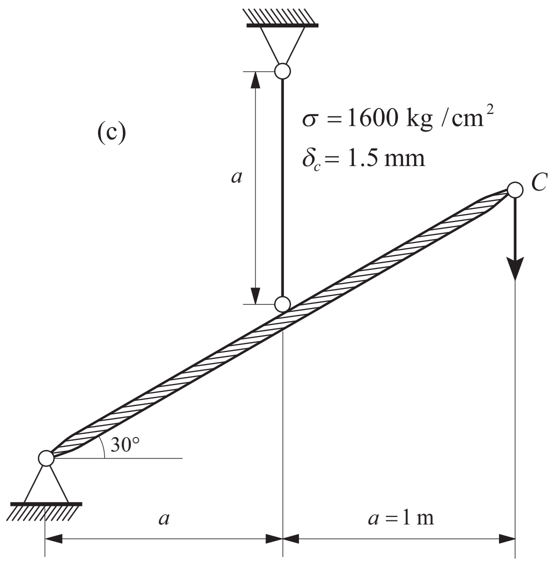


$$A_1 = A_2 = \frac{P}{\sqrt{3}\sigma}$$

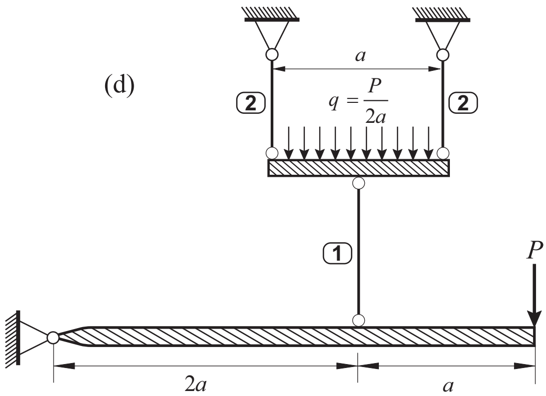
$$A_3 = \frac{P}{2\sqrt{3}\sigma} \quad , \quad A_4 = \frac{P}{2\sigma}$$



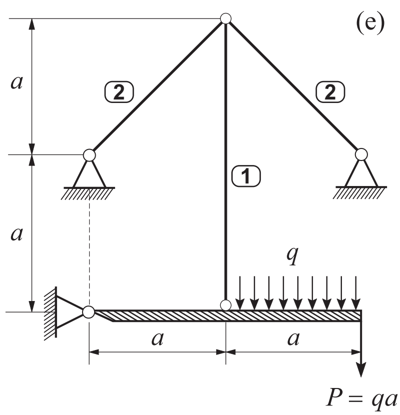
$$A_1 = A_2 = A_3 = \frac{P}{\sigma}$$



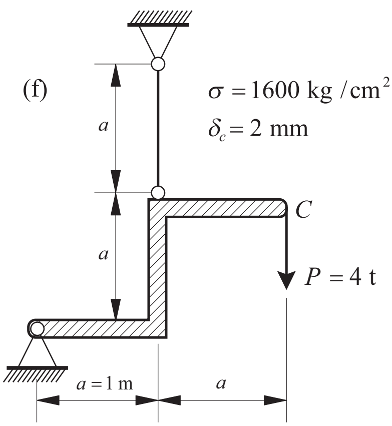
$$A = 5.33\ cm^2$$



$$A_1 = \frac{3P}{2\sigma} \quad , \quad A_2 = \frac{P}{\sigma}$$

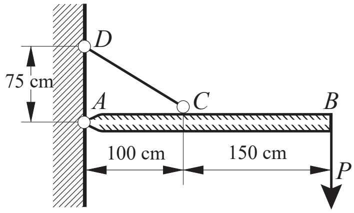


$$A_1 = \frac{7qa}{2\sigma} \quad , \quad A_2 = \frac{7\sqrt{2}qa}{4\sigma}$$



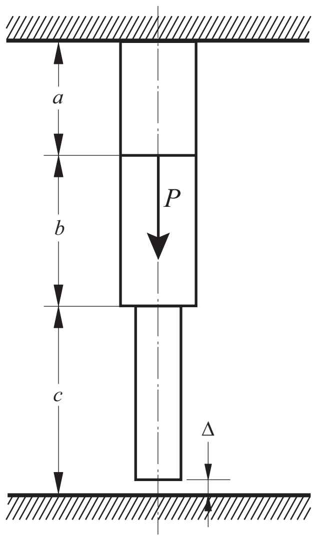
$A = 5\ cm^2$

4- The rigid bar AB is under load P and is supported by the steel bar DC with diameter $20\ mm$. Determine the allowable load P and the displacement of point B. The allowable stress of the steel bar is $1600\ kg/cm^2$.



$P = 1.2\ t \qquad \delta_B = 4.17\ mm$

5- The upper part of the bar with cross-sectional area $A_s = 200\ cm^2$ is made of steel, and the lower part with cross-sectional area $A_c = 100\ cm^2$ is made of copper. The end gap distance is $\Delta = 0.08\ mm$. The values of $c = 1\ m$, $a = 0.5\ m$ and $b = 0.5\ m$ are known. If the load $P = 150\ t$ and temperature change $t = 20^\circ\ C$ are applied simultaneously to the bar, determine the stresses in each section of the bar.

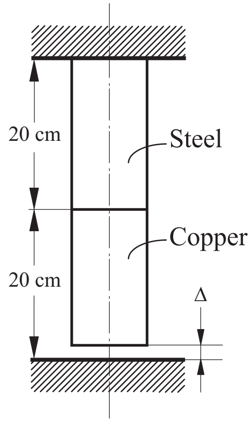


$\sigma_a = 475\ kg/cm^2$

$\sigma_b = -275\ kg/cm^2$

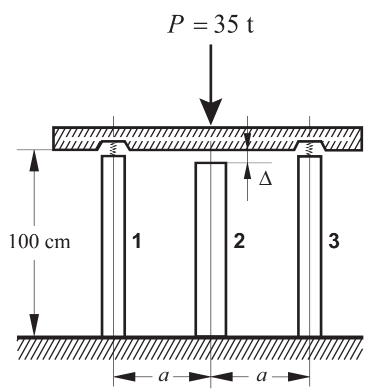
$\sigma_c = -550\ kg/cm^2$

6- As shown in the figure, a bar is suspended from the ceiling at a temperature of $t_1 = -15^\circ\ C$, with a gap of $\Delta = 0.4\ mm$ between the end of the bar and the floor. If the bar is heated to a temperature of $t_2 = 85^\circ\ C$, determine the induced stress in the bar.



$\sigma = -600\ kg/cm^2$

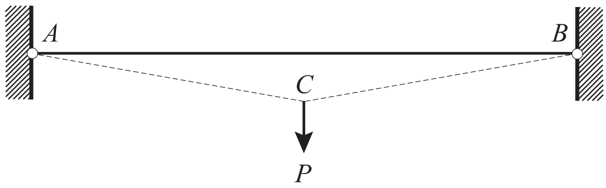
7- A rigid bar is supported on three vertical supports. The end supports are made of steel with a cross-sectional area of $20\ cm^2$. The middle support is made of cast iron with a cross-sectional area of $50\ cm^2$. Identical springs with a constant of $5\times10^{-6}\ kg/cm$ are placed between the end columns and the rigid bar. Before applying the load, there is a gap of size $\Delta = 0.5\ mm$ between the middle support and the rigid bar. Calculate the induced stresses in the supports.



$\sigma_s = -500\ kg/cm^2$

$\sigma_{ci} = -300\ kg/cm^2$

8- A weightless wire with diameter $1\ mm$ is attached between points A and B without initial stress at a temperature of $t_0 = 0^\circ\ C$. The wire temperature can vary between $t_1 = 50^\circ\ C$ and $t_2 = -50^\circ\ C$. The allowable stress of the wire material is $2000\ kg/cm^2$. Determine the maximum permissible load P at each of the three temperatures.

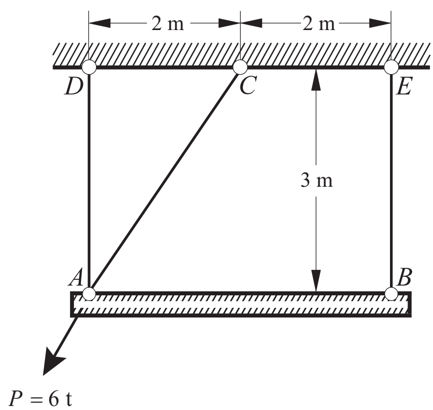


$t_0 = 0^\circ\ C\ , P = 1.4\ kg$

$t_1 = +50^\circ\ C\ ,\ P = 1.79\ kg$

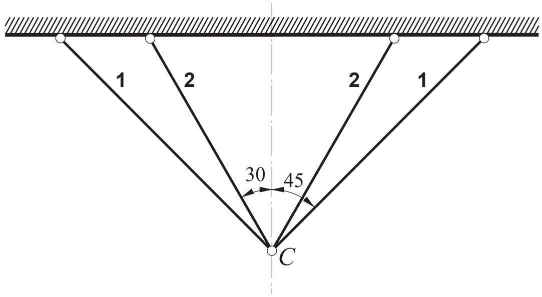
$t_2 = -50^\circ\ C\ ,\ P = 0.32\ kg$

9- The rigid bar AB is supported by three steel bars. The cross-sectional area of these bars is equal and is given as $A = 10\ cm^2$. Detremine the stress induced in the bars and the horizontal and vertical displacement of point B.



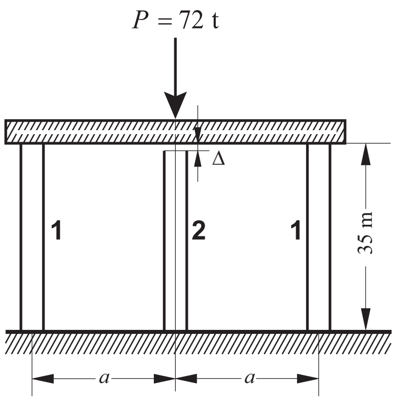
$\sigma_{AD} = \sigma_{BE} = 0$ $\quad \sigma_{AC} = 600\ kg/cm^2$ $\quad \delta_x = 1.95 mm$ $\quad \delta_y = 0\ mm$

10- Steel bars 1 with cross-sectional area $A_s = 20\ cm^2$ and copper bars 2 with cross-sectional area $A_c = 30\ cm^2$ are suspended from the ceiling and connected to each other at point C at a temperature of $t_1 = 15\ ^\circ c$. If the temperature is reduced to $t_2 = -45^\circ\ c$, determine the induced stresses in the bars.



$\sigma_s = 104\ kg/cm^2 \qquad \sigma_b = -57\ kg/cm^2$

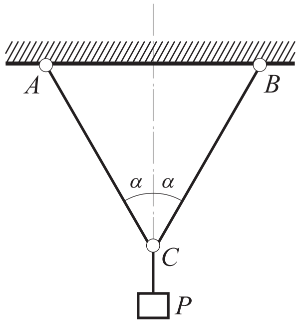
11- A rigid beam, as shown in the figure, is supported by three steel columns with identical cross-sectional areas of $A = 400\ cm^2$. Before applying the load, a small gap of $\Delta = 1.5\ mm$ exists between the middle column and the rigid beam. The columns are made of concrete with a modulus of elasticity of $14 \times 10^4\ kg/cm^2$. Calculate the stresses induced in the columns.



$\sigma_s = -80\ kg/cm^2$

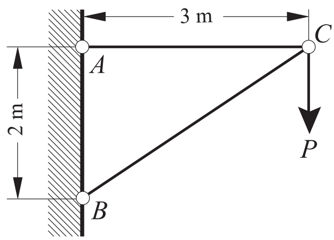
$\sigma_b = -20\ kg/cm^2$

12- In the following figure, the load P is supported by two bars. The angle α is given by 30°. The bar AC is made of steel and has a diameter $30\ mm$, with an allowable stress of $1600\ kg/cm^2$. The bar CB is made of aluminum and has a diameter $40\ mm$, with an allowable stress of $600\ kg/cm^2$. Determine the maximum allowable load P.



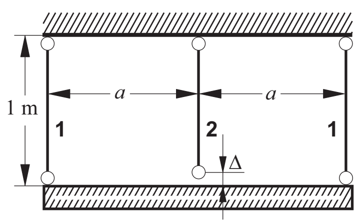
$P = 13\ t$

13- The structure in the following figure is subjected to load $P = 6\ t$. Bar AC is made of steel with an allowable stress of $1600\ kg/cm^2$, and bar BC is made of wood with an allowable stress of $40\ kg/cm^2$. Determine the diameter of the steel bar and the cross-sectional dimensions of the square wooden bar. Then, calculate the vertical, horizontal, and total displacement of point C.



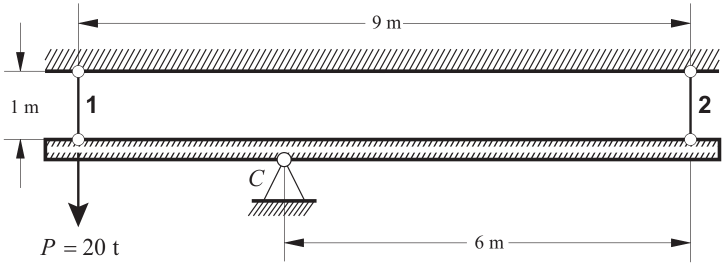
$d = 27\ mm$ $\quad a = 16.4\ cm$ $\quad \delta_{hor} = 1.4\ mm$ $\quad \delta_{ver} = 6.2\ mm$ $\quad \delta = 6.64\ mm$

14- A rigid beam is suspended from three steel bars. The middle bar is manufactured $0.5\ mm$ shorter than its required length and is installed with an initial prestress. Calculate the stresses induced in the bars. The cross-sectional areas of all bars are equal and are denoted as $A = 2\ cm^2$.



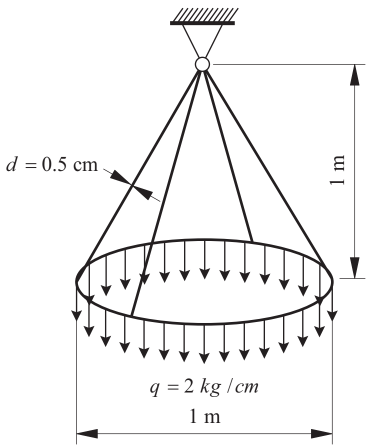
$\sigma_1 = -333\ kg/cm^2$ $\quad \sigma_2 = 667\ kg/cm^2$

15- Calculate the stresses induced in bars 1 and 2, which support the rigid bar. Bar 1 is made of copper with a cross-sectional area of $20\ cm^2$, and bar 2 is made of steel with a cross-sectional area of $10\ cm^2$.



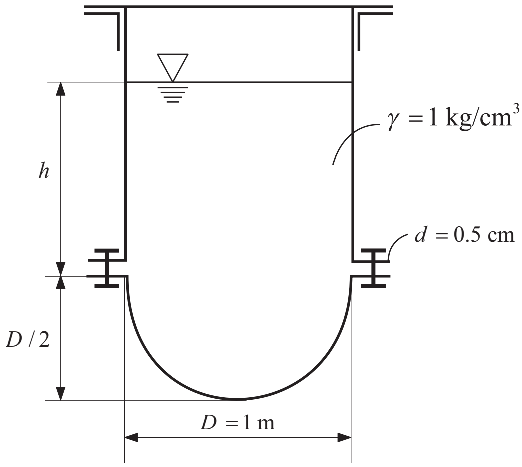
$\sigma_1 = 200\ kg/cm^2 \qquad \sigma_2 = -800\ kg/cm^2$

16- In the following problem, determine the allowable stress of the bars.



$\sigma = 894\ kg/cm^2$

17- In the following problem, determine the value of h. The number of bolts is assumed to be 8, and allowable stress of each bolt is $\sigma = 1200\ kg/cm^2$.

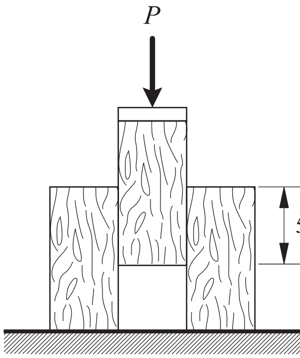


$h = 2.07\ cm$

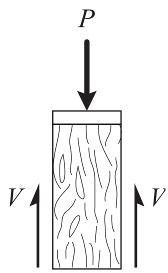
Chapter 3: Pure Shear Stress and Strain

Section 1: Shear Stress in Shear Connection

1– As shown in the figure, three wooden members are bonded together with adhesive. The wooden members have identical cross-sectional areas and lengths measured perpendicular to the plane of the figure is 20 *cm*. If the applied load *P* equals to 10000 *kg*, determine the average shear stress at the bonded interface.



The equilibrium equation for the upper piece is formulated, and the shear force acting on each bonded surface is calculated.

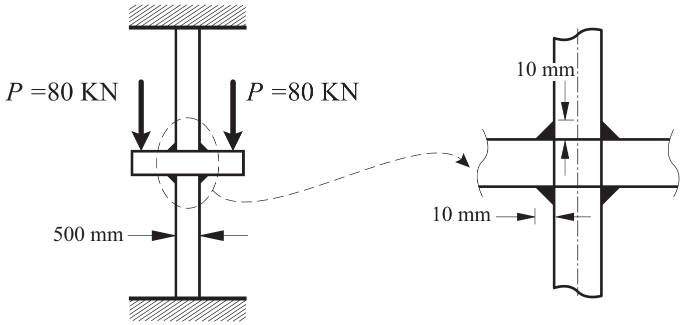


$$\sum F_y = 0$$

$$\rightarrow\ 2V = P\ \rightarrow\ V = \frac{P}{2} = 5000\ kg$$

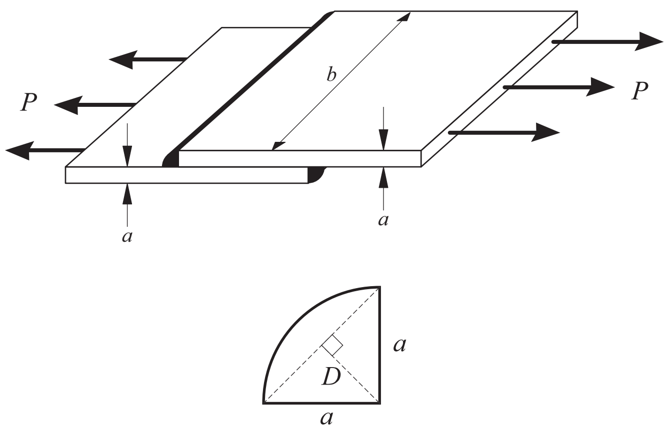
$$\tau_{ave} = \frac{V}{A} = \frac{5000}{(20)(5)} = 50\ kg/cm^2$$

2– As shown in the figure, a load $80\ KN$ is applied to the balcony symmetrically and radially. The central support is a column with diameter of $500\ mm$. The balcony is welded to the column at its top and bottom surfaces, with the length of the fillet weld legs equal to $10\ mm$. Determine the average shear stress developed between the column and the balcony.



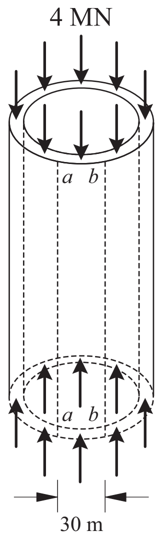
$\tau = 2.54\ MN / m^2$

3- Two plates of equal thickness are connected to each other by two fillet welds as shown in the figure. Determine the maximum shear stress developed in the welds.



$$\tau = \frac{0.707P}{ab}$$

4- A thin-walled vertical cylindrical shell is uniformly loaded along its top edge, but it is unsupported along its entire bottom edge. The cylinder is made of concrete with a wall thickness of 200 *mm*, a height of 10 *m*, and a diameter of 40 *m*. If the total load applied at the top of the cylinder is 4 *MN* and a 30-meter length of the bottom edge is unsupported, determine the average shear stress on each of the sections *a-a* and *b-b*.

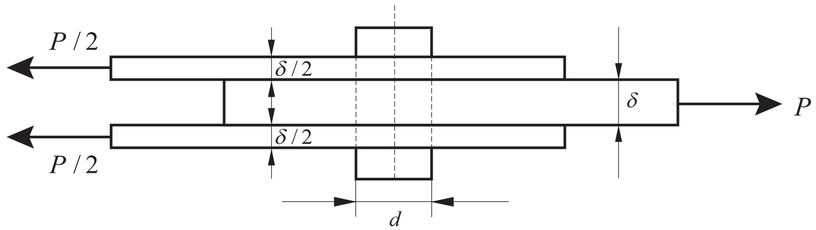


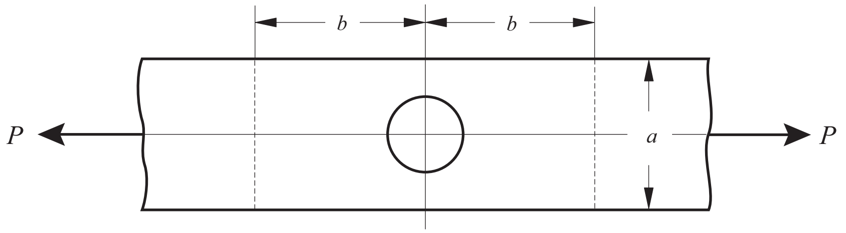
$\tau = 0.24 \; MPa$

Chapter 3: Pure Shear Stress and Strain

Section 2: Design of Shear Connections

1– If, as shown in the figure, the values of $P = 4\ t$, $\sigma = 1600\ kg/cm^2$, $\tau = 1200\ kg/cm^2$, and $\sigma_{br} = 3200\ kg/cm^2$ are given. Determine the values of d, δ, a and b.

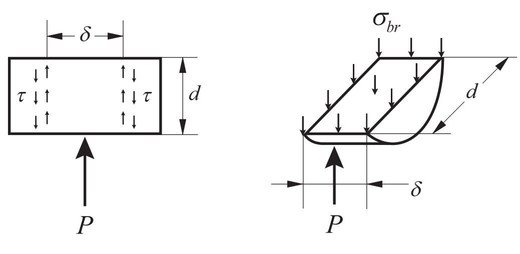




First, the diameter of the bolt d is calculated based on the shear strength.

$$\frac{2\pi d^2}{4} \geq \frac{P}{\tau}$$

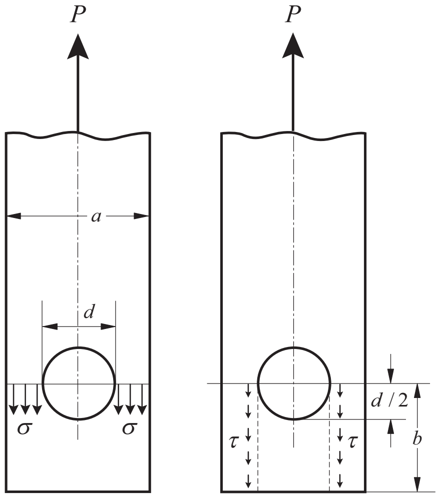
$$\rightarrow \quad d \geq \sqrt{\frac{2P}{\pi\tau}} = \sqrt{\frac{(2)(4\times10^3)}{(\pi)(12\times10^2)}} \simeq 1.46\ cm$$



Next, the thickness of the plate δ is calculated based on bearing strength.

$$\delta d \geq \frac{P}{\sigma_{br}}$$

$$\rightarrow \quad \delta \geq \frac{P}{d\sigma_{br}} = \frac{4\times10^3}{(1.46)(32\times10^2)} \simeq 0.86\ cm$$



The width a is then determined based on tensile strength.

$$(a-d)\delta \geq \frac{P}{\sigma} \quad \rightarrow \quad a=\frac{P}{\delta\sigma}+d=\frac{4\times 10^3}{(0.86)(16\times 10^2)}+1.46 \simeq 2.92\ cm$$

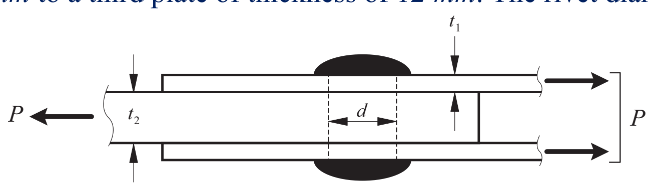
Finally, the width b, measured from the bolt center to the edge of the plate, is calculated based on shear strength.

$$2b'\delta \geq \frac{P}{\tau} \quad \rightarrow \quad b'=\frac{P}{2\delta\tau}=\frac{4\times 10^3}{(2)(0.86)(12\times 10^2)} \simeq 1.94\ cm$$

Accordingly, the following is obtained:

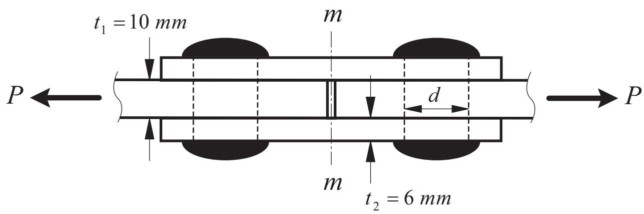
$$b=b'+\frac{d}{2}=1.94+0.73=2.67\ cm$$

2- As shown in the figure below, the tensile force P applied to the joint is 18 t. The allowable shear and bearing stresses are $1000\ kg/cm^2$ and $2800\ kg/cm^2$, respectively. Determine the number of rivets required to connect two plates of thickness of 5 mm to a third plate of thickness of 12 mm. The rivet diameter is 20 mm.



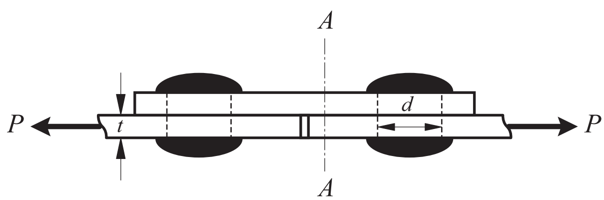
$n=4$

3- As shown in the figure below, a tensile force P equal to 30 t is applied to the joint. The thickness of each plate is 10 mm, and the thickness of the straps is 6 mm. If the allowable shear and bearing stresses are $1000\ kg/cm^2$ and $2800\ kg/cm^2$, respectively, determine the number of rivets with diameter 17 mm required to connect the two plates using the two straps.



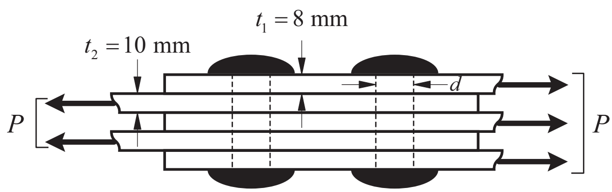
$n = 7$

4- As shown in the figure below, two plates are riveted together using a single strap. A horizontal force P equal to 24 t is applied at the joint. If the thickness of the plates and the strap are10 mm, and the allowable shear and bearing stresses are1400 kg/cm^2 and $3200\ kg/cm^2$, respectively, determine the number of rivets with a diameter of 17 mm required for the shown connection.



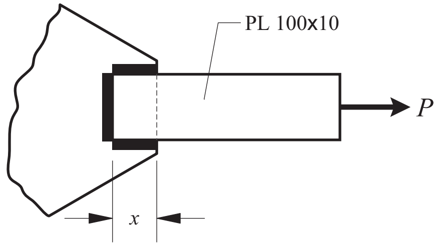
$n = 8$

5- As shown in the figure below, two plates with a thickness of 10 mm are connected to three plates with a thickness of $8\ mm$ using rivets with a diameter of 20 mm. The connection is subjected to a tensile force of 28 t. If the allowable shear and bearing stresses are $1000\ kg/cm^2$ and $2800\ kg/cm^2$, respectively, determine the number of rivets required for the connection.



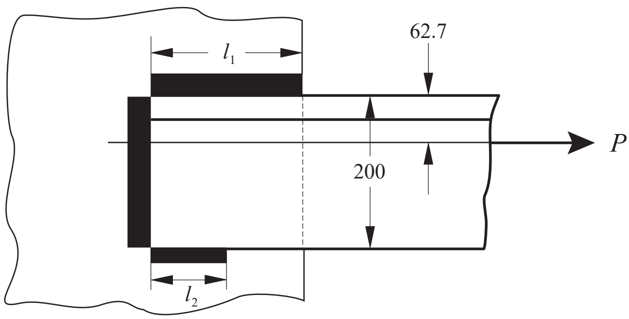
$n = 3$

6- As shown in the figure below, if the allowable tensile stress in the plate is $1400\ kg/cm^2$ and the allowable shear stress of the weld is $800\ kg/cm^2$, determine the minimum required length x for welding the steel plate.



$x = 7.5\ cm$

7- A tensile force of 40 t is applied concentrically to an angle section with dimensions $150\times200\times16\ mm$. As shown in the figure below, the angle is welded to a plate. To reduce the lengths l_1 and l_2, the ends of the angle are also welded to the plate. If the allowable shear stress of the weld is $800\ kg/cm^2$, determine the dimensions l_1 and l_2.



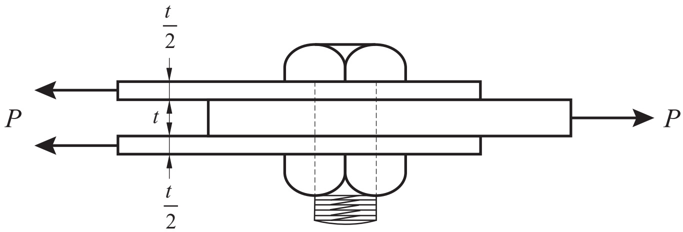
$l_1 = 20.7\ cm$

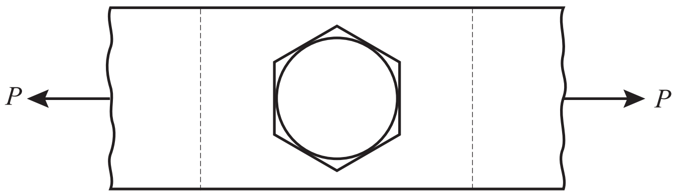
$l_2 = 4\ cm$

Chapter 3: Pure Shear Stress and Strain

Section 3: Unsolved Problems

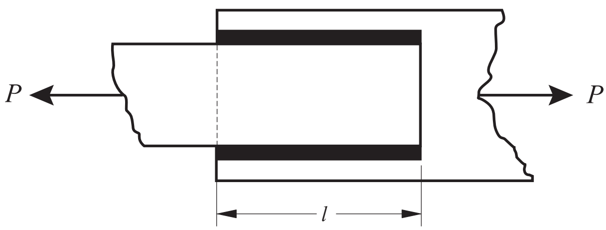
1- Determine the diameter of the bolt shown in the connection below. The tensile force P is 20 t, and the thickness is 2 cm. The allowable shear stress of the bolt is $800\ kg/cm^2$, and the allowable bearing stress is $2000\ kg/cm^2$.





$d = 5\ cm$

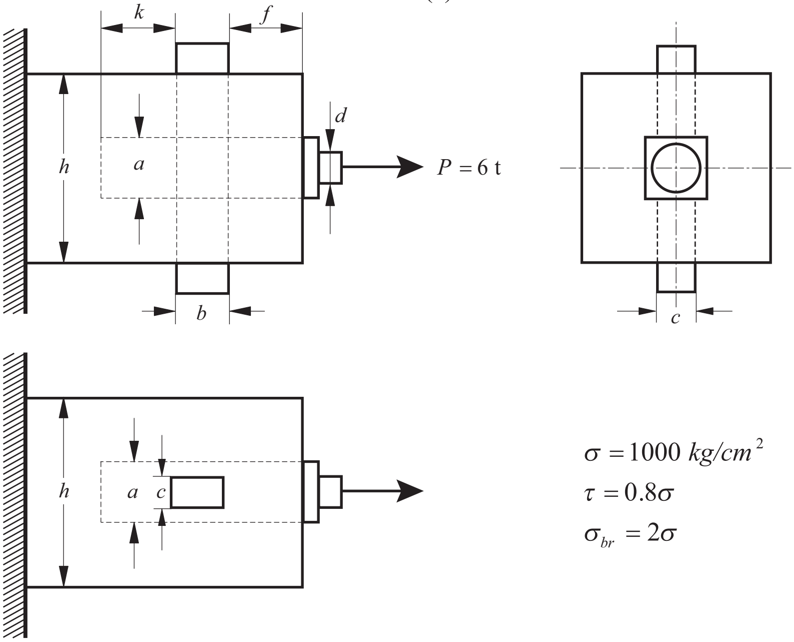
2- As shown in the figure below, two plates connected to each other resist a tensile force of $P = 15\ t$. These two plates are welded together along a length l. The allowable shear stress of the weld is $1100\ kg/cm^2$. The thickness of the plate with smaller width is 10 mm, and the thickness of the wider plate is 8 mm. Determine the required weld length l.



$l \simeq 10\ cm$

3- Determine the dimensions of the connection components shown in the following figures. The design calculations shall be performed assuming identical strength capacities for all members. In examples (b) and (c), the quantities $\sigma = 1600\ kg/cm^2, \tau = 1200\ kg/m^2$ and $\sigma_{br} = 3200\ kg/cm^2$ are provided, while in example (d), the quantities $\sigma = 160\ MN/m^2, \tau = 120\ MN/m^2$, $\sigma_{br} = 320\ MN/m^2$ are given.

(a)



$\sigma = 1000\ kg/cm^2$

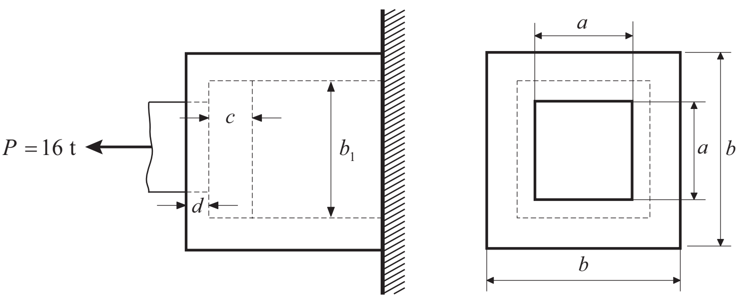
$\tau = 0.8\sigma$

$\sigma_{br} = 2\sigma$

$a = 3\ cm$, $b = 3.75\ cm$, $c = 1\ cm$, $d = 2.8\ cm$

$f = 1.25\ cm$, $k = 1.25\ cm$, $h = 6\ cm$

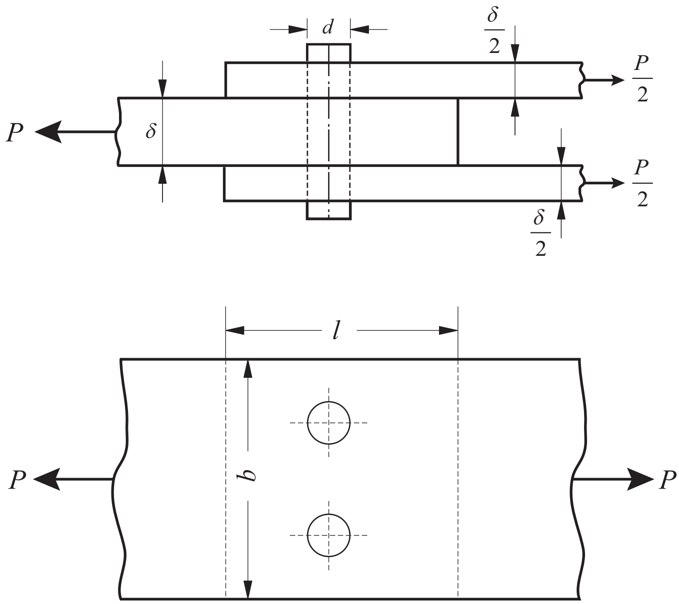
(b)



$a = 3.16\ cm$, $b = 5\ cm$, $c = 1.06\ cm$

$b_1 = 3.87\ cm$, $d = 0.87\ cm$

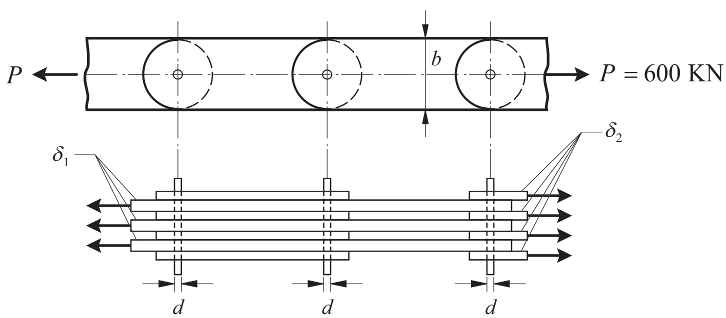
(c)



$b = 21.4\ cm$, $d = 3.57\ cm$

$\delta = 2.1\ cm$, $l = 13.1\ cm$

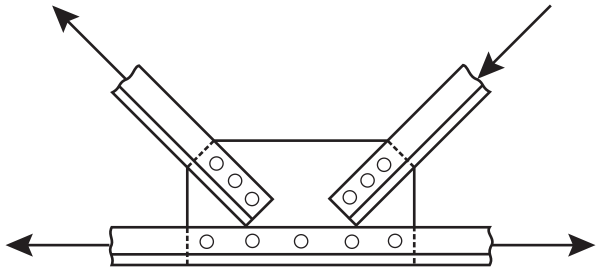
(d)



$b = 9.75\ cm$, $d = 3.25\ cm$

$\delta_1 = 1.92\ cm$, $\delta_2 = 1.44\ cm$

4- The diagonal members of a truss are subjected to a force 5.25 *t*. Each of these members is connected to a gusset plate with a thickness of 10 *mm* using three bolts. The cross-section of the diagonal members is an angle section with dimensions $75 \times 75 \times 75\ mm$. Calculate the shear and bearing stresses in the bolts.



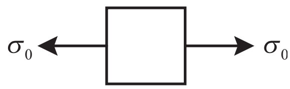
$\tau = 556\ kg/cm^2$, $\sigma_c = 1090\ kg/cm^2$

Chapter 4: Analysis of Stress-Strain

Section 1: Uniaxial and Biaxial Normal Stress and Strain

1- A circular bar is subjected to a tensile force of 15 *t*. Determine the diameter of the bar such that the shear stress on any of its planes does not exceed $600\ kg/cm^2$.

Since the bar is loaded only in one direction, an arbitrary stress element of the bar is represented as follows:



The stress element represents a uniaxial stress state.

$$\begin{cases} \sigma_\theta = \sigma_0 \cos^2\theta \\ \tau_\theta = \sigma_0 \sin\theta\cos\theta = \frac{1}{2}\sigma_0 \sin 2\theta \end{cases}$$

$$\frac{d\tau_\theta}{d_\theta} = 0 \ \rightarrow\ \frac{1}{2}\sigma_0(2\cos 2\theta) = 0 \ \rightarrow\ \cos 2\theta = 0 \ \rightarrow\ 2\theta = k\pi + \frac{\pi}{2} \ \rightarrow\ \theta = \frac{k\pi}{2} + \frac{\pi}{4}$$

$$\rightarrow\ \theta = 45^\circ$$

Therefore, the maximum shear occurs on plane $\theta = 45^\circ$.

$$\rightarrow \tau_{\max} = \sigma_0\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sigma_0}{2}$$

$$\tau_{\max} = 600 \ \rightarrow\ \sigma_{0,\max} = (2)(600) = 1200$$

$$A = \frac{P}{\sigma_{0,\max}} = \frac{15000}{1200} = 12.5\ cm^2$$

$$\rightarrow\ \frac{\pi d^2}{4} = 12.5 \ \rightarrow\ d = 3.99 \simeq 4\ cm$$

2- A steel bar with a square cross-section of side 25 *mm* is subjected to an axial compressive force of 50 *KN*. Determine the normal and shear stresses on a plane that forms an angle of 30° with the line of action of the compressive force.

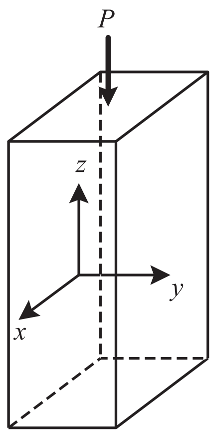
$$\sigma_{30} = -20\ MPa$$

$$\tau_{30} = -34.64\ MPa$$

3- A thin circular brass membrane supported along its entire edge by a rigid circular brass ring is stress-free at the room temperature of $20^\circ\ C$. If the room temperature decreases to $-20^\circ\ C$, what will be the principal stresses σ_x and σ_y in the membrane? The values of $E = 10^6\ kg/cm^2$, $\nu = 0.3$, and $\alpha = 19\times10^{-6}\ C^{-1}$ are assumed to be given.

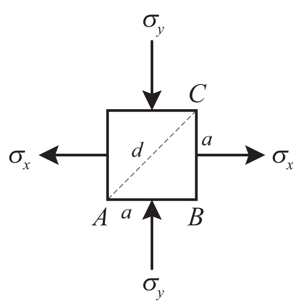
$$\sigma_x = \sigma_y = 1085.7\ kg/cm^2$$

4- If the ratio of the volumetric strain to the area strain of a prismatic bar under axial compression is 0.75, determine the Poisson's ratio.



$$\nu = \frac{2}{7}$$

5- A square steel plate with sides of 15 cm is subjected to a tensile stress of $750\ kg/cm^2$ in one direction and a compressive stress of equal magnitude in the perpendicular direction. Determine the strain developed along the diagonals of the plate.

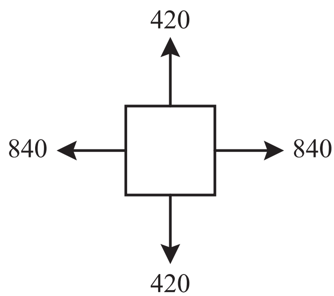


$$\Delta = 0$$

6- For an element subjected to pure shear, determine the volumetric strain.

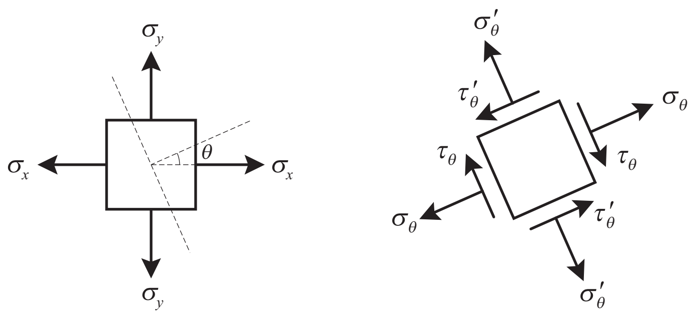
$$\Delta V / V = 0$$

7- As shown in the figure below, the principal stresses are $\sigma_x = 840\ kg/cm^2$ and $\sigma_y = 420\ kg/cm^2$. Draw the corresponding Mohr's circle and determine the angle θ of the plane on which the ratio of normal stress to shear stress is minimum.



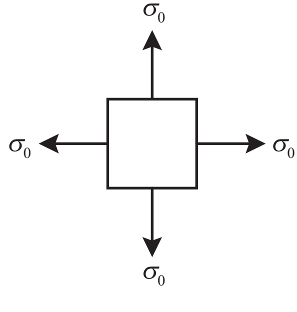
$\theta = 54.74^\circ = 54^\circ, 44'$

8- As shown in the figure below, an element under biaxial stress in the x and y directions has principal strains ε_x and ε_y, respectively. Determine the strain ε_θ along direction σ_θ in terms of ε_x, ε_y, and the angle θ.

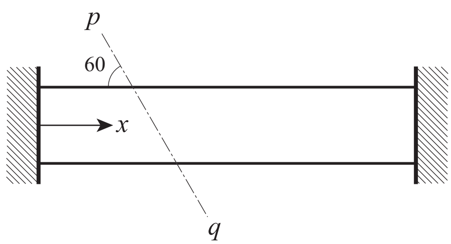


$\varepsilon_\theta = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta$

9- As shown in the figure below, and assuming $\sigma_x = \sigma_y = \sigma_0$, draw Mohr's circle for the biaxial stress state.



10- As shown in the figure below, a metal bar is placed exactly between rigid supports at a temperature of $21^\circ\ C$. The modulus of elasticity is $E = 2.1\times10^6\ kg/cm^2$, and the coefficient of thermal expansion is $\alpha = 11.7\times10^{-6}\ 1/C$. If the temperature increases by $93^\circ\ C$, determine the normal and shear stresses developed on the inclined section pq.



$\sigma_{30} = 1326.75$

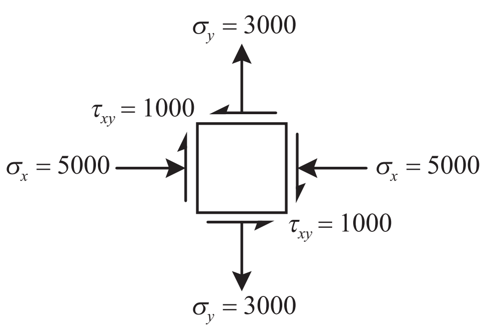
$\tau_{30} = -766$

11- A thin plate is subjected to biaxial stresses of $\sigma_x = 10000\ kg/cm^2$ and $\sigma_y = 20000\ kg/cm^2$. Determine the maximum normal and shear stresses developed in the plate.

$\sigma_{\max} = 20000\ kg/cm^2$

$\sigma_{\min} = 10000\ kg/cm^2$

12- For an element as shown in the figure below, if the stresses are $\sigma_x = -5000\ kg/cm^2$, $\sigma_y = 3000\ kg/cm^2$ and $\tau_{xy} = -1000\ kg/cm^2$, determine the principal stresses and the orientations of the principal planes.



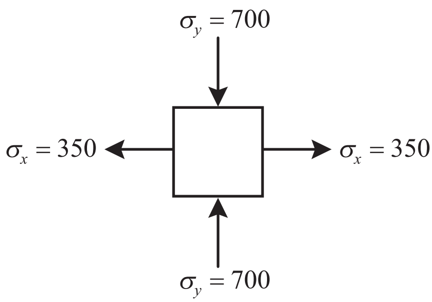
$\sigma_{\max} = 3123.11\ kg/cm^2$

$\sigma_{\min} = -5123.11\ kg/cm^2$

$\theta_P = 7^\circ\ 1'$

13- Solve the previous problem again using Mohr's circle.

14- In the figure below, the principal stresses are $\sigma_x = 350\ kg/cm^2$ and $\sigma_y = -700\ kg/cm^2$. Draw Mohr's circle, determine the angle θ of the plane on which the normal stress is zero, and calculate the shear stress on this plane.



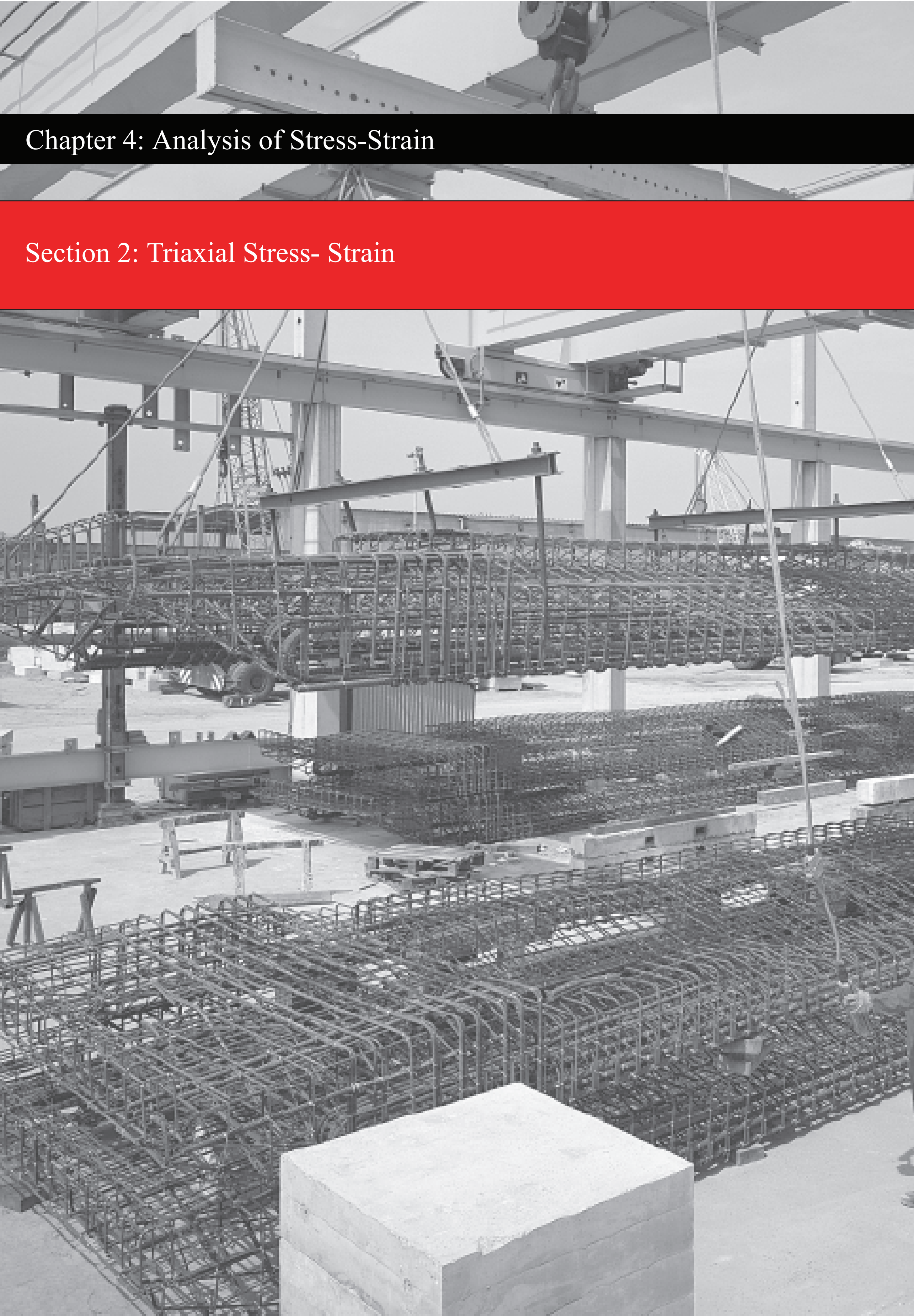
$\theta = 35.26^\circ$

$\tau_{35.26^\circ} = 495\ kg/cm^2$

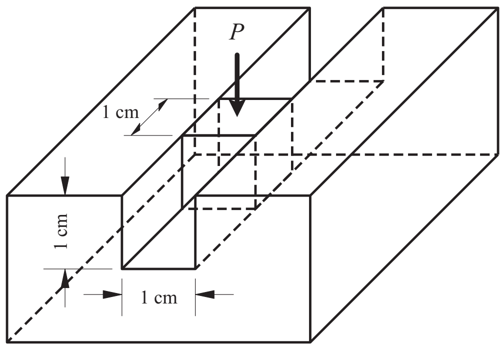
15- Draw Mohr's circle corresponding to the state of pure shear.

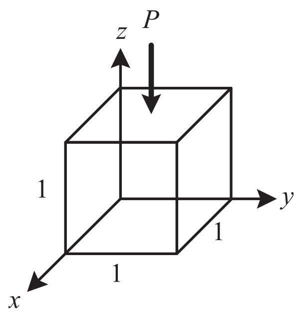
Chapter 4: Analysis of Stress-Strain

Section 2: Triaxial Stress- Strain



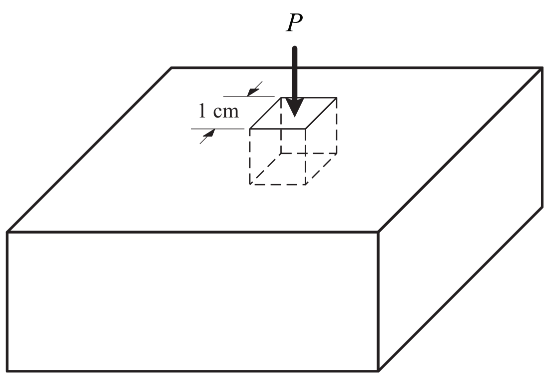
1- As shown in the figure below, a square groove with dimensions 1 *cm* is made inside a thick steel piece. An aluminum cube with dimensions $1\times1\times1\ cm$ is placed in this groove and compressed under a force of $P = 600\ kg$. Assuming the values $\nu = 0.33$ and $E = 0.7\times10^6\ kg/cm^2$ for aluminum and that the aluminum piece undergoes negligible deformation, calculate the principal stresses developed in the aluminum cube.

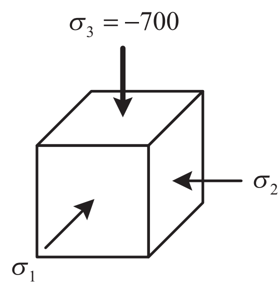




$\sigma_x = -198$, $\sigma_y = -600$, $\sigma_z = 0$

2- As shown in the figure below, a cubic hole with dimensions 1 *cm* is made inside a thick steel slab. A steel cube fits inside the cavity and is compressed by a force of $P = 700\ kg$. Neglecting deformation of the slab, determine the principal stresses developed in the cube.





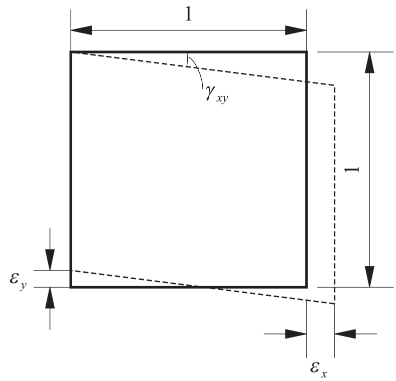
$\sigma_1 = -300\ kg/cm^2$

$\sigma_2 = -300\ kg/cm^2$

Chapter 4: Analysis of Stress-Strain

Section 3: Plane Strain

1- In the figure below, the strains of a steel plate element are given as $\varepsilon_x = 5.32\times10^{-4}$, $\varepsilon_y = -1.82\times10^{-4}$, and $\gamma_{xy} = 1.19\times10^{-3}$. The material properties of the steel are $\nu = 0.3$ and $E = 2\times10^{6}\ kg/cm^2$. Determine the magnitudes and directions of the principal stresses.



$\sigma_1 = 1567.5\ kg/cm^2$

$\sigma_2 = -567.5\ kg/cm^2$

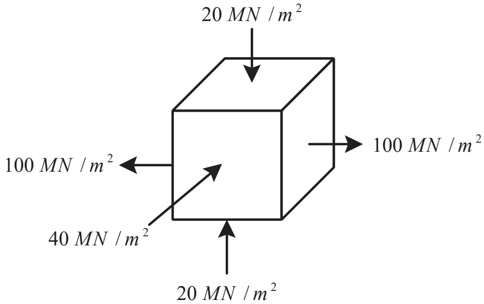
$\theta_p = -29.52^\circ$

2- If the values of $\varepsilon_x = 500\times10^{-6}$, $\varepsilon_y = 140\times10^{-6}$, and $\gamma_{xy} = -360\times10^{-6}$ are known, determine the principal strains ε_1 and ε_2 and their orientations.

$\varepsilon_1 = 574.6\times10^{-6}$

$\varepsilon_2 = 65.44\times10^{-6}$ $\qquad \theta_p = -22.5^\circ$

3- Calculate the values of $\gamma_{1,2,3}$ for the stress state shown in the figure below. The values of $E=2\times10^{5}\ MN/m^2$ and $\nu = 0.25$ are given.

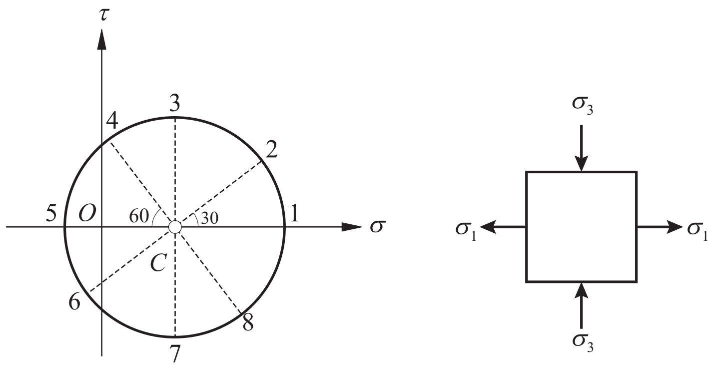


$\gamma_1 = 1.25\times10^{-4}$ $\qquad \gamma_2 = 8.75\times10^{-4}$ $\qquad \gamma_3 = 7.5\times10^{-4}$

Chapter 4: Analysis of Stress-Strain

Section 4: Unsolved Problems

1- The Mohr's circle for an element under plane stress has been drawn. Draw the planes on the element corresponding to points 1, 2, 3, 4, 5, 6, 7, and 8.



2- A bar with a diameter of 7.5 cm is subjected to a tensile force of 35 t . Determine the normal stress developed on the cross-section. Additionally, compute the shear stress and the resultant stress on a plane inclined at an angle of 15° with respect to the bar's axis.

$\sigma_0 = 795\ kg/cm^2$, $P_\alpha = 767\ kg/cm^2$, $\tau_\alpha = 198\ kg/cm^2$

3- A circular bar is subjected to a tensile force of 15 t. The shear stress on any plane must not exceed $600\ kg/cm^2$. Determine the required diameter of the bar.

$d = 4\ cm$

4- In a bar under tension, the normal stress on an inclined plane is $700\ kg/cm^2$ and the shear stress is $500\ kg/cm^2$. Determine the maximum normal and shear stresses.

$\sigma_{\max} = 1056\ kg/cm^2$, $\tau_{\max} = 528\ kg/cm^2$

5- A square steel plate with sides 15 cm is subjected to a tensile stress of $750\ kg/cm^2$ in one direction and an equal compressive stress in the perpendicular direction. Determine the strain along the diagonal of the plate.

$\varepsilon = 0$

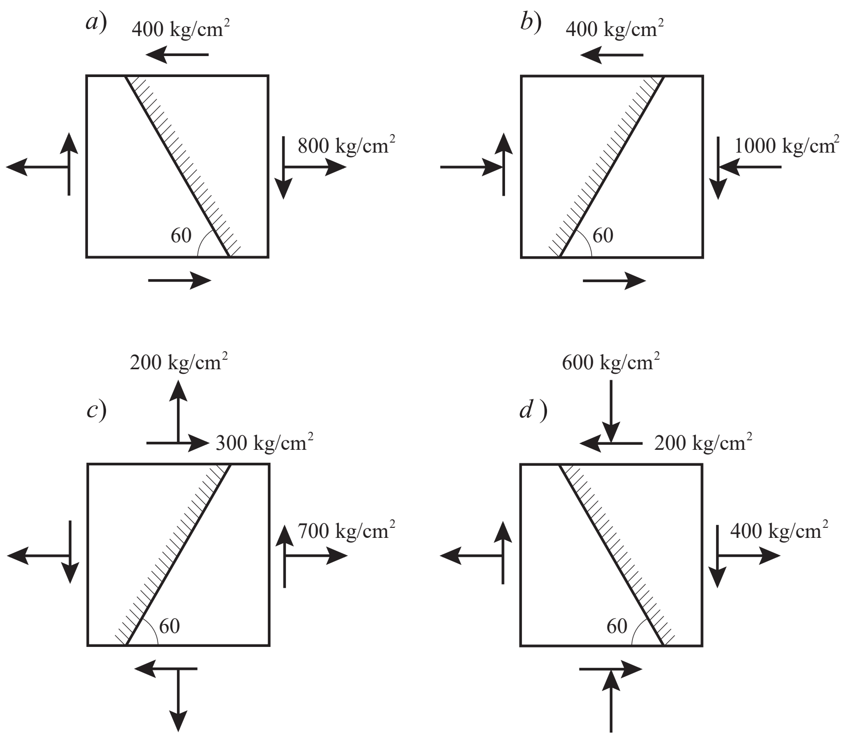
6- In the figures below, using the given values of σ_x, σ_y, and τ_y, determine the values of σ_α and τ_α on the inclined plane shown.

(a) $\sigma_\alpha = 258\ kg/cm^2$ $\tau_\alpha = 546\ kg/cm^2$

(b) $\sigma_\alpha = -402\ kg/cm^2$ $\tau_\alpha = 630\ kg/cm^2$

$(c)\ \sigma_\alpha = 314\ kg/cm^2 \qquad \tau_\alpha = 368\ kg/cm^2$

$(d)\ \sigma_\alpha = -24\ kg/cm^2 \qquad \tau_\alpha = 532\ kg/cm^2$



7- The normal stress on a transverse section of a bar in tension is $500\ kg/cm^2$. The shear stress on an inclined plane is $160\ kg/cm^2$. Determine the angle of this plane and compute the normal stress acting on it.

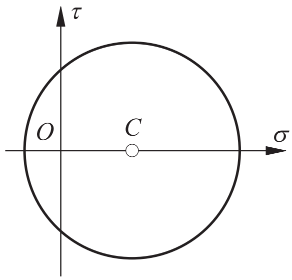
$\alpha_1 = 19^\circ\ 55' \ ,\ \sigma_{\alpha_1} = 442\ kg/cm^2$

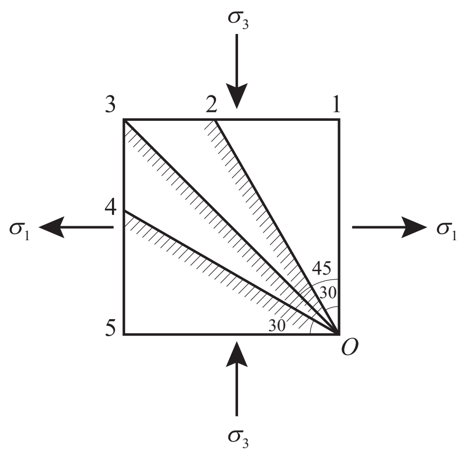
$\alpha_2 = 70^\circ\ 5' \ ,\ \sigma_{\alpha_2} = 58\ kg/cm^2$

8- A short concrete column with dimensions $20\times20\ cm$ is subjected to a compressive load of P. Determine the value of the load P such that the normal compressive stress on a plane inclined at 45° to the column's axis is equal to $15\ kg/cm^2$.

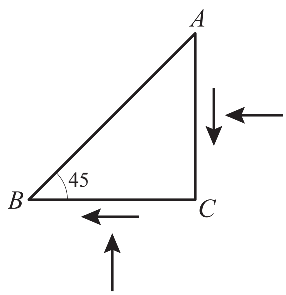
$P = 12\ t$

9- The Mohr's circle for an element under a state of plane stress has been constructed. Identify, on this circle, the points corresponding to the stresses acting on the 1-0, 2-0, 3-0, 4-0 and 5-0 planes.



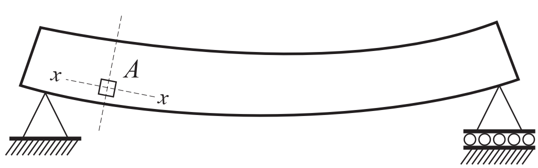


10- In the figure below, an element of a prismatic body is shown. Faces *AB* and the prism base (parallel to face *ABC*) are unloaded. The normal stresses on faces *AC* and *BC* are equal 150 kg/cm^2. Determine the shear stresses on faces *AC* and *BC*, as well as the magnitudes and directions of the principal stresses.



$\tau = 150\ kg/cm^2 \qquad \sigma_1 = 0 \qquad \sigma_3 = -300\ kg/cm^2$

11- Using a strain gauge, the strain induced by a train moving over a bridge was measured at point *A* of the longitudinal girder. The strain in the *xx* direction (along the girder axis) is reported as 0.0004, and in the *yy* direction as -0.00012. Determine the longitudinal stresses corresponding to the measured strains.



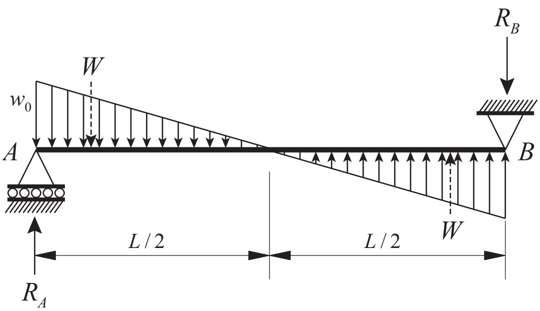
Parallel to the beam axis $\sigma = 800\ kg/cm^2$; perpendicular to this axis $\sigma = 0$

Chapter 5: Bending Moment Effects in Structures

Section 1: Shear forces and Bending Moments in Beams

1- In the figure below, derive the shear force and bending moment at a point located a distance x from support A as functions of x. Determine the shear force at mid span, and identify the location along the beam where the maximum bending moment occurs.

To establish the shear-force and bending-moment expressions, the support reactions must first be determined



$$W = w_0(L/2)(\frac{1}{2}) = \frac{w_0 L}{4}$$

$$\sum M_A = 0$$

$$\rightarrow (\frac{-w_0 L}{4})(\frac{L}{2}\times\frac{1}{3}) + (\frac{w_0 L}{4})(\frac{L}{2}+\frac{2}{3}\times\frac{L}{2}) - (R_B)(L) = 0$$

$$\rightarrow R_B = \frac{w_0 L}{6}\downarrow$$

Due to the symmetry of the loading, the support reactions are equal in magnitude and act in opposite directions.

$$\sum F_y = 0$$

$$\rightarrow R_A - W + W + R_B = 0 \rightarrow R_A = -R_B$$

$$\rightarrow R_A = \frac{w_0 L}{6}\uparrow$$

Using the relationships among loading, shear, and bending moment, together with the calculated support reactions, the following results are obtained:

a) Determination of the shear force

$$\frac{dV}{dx} = -q \quad , \quad w_{(x)} = w_0 - \frac{w_0}{(\frac{L}{2})}x = w_0(1-\frac{2x}{L})$$

$$\frac{dV}{dx} = -w_0(1-\frac{2x}{L}) \rightarrow \int_0^x dV = \int_0^x -w_0(1-\frac{2x}{L})dx$$

$$\rightarrow V_{(x)} - V_{(x=0)} = \left[-w_0(x-\frac{x^2}{L})\right]_0^x$$

$$(V_{(x=0)} = R_A = \frac{w_0}{6}) \rightarrow V_{(x)} = \frac{w_0 L}{6} - w_0 (x - \frac{x^2}{L}) = w_0 \left[\frac{L}{6} - x(1 - \frac{x}{L}) \right]$$

b) Determination of the bending moment

$$\frac{dM}{dx} = V$$

$$\rightarrow \frac{dM}{dx} = w_0 \left[\frac{L}{6} - x(1 - \frac{x}{L}) \right] \rightarrow \int_0^x dM = \int_0^x w_0 (\frac{L}{6} - (x - \frac{x^2}{L}))dx$$

$$\rightarrow M_{(x)} - M_{(x=0)} = w_0 \left[\frac{L}{6}x - (\frac{x^2}{2} - \frac{x^3}{3L}) \right]_0^x$$

$$(M_{(0)} = 0) \rightarrow M_{(x)} = \frac{w_0 x}{6} \left[L - (3x - \frac{2x^2}{L}) \right]$$

The shear at mid span is calculated as follows:

$$V_{(x)} = w_0 \left[\frac{L}{6} - \frac{L}{2}(1 - \frac{(\frac{L}{2})}{L}) \right] = \frac{-w_0 L}{12}$$

To identify the location of the maximum bending moment, the point at which the shear force becomes zero must be determined.

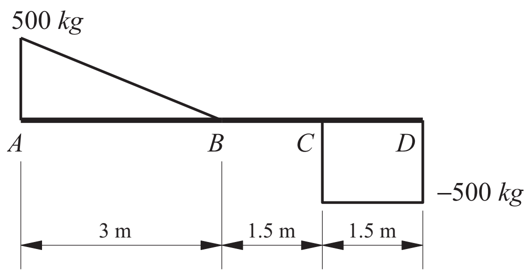
$$V_{(x)} = 0 \rightarrow \frac{dM}{dx} = 0$$

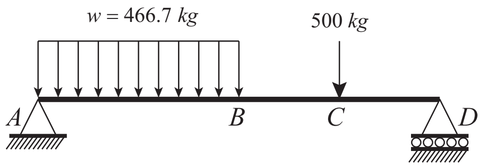
$$\rightarrow V_{(x)} = w_0 \left[\frac{L}{6} - x(1 - \frac{x}{L}) \right] = 0$$

$$\rightarrow \frac{L}{6} - x + \frac{x^2}{L} = 0 \rightarrow x^2 - xL + \frac{1}{6} = 0$$

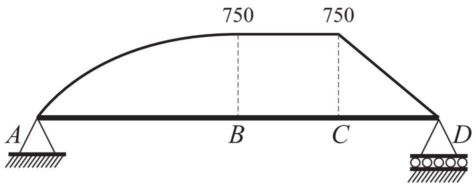
$$\rightarrow x = 0.2114\ L$$

2- In the figure below, the shear-force diagram for a simply supported beam is provided. Determine the loading applied to the beam and construct the corresponding bending-moment diagram.



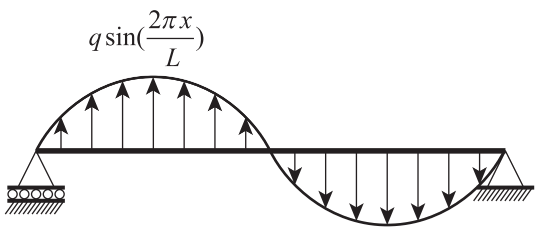


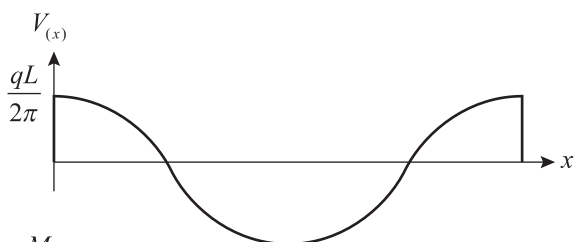
Loading Diagram

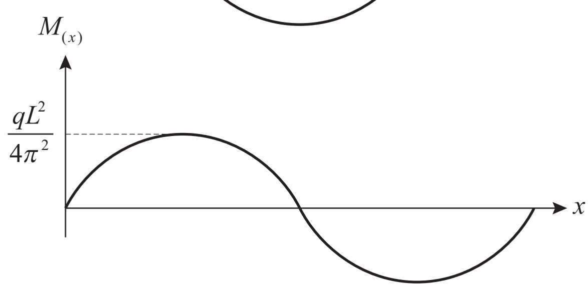


Bending Moment Diagram

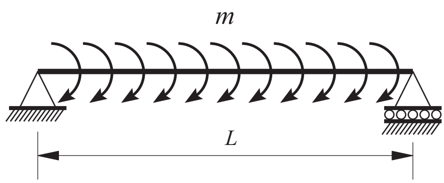
3- Draw the shear-force and bending-moment diagrams for the beam shown in the figure below.

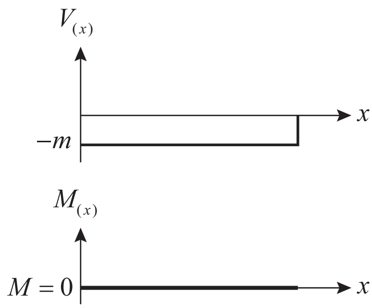




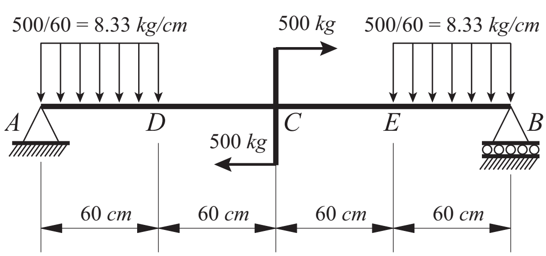


4- Draw the shear-force and bending-moment diagrams for the beam shown in the figure below.



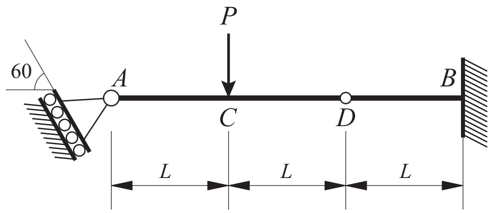


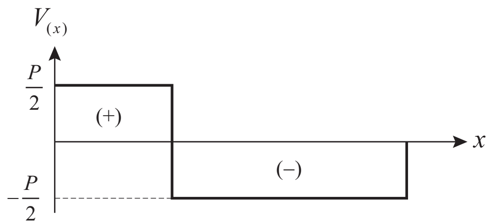
5- Determine the shear force and bending moment at section *D* of the beam shown in the figure below.

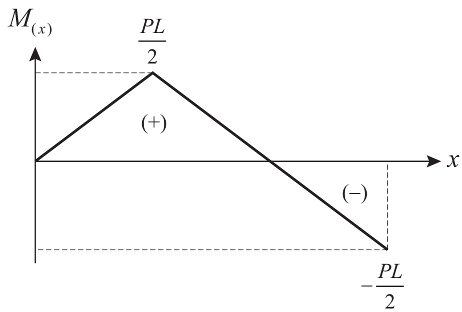


$M_D = 75\ kg.m$, $V_D = -125\ kg$

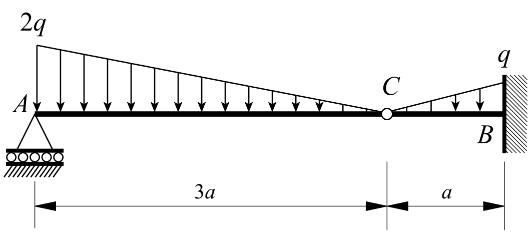
6- Draw the shear-force and bending-moment diagrams for the beam shown in the figure below.

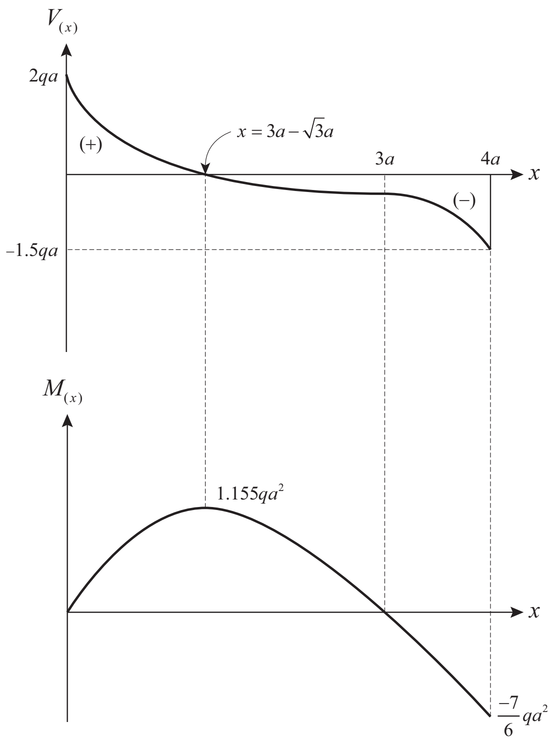




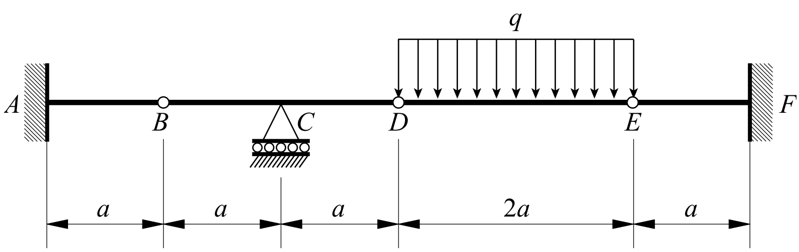


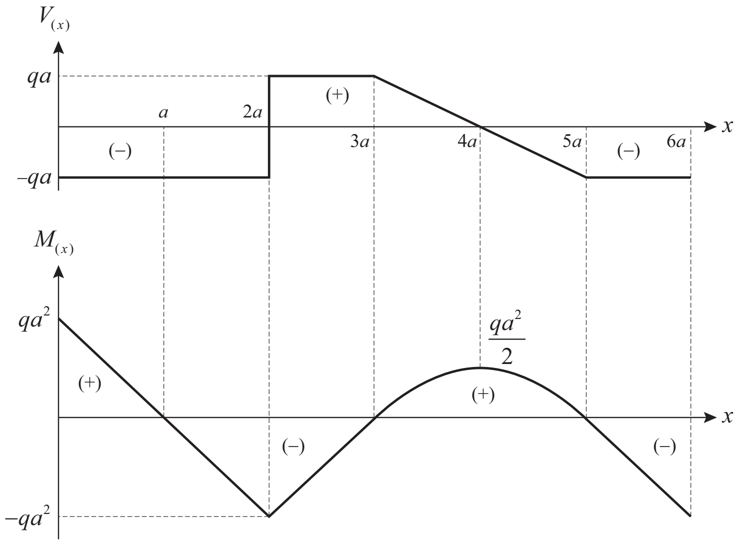
7- Draw the shear-force and bending-moment diagrams for the beam shown in the figure below.



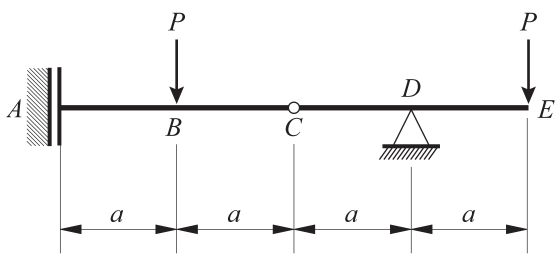


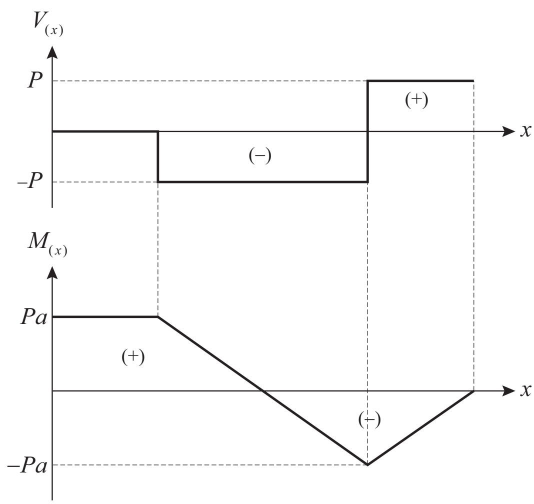
8- Draw the shear-force and bending-moment diagrams for the beam shown in the figure below.



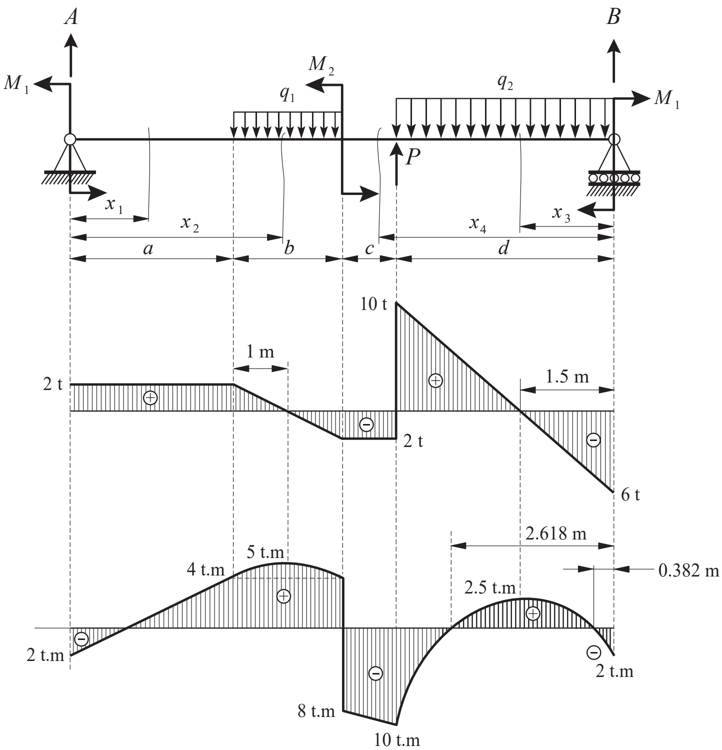


9- Draw the shear-force and bending-moment diagrams for the beam shown in the figure below.

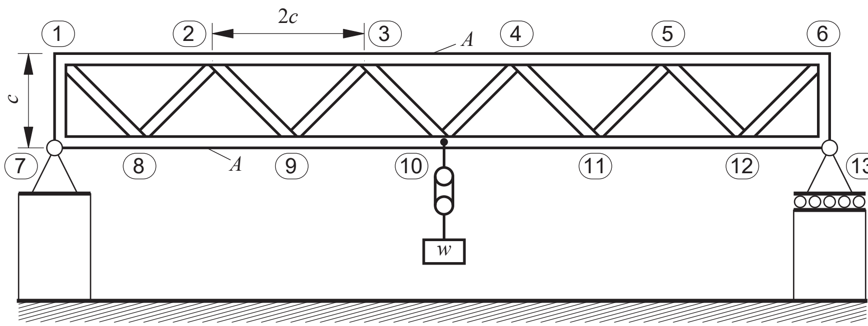


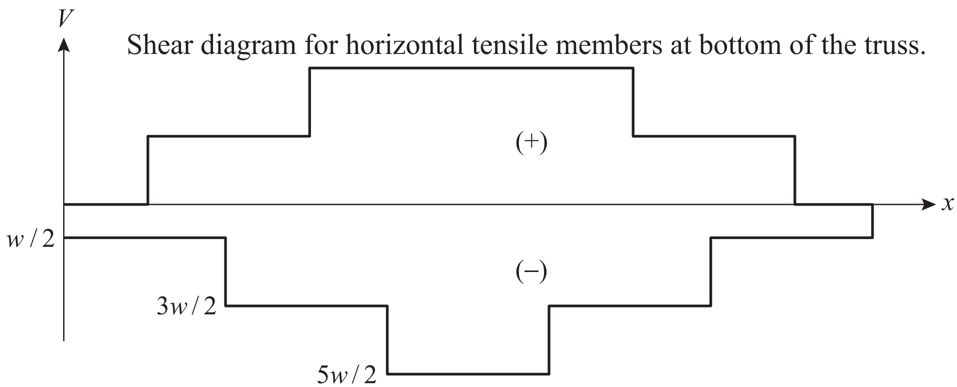


10- Assuming $M_1 = 2\ t.m$, $M_2 = 12\ t.m$, $d = 4\ m$, $c = 1\ m$, $a = 3\ m$, $b = 2\ m$, $P = 12\ t$, $q_2 = 2\ t/m$, and $q_1 = 4\ t/m$, draw the shear-force V and bending-moment M diagrams.



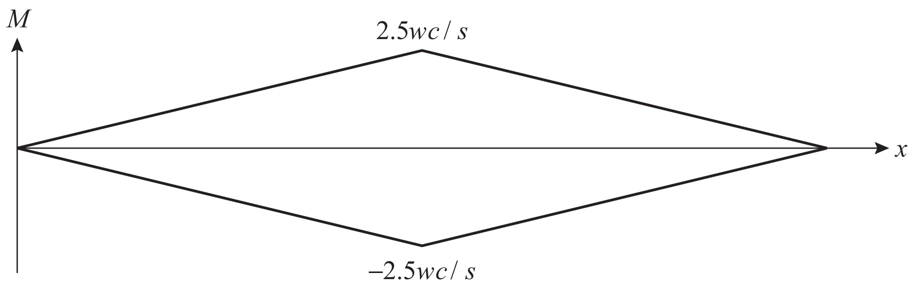
11- In the figure, the load w is carried by the roof truss at mid span. Calculate the forces in the horizontal members at the top and bottom of the truss. Also, determine the stresses induced in these members and plot them as a function of position along the span of the truss. Next, idealize the truss as a continuous beam in which only the top and bottom members are effective in bending. Using beam theory, plot the stresses developed in the top and bottom members as a function of position along the beam span, and compare this curve with the one obtained previously. If the truss is modeled as a beam, what is the role of its diagonal members?





Shear diagram for horizontal tensile members at bottom of the truss.

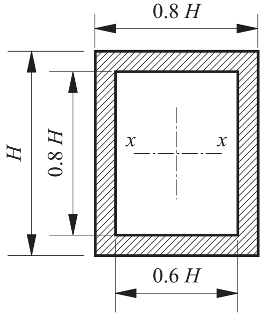
Shear diagram for horizontal compressive members at top of the truss.



Chapter 5: Bending Moment Effects in Structures

Section 2: Section Modulus and Moment of Inertia

1- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section.



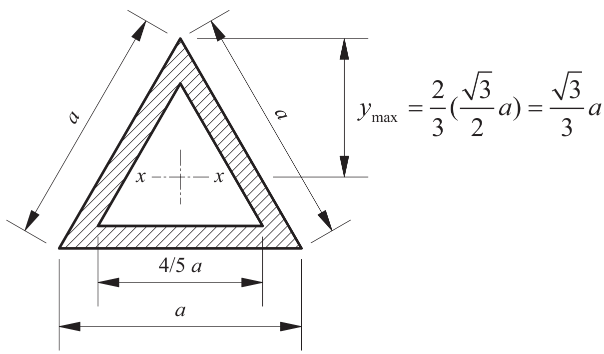
Outer rectangle: $I_x = \left[\frac{1}{12}(0.8H)(H)^3\right]$

Inner rectangle: $I_x = \left[\frac{1}{12}(0.6H)(0.8H)^3\right]$

$$\rightarrow I_x = \left[\frac{1}{12}(0.8H)(H)^3\right] - \left[\frac{1}{12}(0.6H)(0.8H)^3\right] = 0.04106\ H^4$$

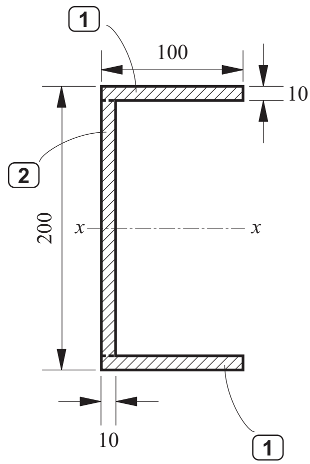
$$S_{\min} = \frac{I_x}{y_{\max}} = \frac{0.04106\ H^4}{(\frac{H}{2})} = 0.0821\ H^3$$

2- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section.



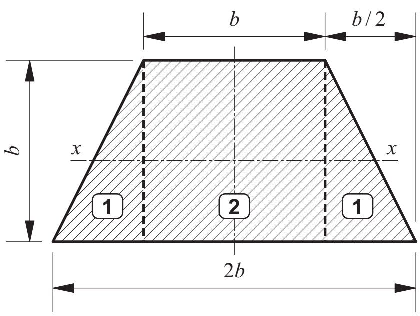
$$S_{\min} = 0.01845\ a^3$$

3- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section. The dimensions shown are in millimeters.



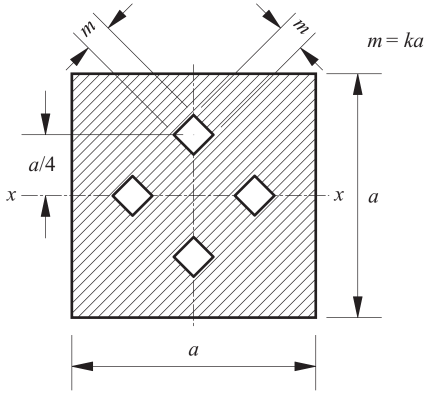
$S_{\min} = 229\ cm^3$

4- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section.



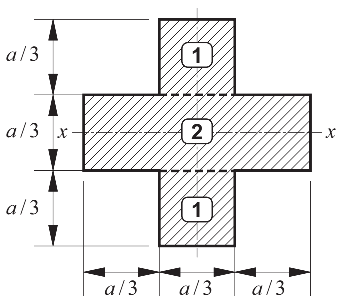
$S_{\min} = \dfrac{13}{60} b^3$

5- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section.



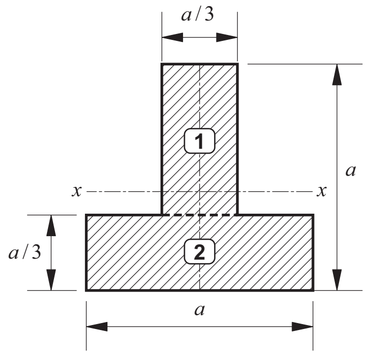
$$S_{\min} = \frac{a^3}{6}(1 - 4k^4 - \frac{3}{2}k^2)$$

6- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section.



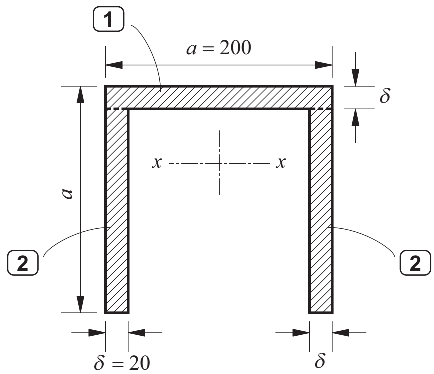
$$S_{\min} = 0.0597a^3$$

7- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section.



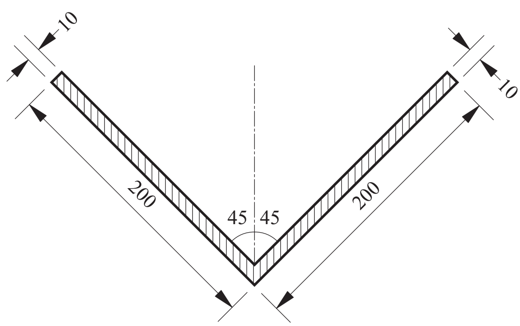
$S_{\min} = 0.0705a^3$

8- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section. The dimensions shown are in millimeters.



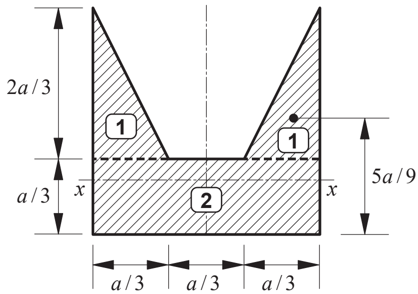
$S_{\min} = 360.25\ \text{cm}^3$

9- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section. The dimensions shown are in millimeters.



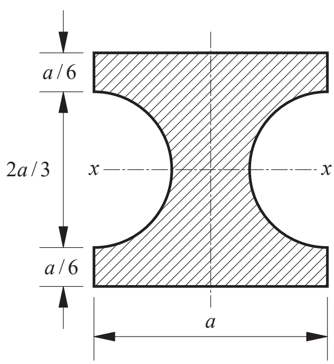
$S_{\min} = 81.87\ cm^3$

10- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section.



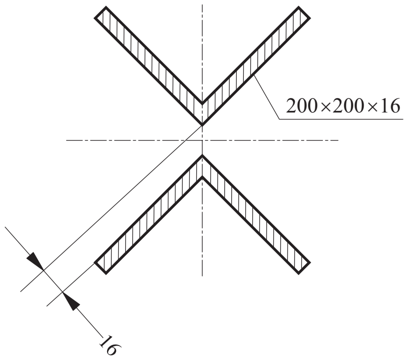
$S_{\min} = 0.0424a^3$

11- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section.



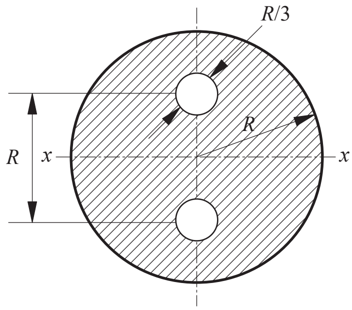
$S_{\min} = 0.147a^3$

12- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section. The dimensions shown are in millimeters.



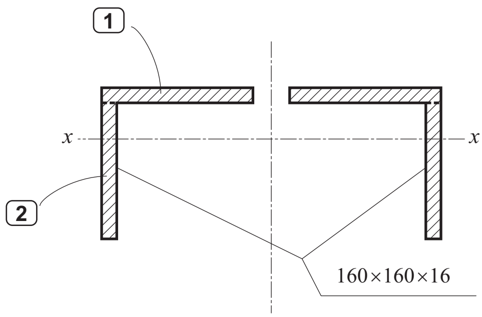
$S_{\min} = 729.08\ cm^3$

13- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section.



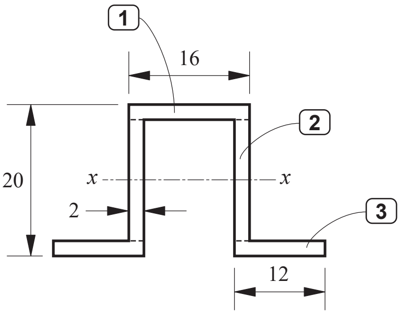
$S_{\min} = 0.74R^3$

14- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section. The dimensions shown are in millimeters.



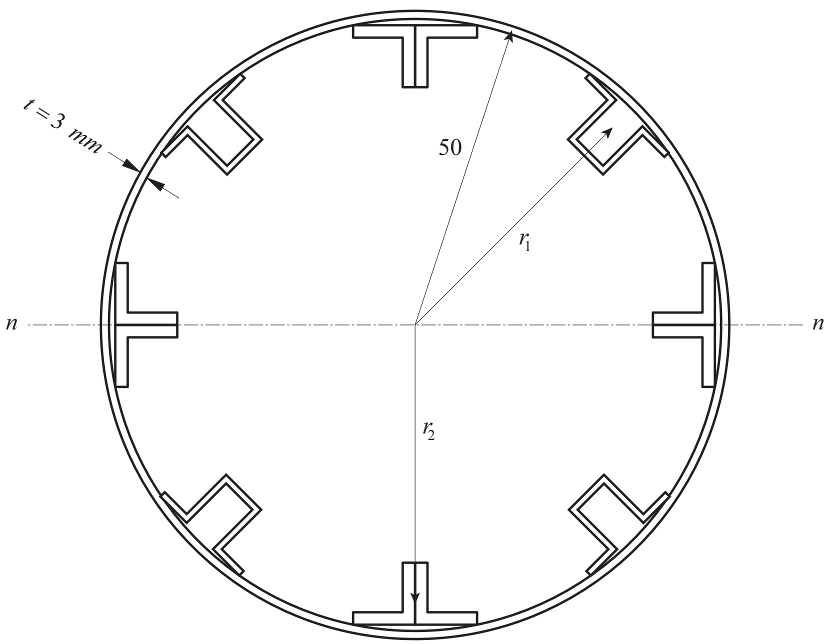
$S_{\min} = 206.8\ cm^3$

15- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section. The dimensions shown are in millimeters.



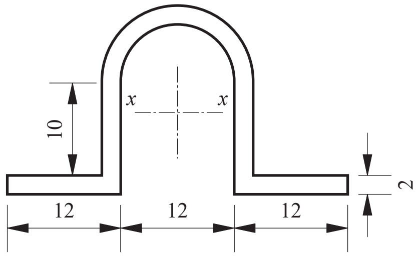
$S_{\min} = 0.703\ cm^3$

16- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section. The dimensions shown are in millimeters.



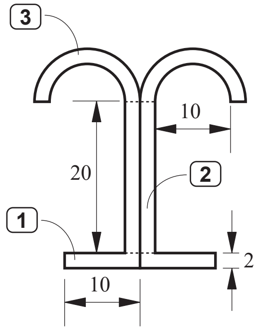
$S_{\min} = 50.84\ cm^3$

17- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section. The dimensions shown are in millimeters.



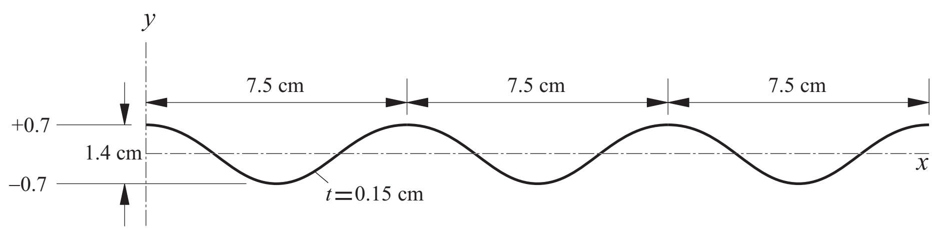
$S_{\min} = 0.51\ cm^3$

18- Calculate the section modulus ($S_{\min}$) with respect to the horizontal axis passing through the centroid of the cross-section. The dimensions shown are in millimeters.



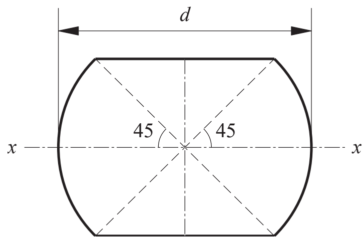
$S_{\min} = 1.35\ cm^3$

19- In the figure below, the cross-section of a corrugated sheet with a thickness of 1.5 *mm* is shown. If the section is assumed to form a sinusoidal wave, determine the section modulus per unit width of the sheet. The width 1.4 *cm* indicated in the figure represents the distance from the outer surface to the inner surface of the sheet.



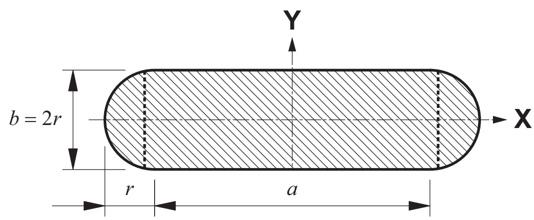
$S_0 = 0.0433\ cm^3 / cm$

20- According to the figure, a cross-section obtained by cutting the upper and lower portions of a tree trunk with diameter $d = 30\ cm$ is shown. Determine the section modulus.



$S_{\min} = 1874.4\ cm^3$

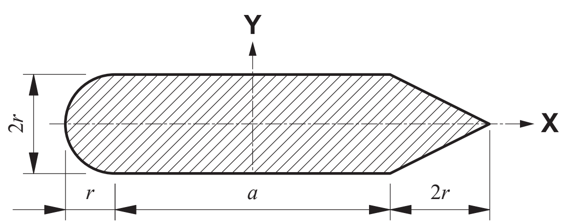
21- Determine the moments of inertia about the x and y axes for the section shown below.



$$I_x = \frac{\pi}{64}b^4 + \frac{1}{12}ab^3$$

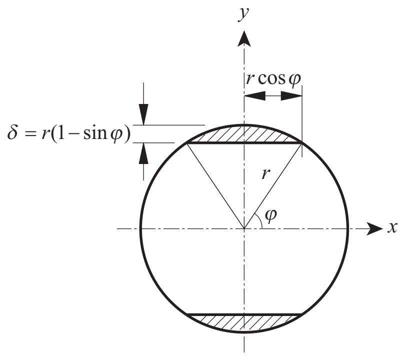
$$I_y = \frac{1}{12}ab(a^2 + 2b^2) + \frac{\pi b^2}{64}(b^2 + 4a^2)$$

22- Determine the moments of inertia about the x axis for the section shown below.



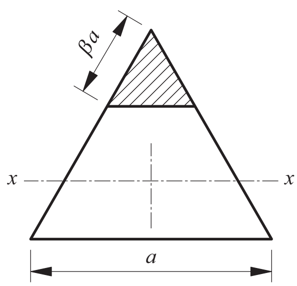
$$I_x = \frac{r^3}{24}(16a + 3\pi r + 8r)$$

23- According to the figure, a beam with a circular cross-section is subjected to bending about the x-axis. The cross-section has a diameter d. To increase the section modulus, two hatched segments of height δ are removed from the beam, as shown. Determine the value of δ that results in the maximum section modulus.

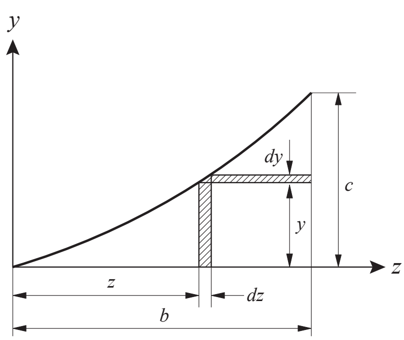


$\delta = 0.011d$

24- According to the figure, the hatched portion is removed from a triangular beam with equal sides to increase the section modulus. Determine the value of β such that the section modulus of the beam is maximized.



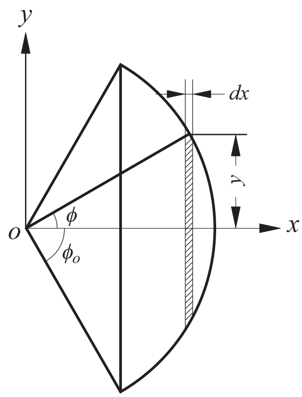
25- Calculate the coordinates of the center of the surface shown in the figure below, which is bounded by lines b and c and the parabola $y = az^n$



$$z_c = \frac{(n+1)}{(n+2)} b$$

$$y_c = \frac{(n+1)}{2(2n+2)} c$$

26- As shown in the figure below, calculate the moment of inertia of a circular sector with radius r about the x-axis.

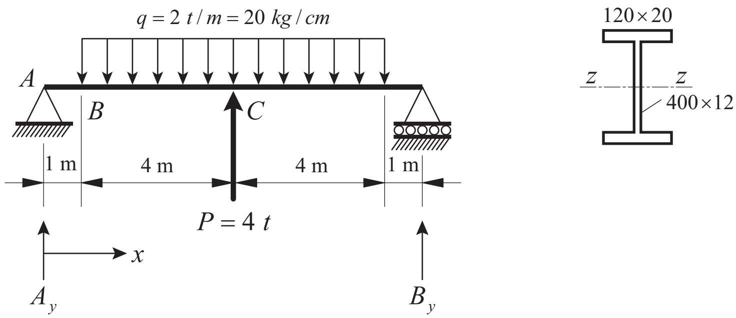


$$I_x = \frac{r^4}{48}(12\phi_0 - 8\sin 2\phi_0 + \sin 4\phi_0)$$

Chapter 5: Bending Moment Effects in Structures

Section 3: Determination of Normal Stress

1- In the figure below, determine the maximum normal stress in the critical section.



$$\begin{cases} \sum F_y = 0 \\ \sum M_A = 0 \end{cases} \rightarrow A_y = B_y = 6000\ kg$$

$$M_{BC} = 6000x - 20\frac{(x-100)^2}{2}$$

$$V = M'_{BC} = 0 \rightarrow 6000 - 2(10)(x - 100) = 0$$

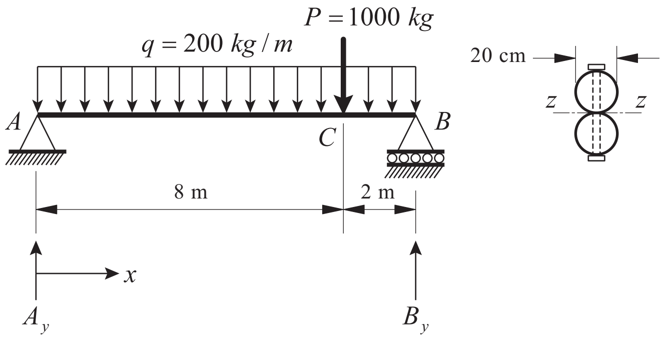
$$\rightarrow x = 400\ cm \rightarrow M_{\max} = 15\times10^5\ kg.cm$$

$$I_z = 2\left[\frac{1}{12}(120)(20)^3 + (120\times20)(210)^2\right] + \frac{1}{12}(12)(400)^3 = 275840000\ mm^4 = 27584\ cm^4$$

$$S_{\min} = \frac{27584}{22} = 1253.8\ cm^3$$

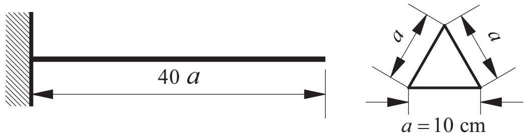
$$\sigma_{\max} = \frac{M_{\max}}{S_{\min}} = \frac{15\times10^5}{1253.8} = 1196.4\ kg/cm^2$$

2- In the figure below, determine the maximum normal stress in the critical section.



$$\sigma_{\max} = 91.7\ kg/cm^2$$

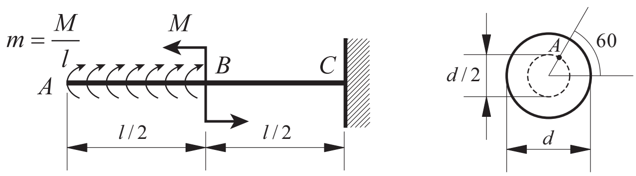
3- In the figure below, determine the ratio of the maximum normal stress to the minimum normal stress in the critical section.



$\gamma = 7.8\ g/cm^3$

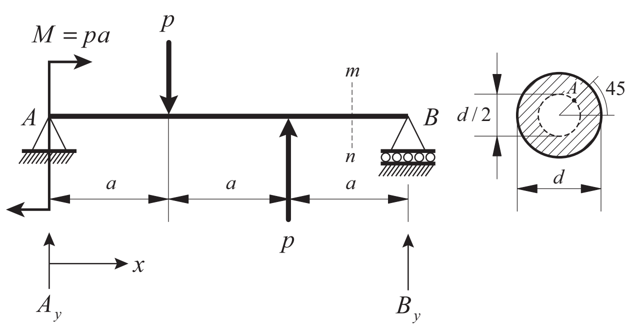
$$\frac{\sigma_{t,\max}}{\sigma_{c,\min}} = 2$$

4- In the figure below, determine the normal stress at point A of the critical section.



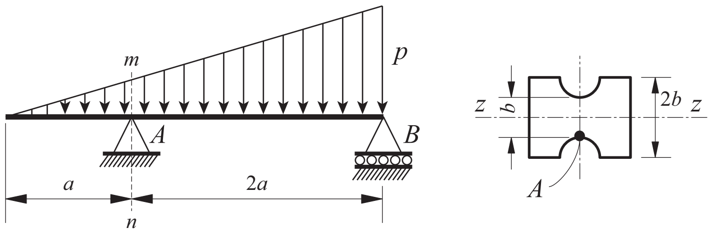
$$\sigma_A = \frac{4\sqrt{3}M}{\pi d^3}$$

5- In the figure below, determine the normal stress at point A of the critical section and point A of section m-n.



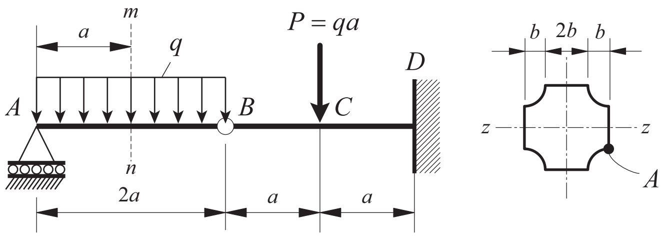
$$\sigma_{A_{mn}} = 0$$

6- In the figure below, determine the maximum normal stress at point A in the m-n section, assuming $\sigma_{\max} = \sigma_0$.



$\sigma_{A_{mn}} = 0.0894\sigma_0$

7- In the figure below, determine the maximum normal stress assuming $\sigma_{A_{mn}} = \sigma_0$ at point A in the m-n section.

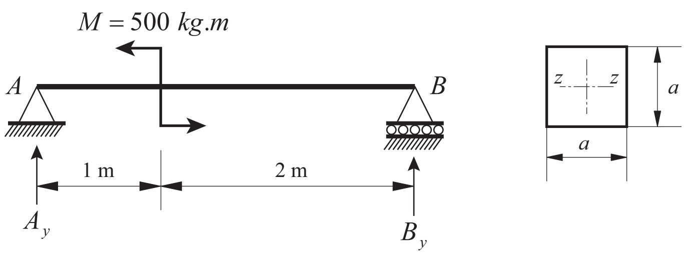


$\sigma_{\max} = 12\sigma_0$

Chapter 5: Bending Moment Effects in Structures

Section 4: Design of Beams

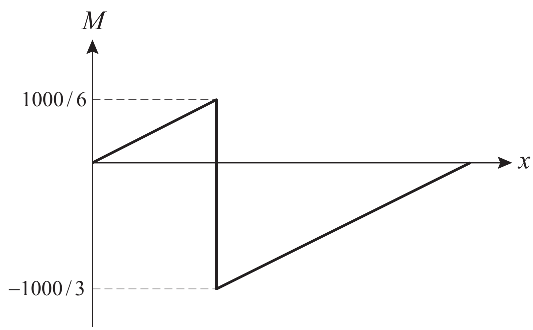
1- Determine the required cross-sectional dimensions of the beam shown in the figure under the indicated loading ($\sigma_w = 1600\ kg/cm^2$).



$$\begin{cases} \sum M_A = 0 \ \rightarrow \ 500 - B_y(3) = 0 \\ \sum F_y = 0 \ \rightarrow \ A_y + B_y = 0 \end{cases} \rightarrow \begin{cases} B_y = \dfrac{500}{3}\ kg \\ A_y = \dfrac{500}{3}\ kg \end{cases}$$

$$\begin{cases} 0 \le x \le 1 \ \rightarrow \ M_x = \dfrac{500}{3}x \\ 1 \le x \le 3 \ \rightarrow \ M_x = \dfrac{500}{3}x - 500 \end{cases}$$

The bending-moment diagram of the beam is shown as follows:

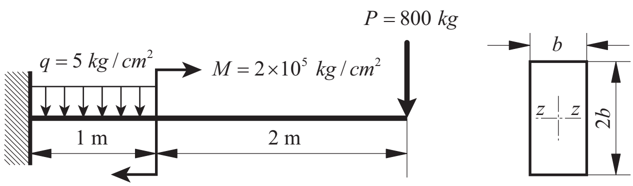


$$M_{\max} = \frac{1000}{3}\ kg.m = \frac{10^5}{3}\ kg.cm$$

$$\sigma = \frac{M_{\max} y_{\max}}{I_z} \ \rightarrow \ 1600 = \frac{\left[(\frac{10^6}{3})(\frac{a}{2}) \right]}{\left[\frac{1}{12} a^4 \right]}$$

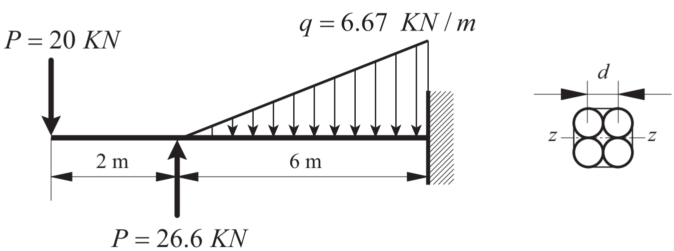
$\rightarrow \ a = 5\ cm$

2- Determine the required cross-sectional dimensions of the beam shown in the figure under the indicated loading ($\sigma_w = 100\ kg/cm^2$).



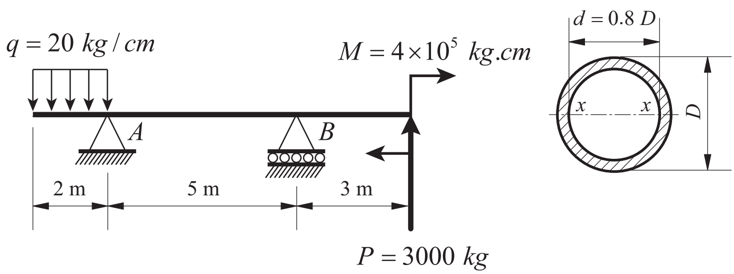
$b \simeq 19\ cm$

3- Determine the required cross-sectional dimensions of the beam shown in the figure under the indicated loading ($\sigma_w = 10\times10^6\ N/m^2$).



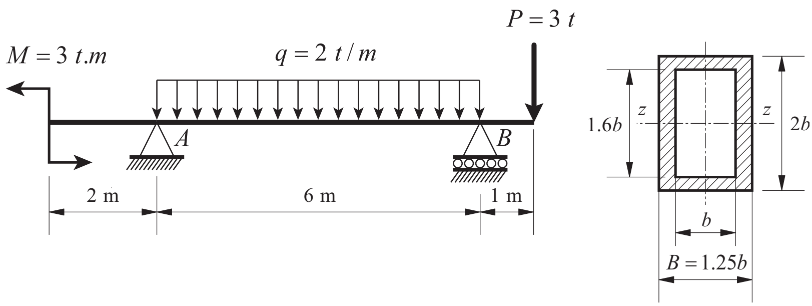
$d = 16\ cm$

4- Determine the required cross-sectional dimensions of the beam shown in the figure under the indicated loading ($\sigma_w = 1000\ kg/cm^2$).



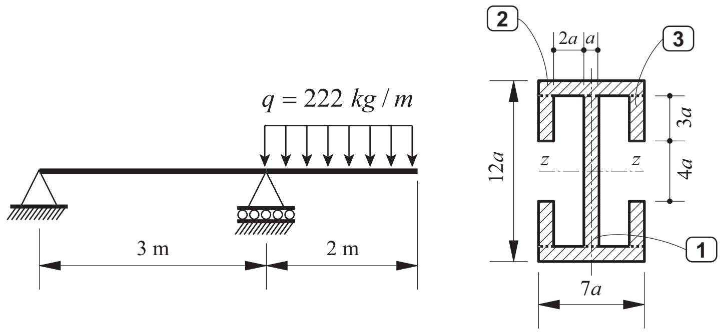
$D = 20.5\ cm$

5- Determine the required cross-sectional dimensions of the beam shown in the figure under the indicated loading ($\sigma_w = 1000\ kg/cm^2$).



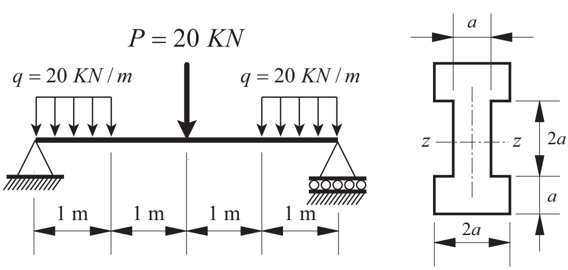
$B = 13.4\ cm$

6- Determine the required cross-sectional dimensions of the beam shown in the figure under the indicated loading. Assume that $\sigma_t = 400\ kg/cm^2$ and $\sigma_c = 1000\ kg/cm^2$.



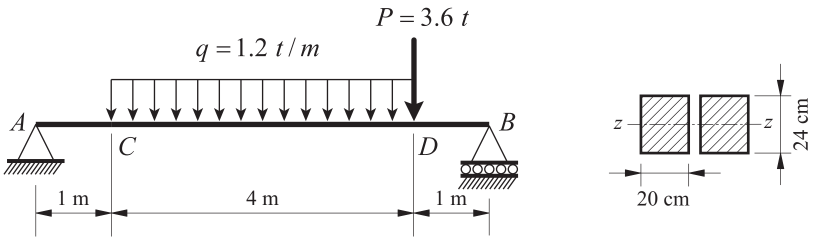
$a = 1\ cm$

7- Determine the required cross-sectional dimensions of the beam shown in the figure under the indicated loading ($\sigma_w = 150\ MN/m^2$).



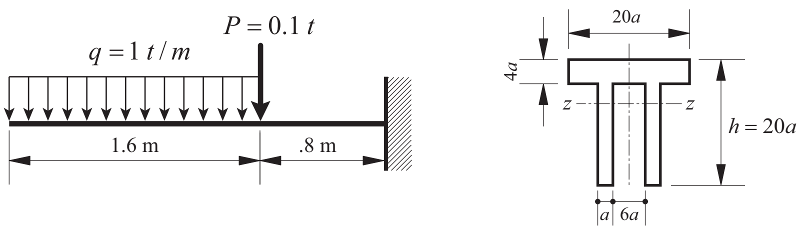
$a = 3.41\ cm$

8- Determine the number of required section for the beam shown in the figure under the indicated loading ($\sigma_w = 90\ kg/cm^2$).



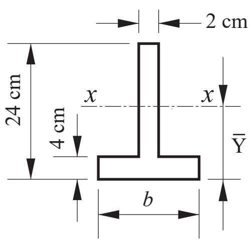
$n \approx 4$

9- Determine the required cross-sectional dimensions of the beam shown in the figure under the indicated loading. Assume that $\sigma_t = 300\ kg/cm^2$ (tensile stress), $\sigma_c = 900\ kg/cm^2$ (compressive stress) and $h = 20a$.



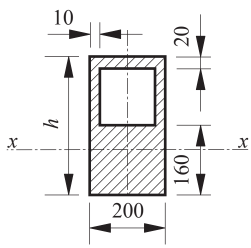
$a = 1.13\ cm$

10- In the figure below, determine the value of b such that the tensile and compressive strengths of the section are equal. Assume that the allowable compressive stress is three times the allowable tensile stress ($\sigma_c = 3\sigma_t$), with the bottom of the section in tension and the top in compression.



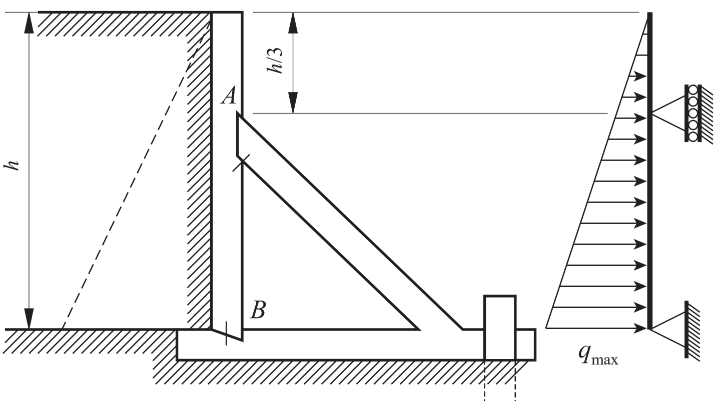
$b = 20\ cm$

11- In the figure below, determine the value of h such that the tensile and compressive strengths of the section are equal. Assume that the allowable compressive stress is twice the allowable tensile stress ($\sigma_c = 2\sigma_t$), with the bottom of the section in tension and the top in compression.



$h = 177.7\ cm \quad , \quad 38.3\ cm$

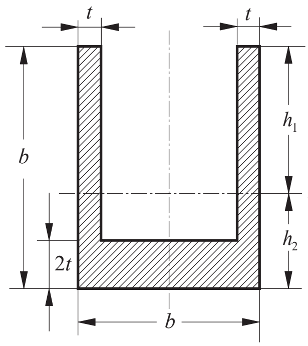
12- According to the figure, the vertical beams are spaced at intervals of 2 m (units as given in the original figure) on the foundation beams and are supported by inclined column. Consequently, the supports of the vertical beam at points A and B may be idealized as pinned. The sheeting members which are placed horizontally and perpendicularly to the vertical beams transfer the earth pressure to these supports. The axial forces in the vertical beam may be neglected. The sheeting planks, each with a width of $b = 20\ cm$, may also be idealized as pinned at two ends. The backfill height is h and equalt to 1.5 m, and the lateral earth pressure varies linearly from zero at the top to $q_{\max} = 1.3\ t/m^2$ at the base. The allowable tensile and compressive stresses are both equal to $100\ kg/cm^2$. Based on these assumptions, determine the required dimensions of the square cross-section of the vertical beam and the required thickness of the sheeting members used as the retaining wall.



$t = 6.25\ cm$

$a \simeq 11\ cm$

13- A beam with a U-shaped cross-section is made of a material with the property $\sigma_c = 3\sigma_t$. The known cross-sectional dimensions are $b = 20\ cm$ and $t = 1\ cm$. Calculate the ratio of the values of h.

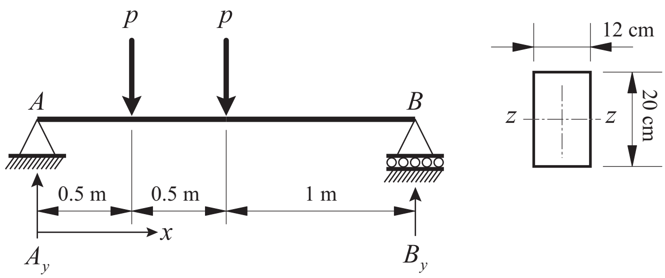


$h = 9 \pm 3$

Chapter 5: Bending Moment Effects in Structures

Section 5: Determination of Allowable Load for Beams

1- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_w = 100\ kg/cm^2$.



$$\begin{cases} \sum M_B = 0 \;\rightarrow\; A_y = 1.25p \uparrow \\ \sum F_y = 0 \;\rightarrow\; B_y = 0.75p \uparrow \end{cases}$$

$$M_z = (1.25p)(x) - (p)(x - 50) = 0.25px + 50p$$

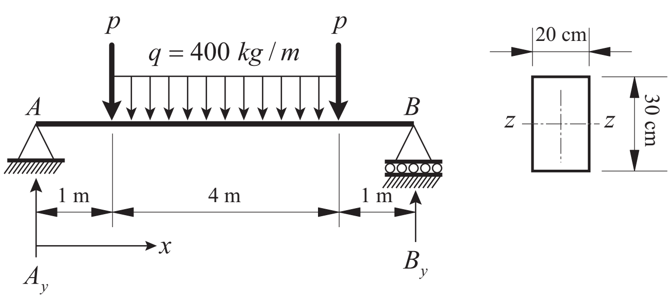
$$x = 100 \;\rightarrow\; M_{\max} = 75p$$

$$I_z = \frac{1}{12}(12)(20^3) = 8000\ cm^4$$

$$\sigma = \frac{M_{\max} y_{\max}}{I_z} \;\rightarrow\; 100 = \frac{(75p)(10)}{8000}$$

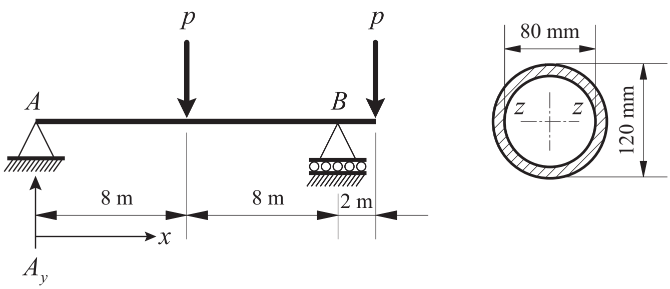
$$\rightarrow p = 1066.7\ kg$$

2- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_w = 100\ kg/cm^2$.



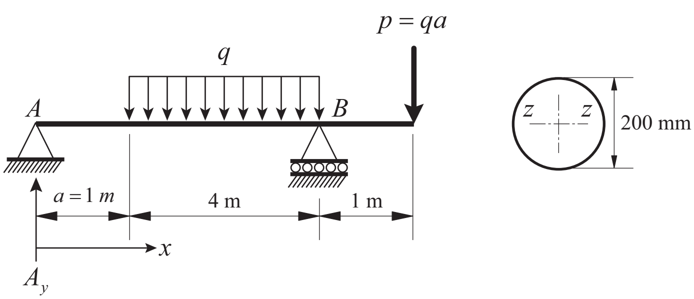
$p = 1400\ kg$

3- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_w = 1000\ kg/cm^2$.



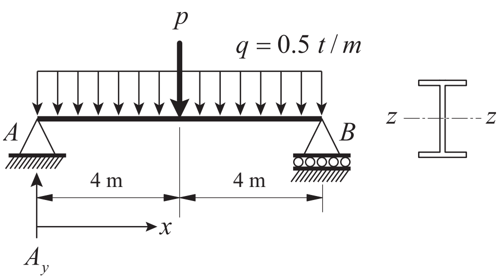
$p = 453.8\ kg$

4- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_w = 100\ kg/cm^2$.



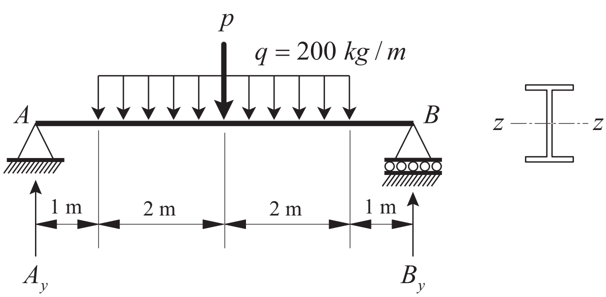
$q = 330\ kg/m$

5- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_w = 100\ kg/cm^2$ and $S_z = 518\ cm^3$.



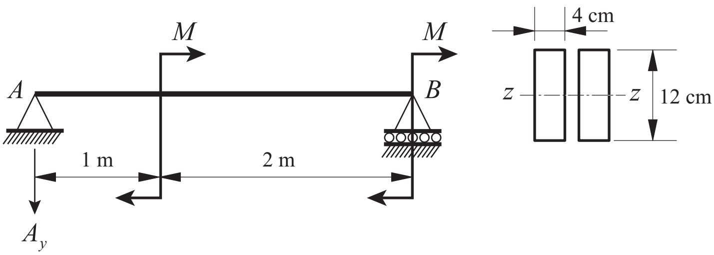
$p = 2.14\ t$

6- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_w = 1400\ kg/cm^2$ and $S_z = 289\ cm^3$.



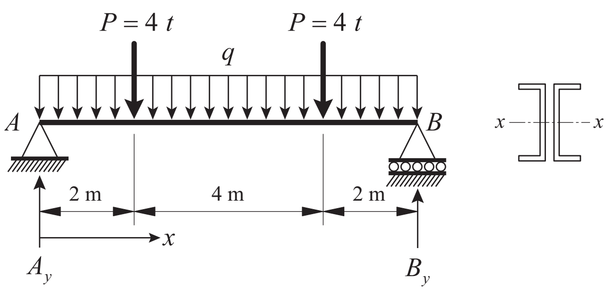
$p = 2.16\ t$

7- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_w = 10^7\ kg / cm^2$.



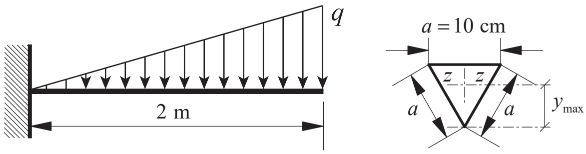
$M = 1920\ N.m$

8- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_w = 160\ MN / m^2$. The section modulus of each of the channel sections about the horizontal axis passing through the centroid is $S_x = 484\ cm^3$.



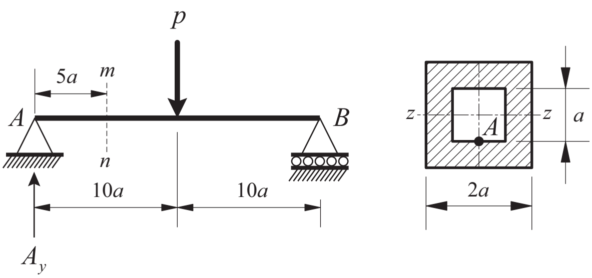
$q = 9.36\ kN / m$

9- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_w = 160\ MN / m^2$.



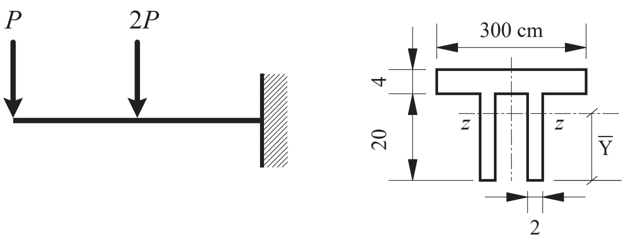
$q = 3.75\ kN/m$

10- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_{A_{mn}} = \sigma_0$.



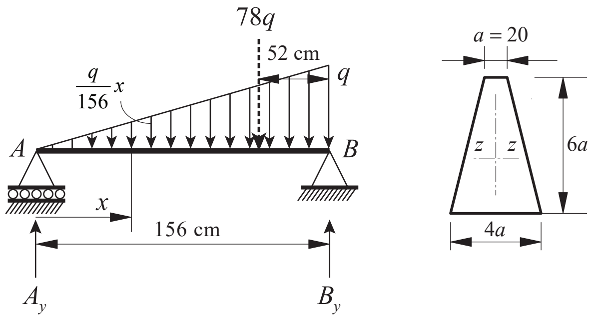
$p = a^2\sigma_0$

11- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_t = 480\ kg/cm^2$ and $\sigma_c = 1200\ kg/cm^2$.



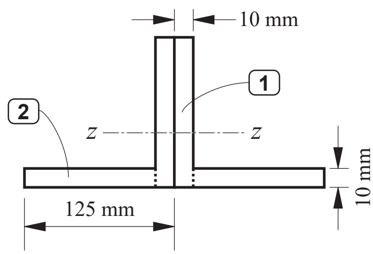
$P = 1.7\ t$

12- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_t = 400\ kg/cm^2$ and $\sigma_c = 1200\ kg/cm^2$.



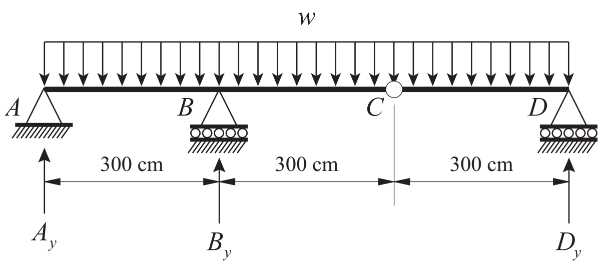
$q = 33.82\ kg / cm^2$

13- According to the figure, two angle sections of size $125 \times 125 \times 10\ mm$ are welded together. This composite section is used to resist loads acting in the vertical plane, such that bending occurs about the horizontal neutral axis. Given that the allowable tensile and compressive bending stresses are both 150 MPa, determine the maximum bending moment that the beam can support.



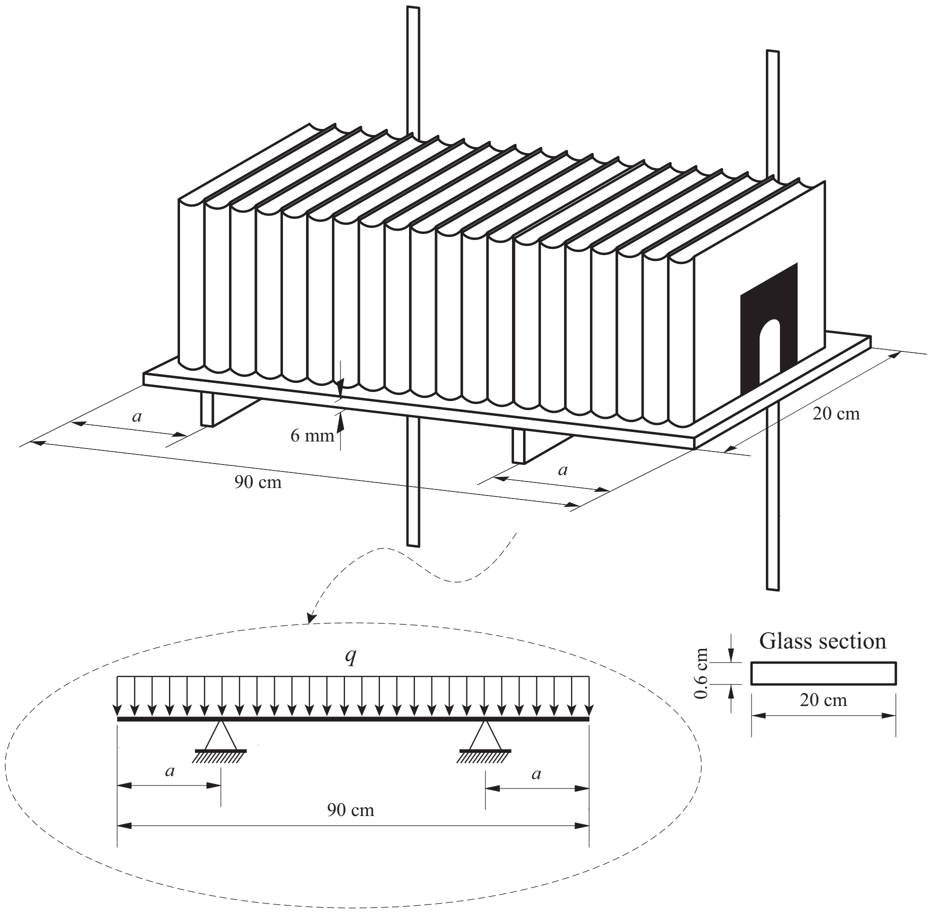
$M_{\max} = 12000\ N.m$

14- According to the figure below, calculate the allowable load on the beam, assuming $\sigma_w = 1400\ kg / cm^2$ and $S_z = 71.8\ cm^2$.



$w \simeq 112\ kg / m$

15- According to the figure, a bookshelf is constructed from glass panels with a thickness of 6 mm. For long-term use, the tensile stress on ordinary glass panels must not exceed $70\ kg/cm^2$. If the supports are placed at optimal intervals, determine the average weight of books distributed along the length of the shelf.

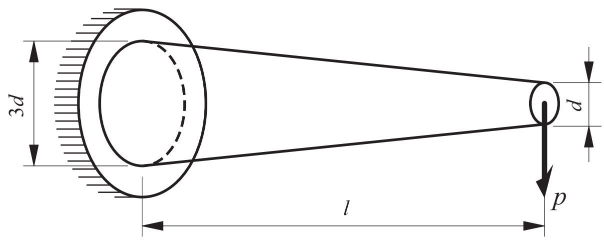


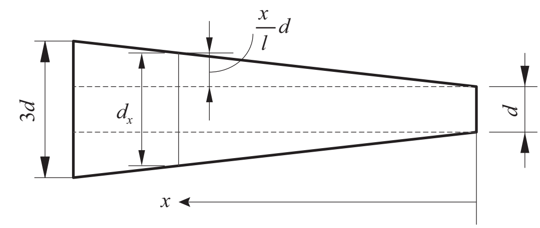
$q = 48.4\ kg/m$

Chapter 5: Bending Moment Effects in Structures

Section 6: Normal Stress in Nonprismatic Beams

1- As shown below, determined the location of the cross-section where the maximum normal stress occurs from the left end of the beam.





$$d_x = d + 2(\frac{x}{l})d$$

$$\rightarrow\ r_x = \frac{d_x}{2} = \frac{d}{2} + \frac{d}{l}x = \frac{ld + 2dx}{2l}$$

$$M_z = px$$

$$\sigma_x = \frac{M_z y_{\max}}{I_z} = \frac{(px)(r_x)}{(\frac{\pi}{4} r_x^{\,4})} = \frac{4px}{\pi r_x^{\,3}}$$

$$\rightarrow\ \sigma_x = \frac{4p}{\pi} x (\frac{2l}{2dx + ld})^3$$

$$\frac{d\sigma_x}{dx} = \frac{4p}{\pi}\left[(\frac{2l}{2dx + ld})^3 + 3\left(\frac{-2d(2l)}{(2dx + ld)^2} (\frac{2l}{2dx + ld})^2 \right) x \right]$$

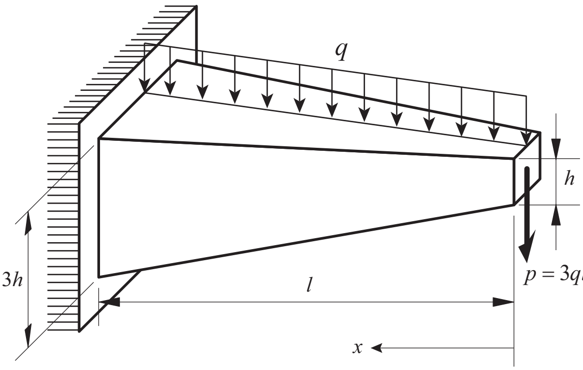
$$\frac{d\sigma_x}{dx} = 0\ \rightarrow\ (\frac{2l}{2dx + ld})^3 + (-6d)(\frac{2l}{2dx + ld})^3 \frac{x}{2dx + ld} = 0$$

$$\rightarrow\ \frac{(2l)^3 (2dx + ld) - (6d)(2l)^3 x}{(2dx + ld)^4} = 0$$

$$\rightarrow\ (2dx + ld) - 6dx = 0\ \rightarrow\ -4dx + ld = 0$$

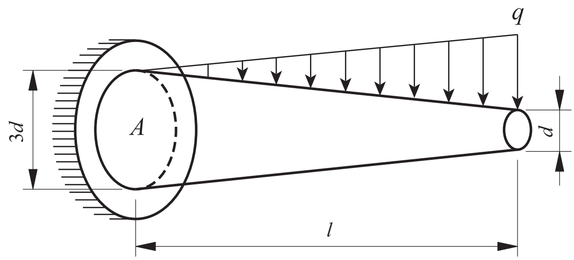
$$\rightarrow\ x = \frac{l}{4}$$

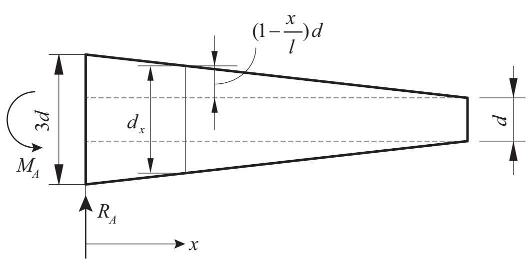
2- As shown below, determined the location of the cross-section where the maximum normal stress occurs from the left end of the beam (The cross-sectional width is taken as b).



$$x = \frac{2}{5}l$$

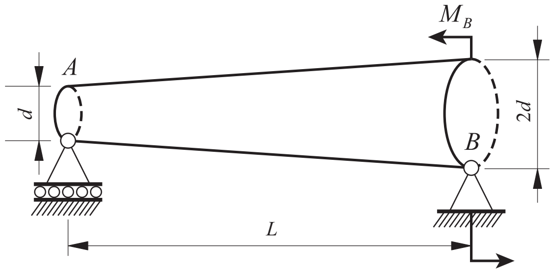
3- As shown below, determined the location of the cross-section where the maximum normal stress occurs from the left end of the beam.





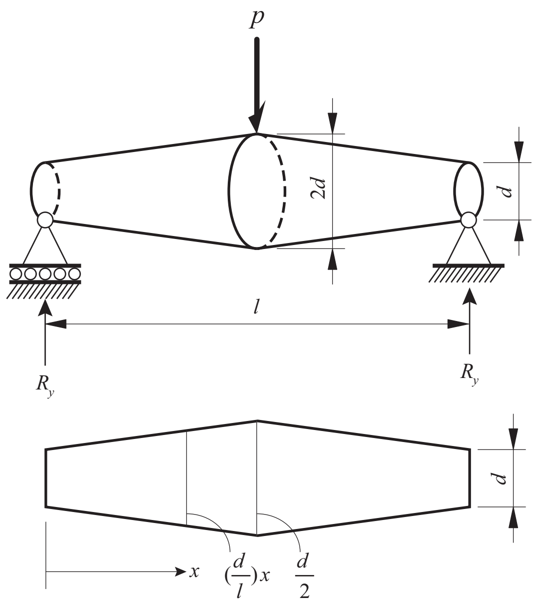
$$x = \frac{l}{3}$$

4- As shown below, determined the location of the cross-section where the maximum normal stress occurs from the left end of the beam



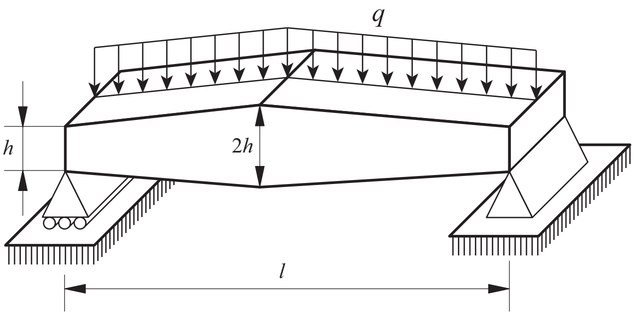
$$x = \frac{L}{2}$$

5- As shown below, determined the location of the cross-section where the maximum normal stress occurs from the left end of the beam



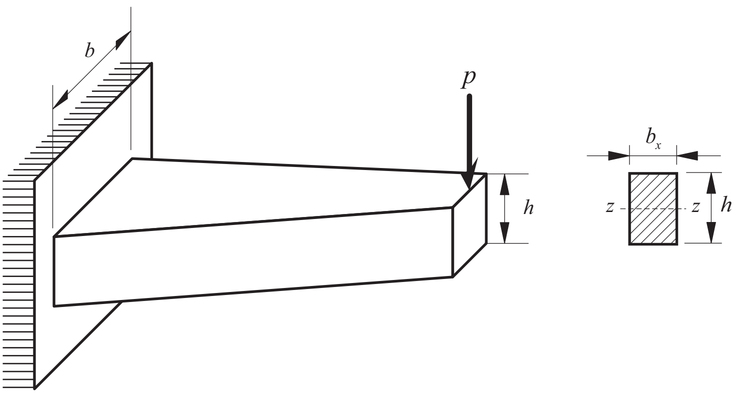
$$x = \frac{l}{4}$$

6- As shown below, determined the location of the cross-section where the maximum normal stress occurs from the left end of the beam



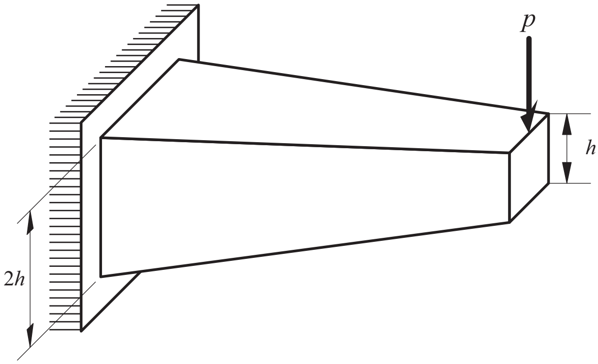
$$x = \frac{l}{4}$$

7- As shown below, a concentrated load p is applied at the free end of a nonprismatic cantilever beam with constant depth h and variable width b. If the maximum normal stress along the beam is to remain constant and equal to σ_w, express the beam width as a function of x measured from the free end.



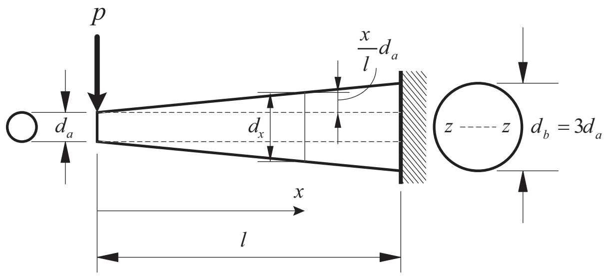
$$b_x = \frac{6px}{\sigma_w h^2}$$

8- As illustrated below, a concentrated load p is applied at the free end of a nonprismatic cantilever beam with a square cross-section. The beam's cross-sectional width and depth vary linearly from h at the free end to $2h$ at the fixed end. Determine the maximum normal stress induced by bending.



$\sigma_{max} \frac{8}{9}\frac{pl}{h^3}$

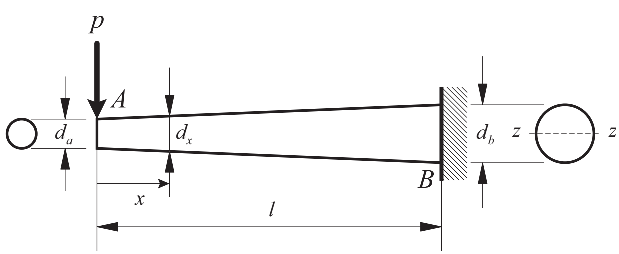
9- If in the non-prismatic cantilever beam shown, the ratio $d_b / d_a = 3$ is satisfied, calculate the maximum normal stress developed due to bending. Also, compare the obtained stress with the maximum stress σ_b at the support.



$$\sigma_{max} = \frac{64}{27}\frac{pl}{\pi d_a^3}$$

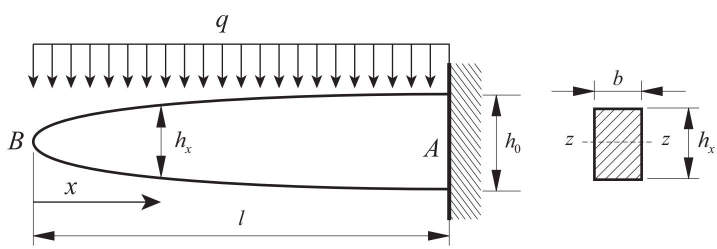
$$\sigma_{max} = 2\sigma_b$$

10- In the beam shown, for which values of the ratio d_b / d_a will the maximum normal stress occur at the support?



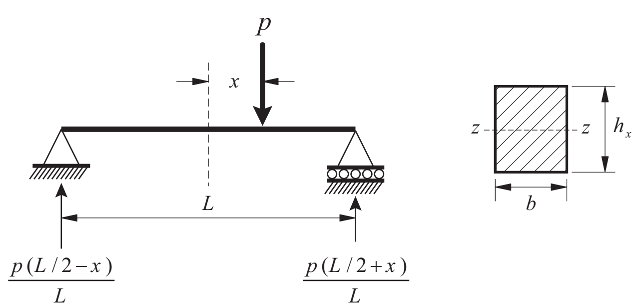
$$\frac{d_b}{d_a} = \frac{3}{2}$$

11- A nonprismatic cantilever beam with a rectangular cross-section of constant width b and variable depth h_x is subjected to a uniformly distributed load of intensity q, as shown. Determine the variable depth h_x such that the maximum normal stress along the beam is kept constant and equal to the allowable stress σ_w.



$$h_x = x\sqrt{\frac{3q}{b\sigma_w}}$$

12- As shown below, a simply supported beam of span L with a rectangular cross-section of constant width b and variable depth h is subjected to a concentrated load p. If the maximum normal stress along the beam is to be kept constant and equal to σ_w, expressed the variable depth of the beam as a function of x measured from the free end.



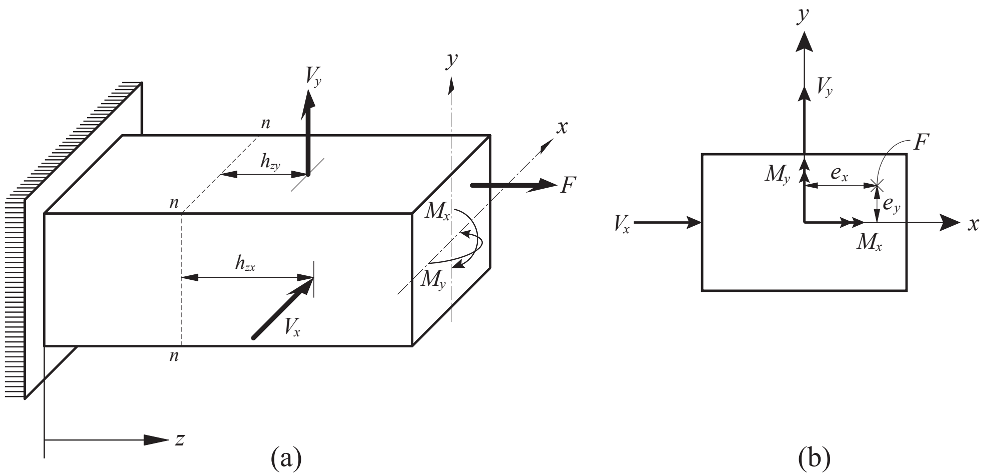
$$h_x = \left[\frac{3pL}{2b\sigma_w}(1-\frac{4x^2}{L^2})\right]^{1/2}$$

Chapter 5: Bending Moment Effects in Structures

Section 7: Combined Loading

Extreme stress in polygonal sections

As shown in the figure below, the cross-section *n-n* is subjected to two bending moments M_x and M_y, a tensile force F with eccentricities e_x and e_y, and shear forces V_x and V_y.



(a) (b)

According to Figure (b), the force F is assumed to be tensile. The bending moments M_x and M_y, as well as the shear forces V_x and V_y, are assumed to act in the positive directions of the *x* and *y* axes. Under these conditions, the normal stress σ_z at any point of the cross-section *n-n* is obtained from the following general relationship:

$$\sigma_z = \frac{F}{A} + \left(\frac{M_x - V_y h_{zy} + F e_y}{I_x}\right) y + \left(\frac{-M_y - V_x h_{zx} + F e_x}{I_y}\right) x$$

Extreme stress in circular sections

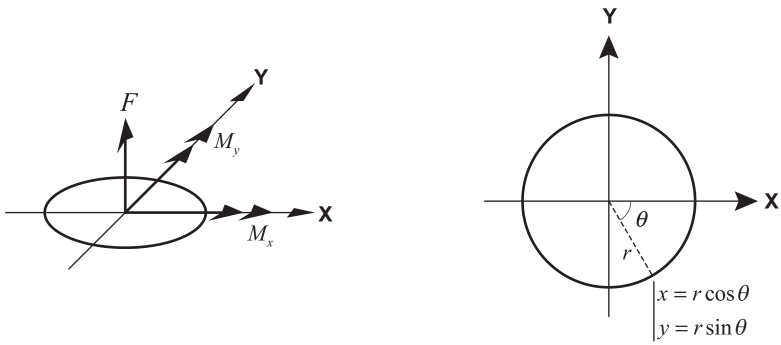
Unlike polygonal sections, circular sections do not have corners. Assuming positive directions for the forces and moments (as shown in the figure below), it is assumed that the extreme compressive stress occurs at an angle denoted by θ. This angle is calculated as follows.

$$\sigma_z = \frac{F}{A} + \left(\frac{M_x}{I_x}\right) y + \left(\frac{-M_y}{I_y}\right) x$$

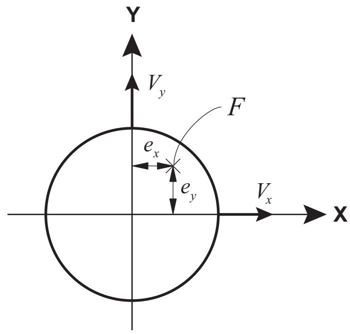
$$\rightarrow \ \sigma_z = \frac{F}{A} + \left(\frac{M_x}{I_x}\right)(r\sin\theta) + \left(\frac{-M_y}{I_y}\right)(r\cos\theta)$$

$$\frac{d\sigma_z}{d\theta} = 0 \ \rightarrow \ \left(\frac{M_x}{I_x}\right)(r\cos\theta) + \left(\frac{-M_y}{I_y}\right)(-r\sin\theta) = 0$$

$$I_x = I_y \ \rightarrow \ \tan\theta = \frac{-M_x}{M_y} \ \rightarrow \ \theta = \tan^{-1}\left(\frac{-M_x}{M_y}\right)$$

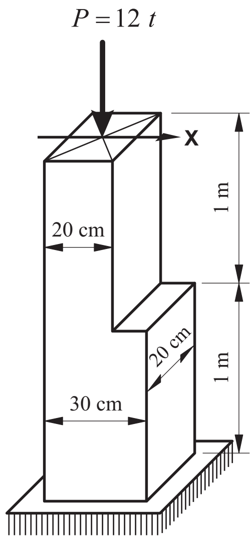


If the circular section is loaded as shown below, the above relationship takes the following form:



$$\theta = \tan^{-1}\left(\frac{M_x - V_y h_{zy} + F e_y}{-M_y - V_x h_{zx} + F e_x}\right)$$

1- Calculate the minimum and maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section.



$A = (20)(30) = 600\ cm^2$

$I_x = \dfrac{1}{12}(30)(20)^3 = 20000\ cm^4 \quad , \quad I_y = \dfrac{1}{12}(20)(30)^3 = 45000\ cm^4$

$e_x = -5\ cm \quad , \quad e_y = 0$

$\begin{cases} F = -12\ t = -12000\ kg \\ M_x = M_y = V_x = V_y = 0 \end{cases}$

$$\sigma_z = \frac{F}{A} + \left[\frac{M_x - V_y h_{zy} + Fe_y}{I_x}\right] y + \left[\frac{-M_y - V_x h_{zx} + Fe_x}{I_y}\right] x$$

In the above equation, the *x*-coordinates of points *A* to *D* are substituted:

$$\sigma_z = \frac{-12000}{600} + \frac{(-12000)(-5)}{4500} x$$

$A\left|\begin{matrix} 15 \\ 10 \end{matrix}\right. \rightarrow \sigma_A = \dfrac{-12000}{600} + \dfrac{(-12000)(-5)}{45000}(15) = 0 = \sigma_{\max}$

$B\left|\begin{matrix} -15 \\ 10 \end{matrix}\right. \rightarrow \sigma_B = \dfrac{-12000}{600} + \dfrac{(-12000)(-5)}{45000}(-15) = -40\ kg/cm^2 = \sigma_{\min}$

$$C\left|\begin{matrix}-15\\-10\end{matrix}\right. \rightarrow \sigma_C = \frac{-12000}{600} + \frac{(-12000)(-5)}{45000}(-15) = -40\ kg/cm^2$$

$$D\left|\begin{matrix}15\\-10\end{matrix}\right. \rightarrow \sigma_D = \frac{-12000}{600} + \frac{(-12000)(-5)}{45000}(15) = 0$$

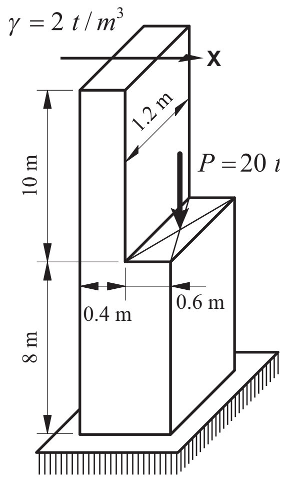
The neutral axis equation is calculated as follows:

$$\sigma_z = 0$$

$$\rightarrow \frac{-12000}{600} + \frac{(-12000)(-5)}{4500}x = 0$$

$$\rightarrow x = 15\ cm$$

2- Calculate the minimum and maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section.

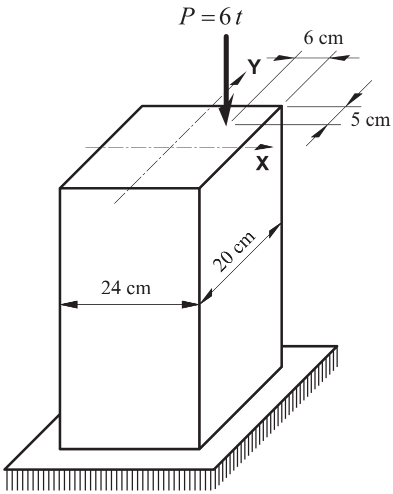


$$\sigma_{max} = -4.63\ kg/cm^2$$

$$\sigma_{min} = -4.51\ kg/cm^2$$

$$x = -363\ cm$$

3- Calculate the minimum and maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section.



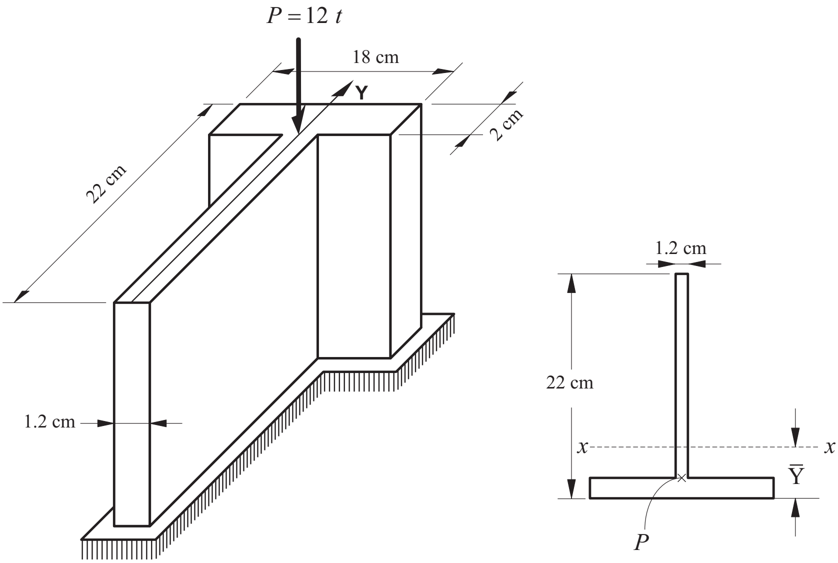
$\sigma_A = -50\ kg/cm^2 = \sigma_{\min}$

$\sigma_B = -12.5\ kg/cm^2$ $\qquad y = 0$

$\sigma_C = +25\ kg/cm^2 = \sigma_{\max}$ $\qquad x = -8\ cm$

$\sigma_D = -12.5\ kg/cm^2$

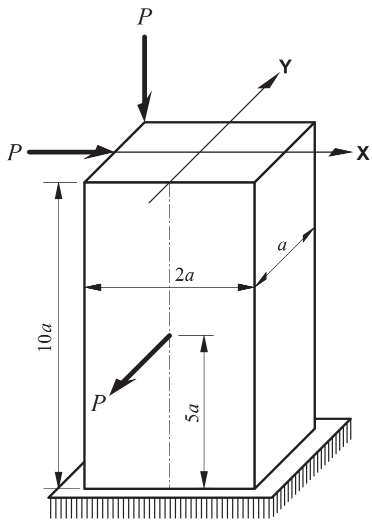
4- Calculate the minimum and maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section.



$\sigma_{\min} = -286.25\ kg/cm^2 \qquad \sigma_{\max} = +65.14\ kg/cm^2$

$y = -12.52\ cm$

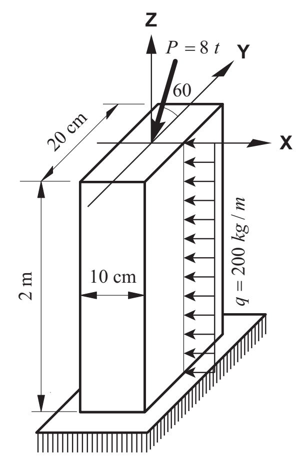
5- Calculate the minimum and maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section.



$\sigma_A = -0.5\dfrac{P}{a^2} \qquad \sigma_B = +26.5\dfrac{P}{a^2} = \sigma_{\max} \qquad \sigma_C = -0.5\dfrac{P}{a^2} \qquad \sigma_D = -27.5\dfrac{P}{a^2} = \sigma_{\min}$

$(0, a/54) \quad (-a/27, 0)$

6- Calculate the minimum and maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section.

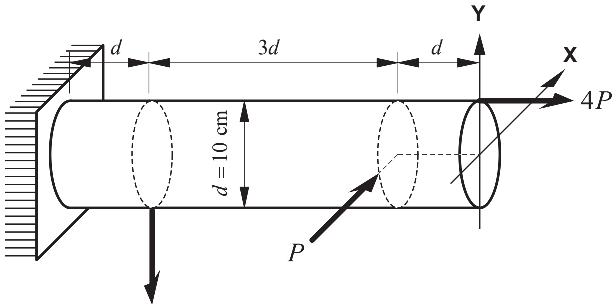


$$\sigma_C = 1285.35\ kg/cm^2$$

$$\sigma_A = -1354.65\ kg/cm^2$$

$(0, 0.29)$ $(1.44, 0)$

7- Calculate the minimum and maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section.

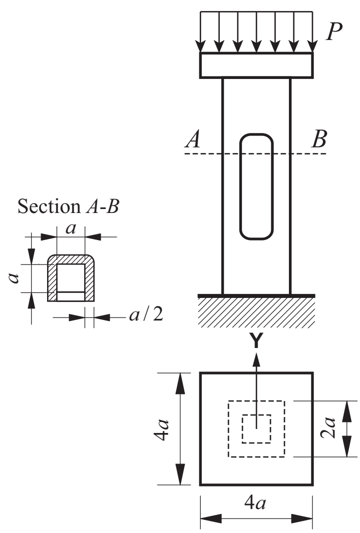


$$\sigma_{z,\min} = -14.4\ MN/m^2$$

$$\sigma_{z,\max} = 17.6\ MN/m^2$$

$(0, -0.833)$ $(0.625, 0)$

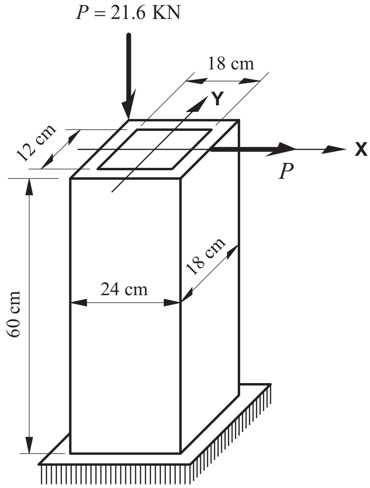
8- Calculate the minimum and maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section.



$\sigma_{\min} = -9.46P \qquad \sigma_{\max} = -4.14P$

$y = 2.41a$

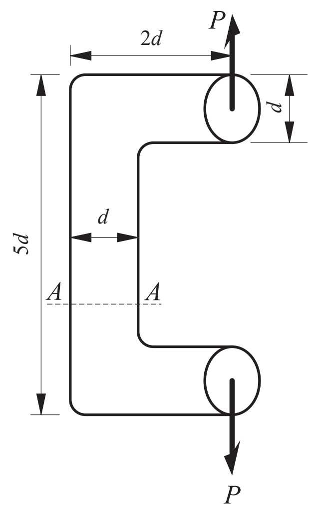
9- Calculate the minimum and maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section.



$\sigma_{\min} = -11.3\ MN/m^2 \qquad \sigma_{\max} = +9.3\ MN/m^2$

$(0, -4.67) \quad (-1.43, 0)$

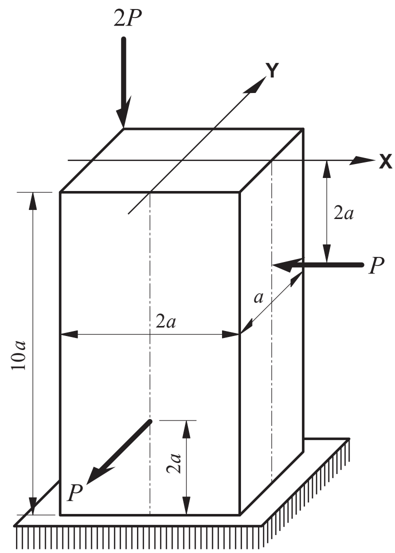
10- Calculate the maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section. The minimun normal stress is $\sigma_{\min} = 550\ kg/cm^2$.



$$\sigma_{\max} = 650\ kg/cm^2\ (T)$$

$$x = \frac{-d}{24}$$

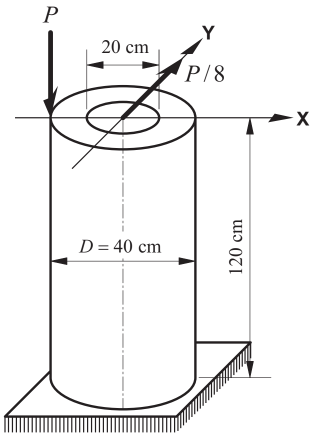
11- Calculate the maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section. The minimun normal stress is $\sigma_{\min} = 50\ kg/cm^2$.

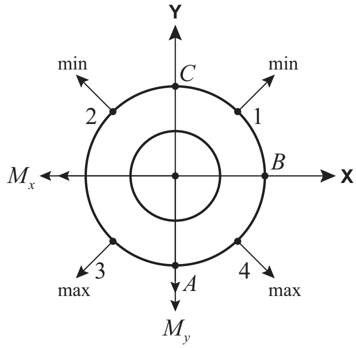


$$\sigma_{\max} = 44.73\ kg/cm^2$$

$(0, a/6)\quad (a/15, 0)$

12- Calculate the maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section. The minimun normal stress is $\sigma_{\min} = 500\ kg/cm^2$.

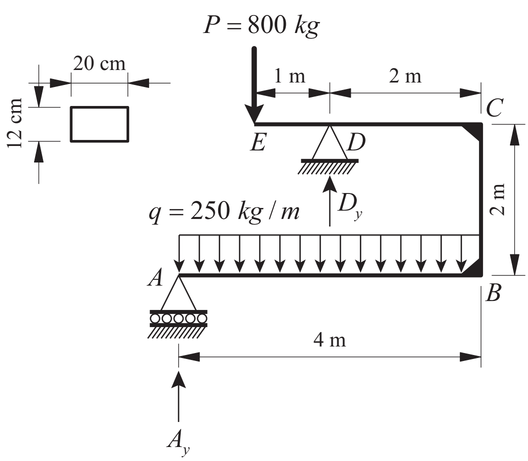




$\sigma_{max} = 298.4\ kg/cm^2$

$(0, -8.33) \quad (6.25, 0)$

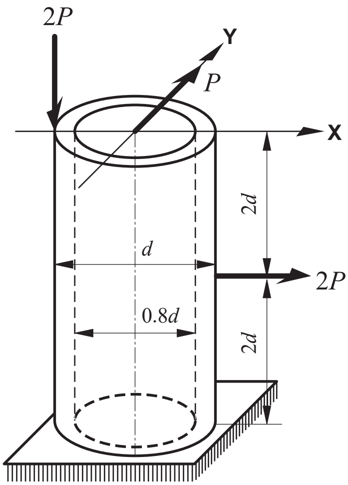
13- Calculate the minimum and maximum normal stresses developed in the most critical section of the bars.



$\sigma_{max} = 85.8\ kg/cm^2$

$\sigma_{min} = -80.8\ kg/cm^2$

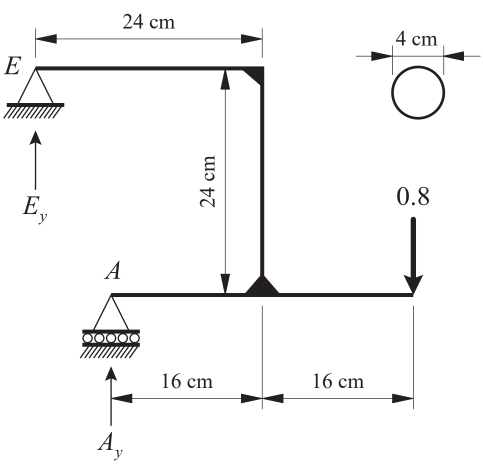
14- Calculate the maximum normal stresses developed in the structure shown below and determine the location of the neutral axis in its most critical section.



$$\sigma_{\min} = \frac{-93.3P}{d^2}$$

$$\sigma_{\max} = \frac{79.2P}{d^2}$$

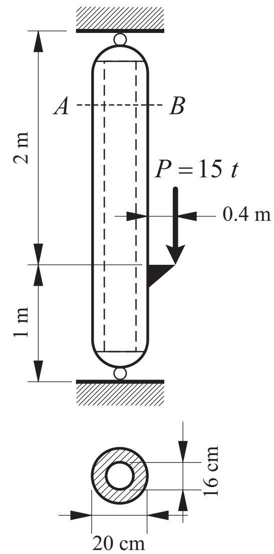
15- Calculate the minimum and maximum normal stresses developed in the most critical section of the bars.



$$\sigma_{\max} = 124.8\ KN/m^2$$

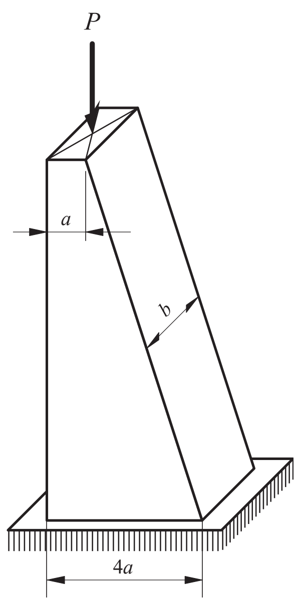
$$\sigma_{\min} = -119.7\ KN/m^2$$

16- Calculate the minimum and maximum normal stresses developed in the most critical section.



$\sigma_{\max} = 1123\ kg / cm^2 \qquad \sigma_{\min} = -1034.6\ kg / cm^2$

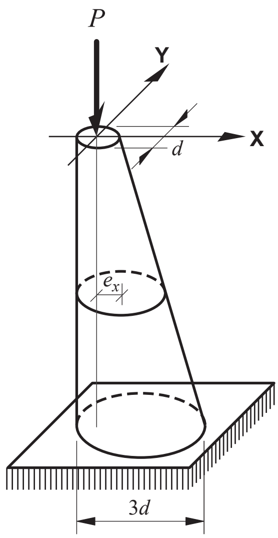
17- Calculate the minimum and maximum normal stresses developed in the most critical section.



$$\sigma_z = \frac{P}{3ab} \ (T)$$

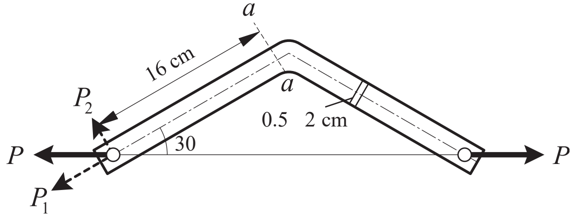
$$\sigma_z = \frac{-4P}{3ab} \ (C)$$

18- Calculate the minimum and maximum normal stresses developed in the most critical section.



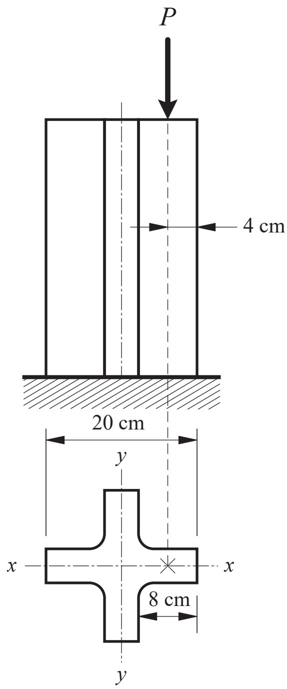
$$\sigma_z = \frac{P}{\pi d^2} \ (T)$$

19- Calculate the allowable load P in the figure below assuming $\sigma = 1600\ kg / cm^2$.



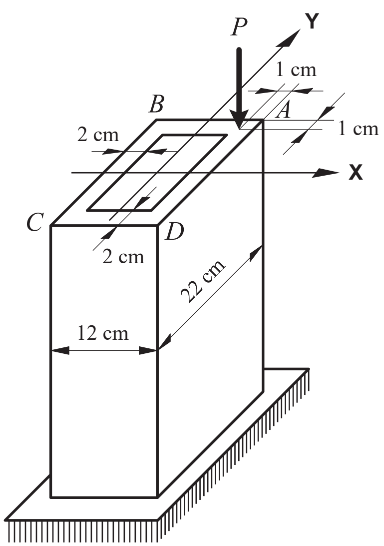
$P_{all} = 64.3\ kg$

20- Calculate the allowable load P in the figure below assuming $\sigma = 800\ kg / cm^2$.



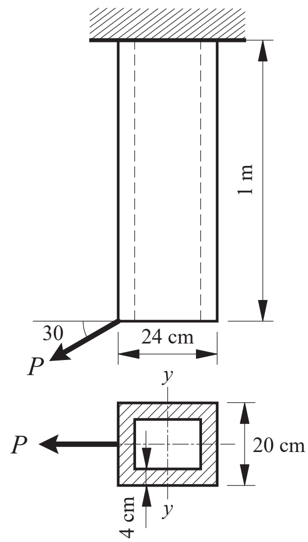
$P_{all} = 27874\ kg$

21- Calculate the allowable load P in the figure below assuming $\sigma_c = -1000\ kg/cm^2$ and $\sigma_t = 400\ kg/cm^2$.



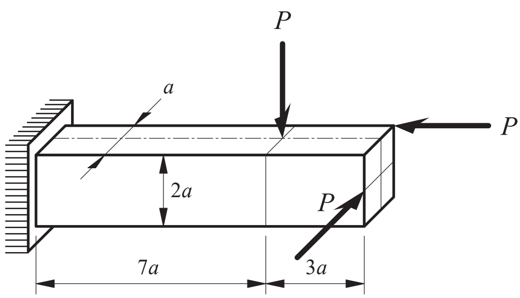
$P_{all} = 19570\ kg$

22- Calculate the allowable load P in the figure below assuming $\sigma_w = 100\ kg/cm^2$.



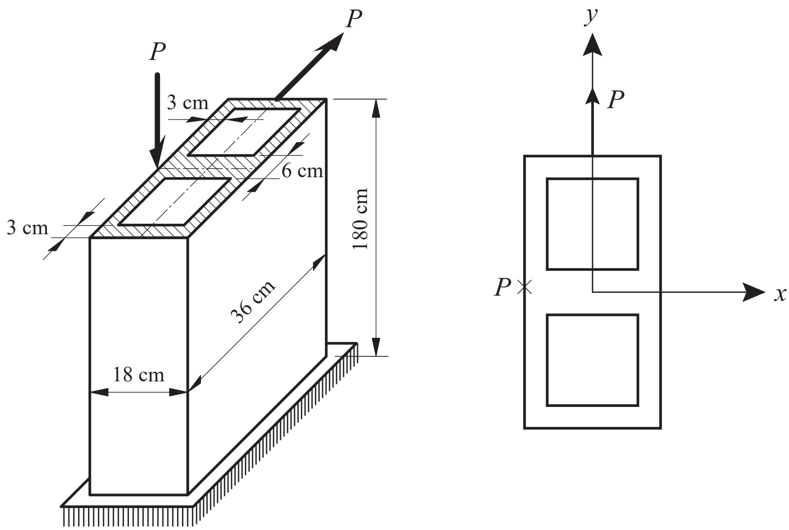
$P_{all} = 1894\ kg$

23- Calculate the allowable load P in the figure below assuming $\sigma_c = 120\ kg/cm^2$, $\sigma_t = 100\ kg/cm^2$ and $a = 12\ cm$.



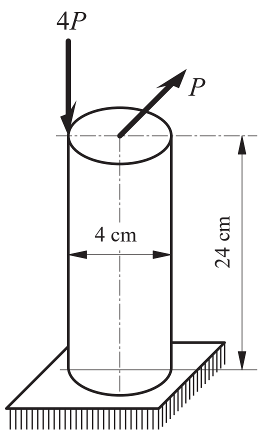
$P_{all} = 360\ kg$

24- Calculate the allowable load P in the figure below assuming $\sigma = 500\ kg/cm^2$.



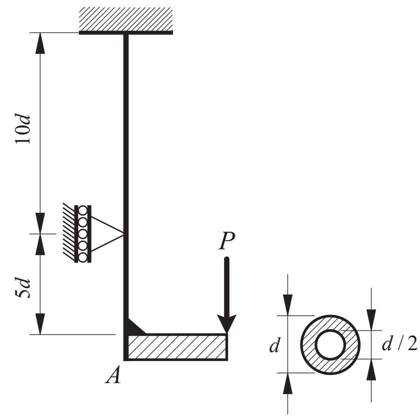
$P_{all} = 5988\ kg$

25- Calculate the allowable load P in the figure below assuming $\sigma_w = 1600\ kg/cm^2$.



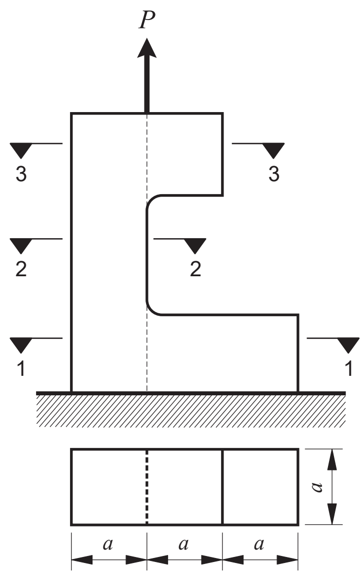
$P_{all} = 368.3\ kg$

26- Calculate the allowable load P in the figure below.



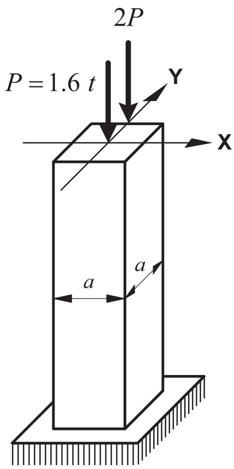
$$P_{all} = \frac{\pi d^2}{170.6}\sigma$$

27- Determine length a such that the allowable tensile and compressive stresses are both equal to σ_w.



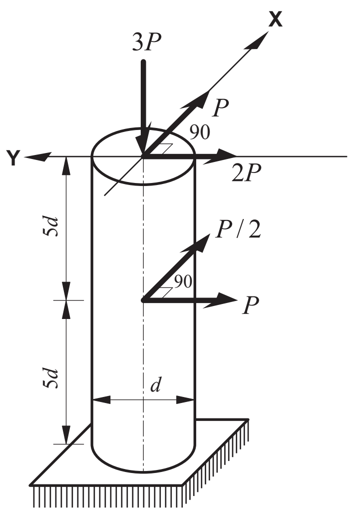
$$a \geq \sqrt{\frac{P}{\sigma_w}}$$

28- For the column shown below, calculate the side length of the column (a) such that the allowable tensile and compressive stresses are $300\ kg/cm^2$ and $1200\ kg/cm^2$, respectively.



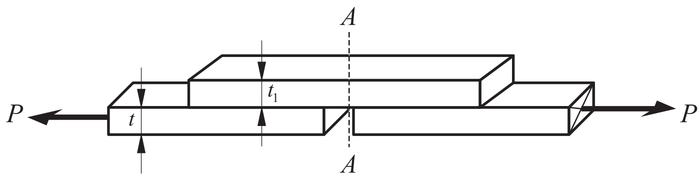
$a = 4.0\ cm$

29- For the circular column shown below, calculate the required diameter of the column such that the allowable tensile and compressive stresses are equal to σ_w.



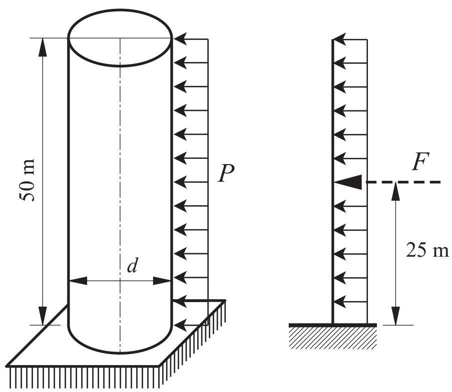
$$d = 17\sqrt{\frac{P}{\sigma_w}}$$

30- In the figure below, determine the thickness t_1 such that the maximum normal stress in the straps and the top plate is equal.



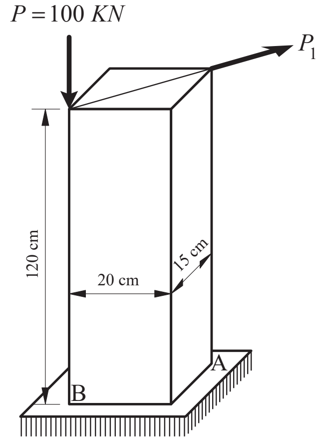
$t_1 = 4.65t$.

31- In the column shown below, the unit weight of the material forming the column is $\gamma = 2.5\ t/m^3$, and the lateral pressure acting on a diagonal section is $P = 90\ kg/m^2$. Calculate the column diameter such that no tensile stress develops in any horizontal section.



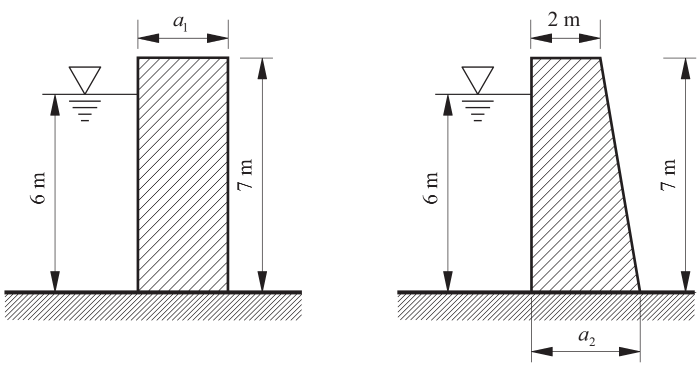
$d = 3\ m$

32- Determine the load P_1 such that no tensile stress develops at the lower part of the column.



$8.42\ KN \le P_1 \le 11.78\ KN$

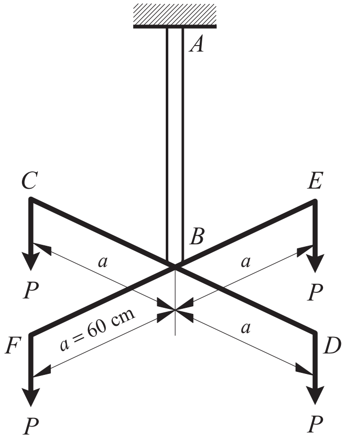
33- Two different cross-sections for a concrete dam are shown below. The dam height is 7 m and the unit weight of concrete is $\gamma_c = 2\ t/m^3 = 2000\ kg/m^3$. Determine the widths a_1 and a_2 of the dam such that no tensile stress develops at the base of the dam. The unit weight of water is $\gamma_w = 1000\ kg/m^3$.



(a) (b)

$a_1 = 3.93\ m$
$a_2 = 3.52\ m$

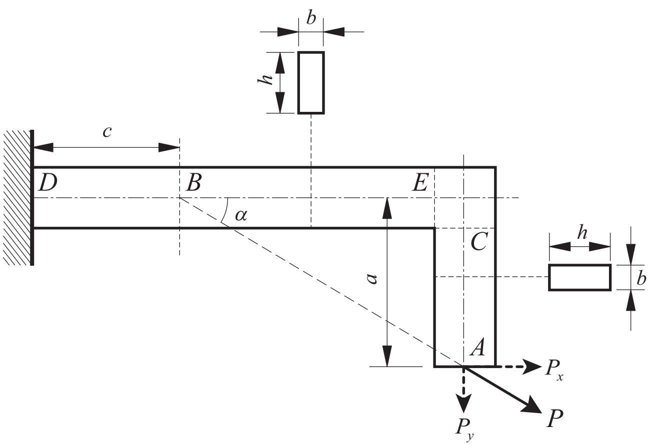
34- As shown below, four loads $P = 100\ kg$ are symmetrically suspended from the cross-shaped piece $CDEF$, which is rigidly attached to the steel pipe AB. The pipe AB has an outer diameter of 60 mm and a wall thickness of 2 mm. Calculate the maximum tensile stress in the pipe due to the four loads. Analyze the problem for the case when one of the loads is missing.



$\sigma = 109.9\ kg/cm^2$

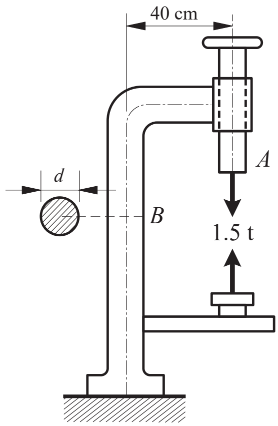
$\sigma_{\max} = 1256.6\ kg/cm^2$

35- As shown below, a rectangular cross-section bar with dimensions $a = 90\ cm$, $b = 6\ cm$, $c = 80\ cm$ and a length L (horizontal part of the bar) equal to 2 m is subjected to a force $P = 2\ t$. If the line of action of force P passes through the centroids of surfaces A and B of the section, calculate the maximum normal stress developed in the section.



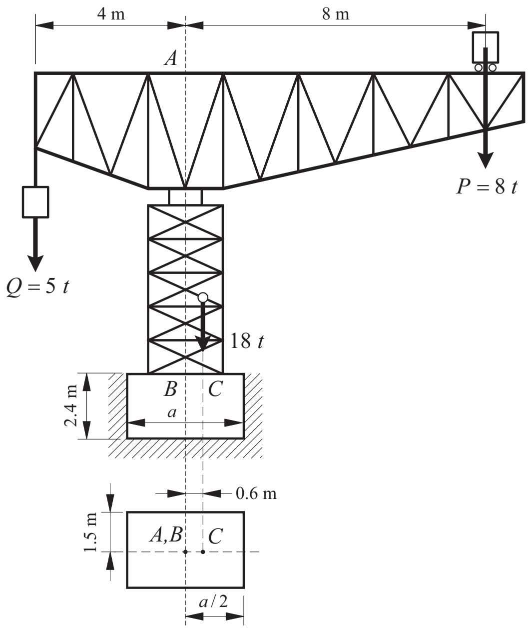
$\sigma_E = 1406.7\ kg/cm^2$

36- As shown below, during drilling a piece, a force of $1.5\ t$ is transmitted to the drill machine. If the allowable tensile stress of cast iron is $350\ kg/cm^2$, determine the diameter of the cast iron column of the drill machine.



$d = 122\ mm$

37- As shown below, a crane is used to lift and move loads of up to 8 t. This crane is mounted on a concrete foundation. The AB axis of the crane passes through the centroid of the foundation surface. Assuming that the weight of the crane without the load P and the counterweight Q equal to 18t is applied at point C, at a distance of 0.6 m from the AB axis to the foundation, and the unit weight of concrete is $2.2\ t/m^3$, determine the side length a of the foundation such that no tensile stress develops at the base. Calculate the maximum pressure exerted on the ground for the chosen value of a.

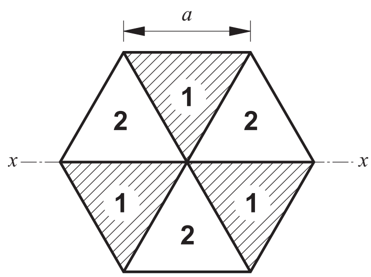


$a = 3.68\ m \qquad \sigma_c = -16.18\ t/m^2$

Chapter 5: Bending Moment Effects in Structures

Section 8: Composite Sections

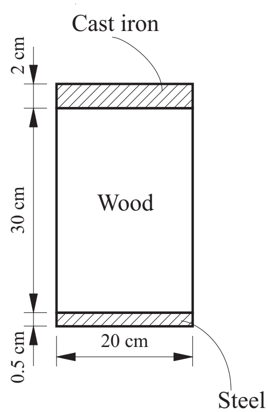
1- As shown below, a prismatic bar with a hexagonal cross-section is constructed from six equilateral triangular wooden segments bonded together. The segments are made of two different types of wood with elastic modulus of $E_1 = 84000\ kg/cm^2$ and $E_2 = 70000\ kg/cm^2$. The side length a is 2.5 cm. If the bar is subjected to pure bending $M = 1000\ kg.cm$ in its longitudinal symmetry plane, calculate the maximum normal stress induced by bending.



$\sigma_{1x} = 111.71\ kg/cm^2$

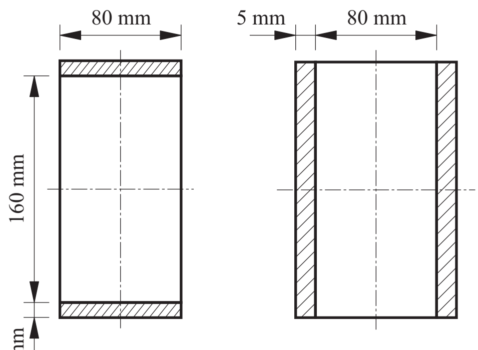
$\sigma_{2x} = 93.1\ kg/cm^2$

2- As shown below, a wooden beam with a rectangular cross-section of width 20 cm, height 30 cm, and elastic modulus $E_W = 10^5\ kg/cm^2$ is reinforced with two metal plates at the top and bottom. The top plate is made of cast iron with a thickness of 2 cm and an elastic modulus of $E_c = 10^6\ kg/cm^2$, and the bottom plate is made of steel with a thickness of 0.5 cm and an elastic modulus of $E_s = 2\times10^6\ kg/cm^2$. If the beam is loaded such that the maximum tensile stress developed in the steel is 1120 kg/cm^2, calculate the maximum compressive stress developed in the cast iron.



$\sigma_{c,c} = 435\ kg / cm^2$

3- As shown below, a wooden beam with a width of 80 *mm* and a height of 160 *mm* is reinforced with two steel plates of 5 *mm* thickness. If the maximum normal stress developed in the wood is the same in both cases (a) and (b) below, calculate the ratio of the maximum moments that the beam can sustain in these two cases. Also, assume that the elastic modulus of steel is 20 times that of wood.



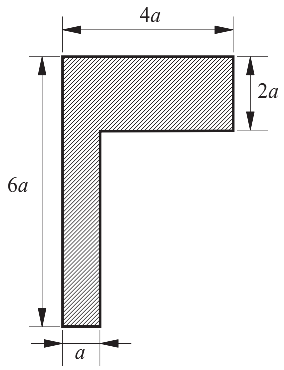
(a) (b)

$n = 1.342$

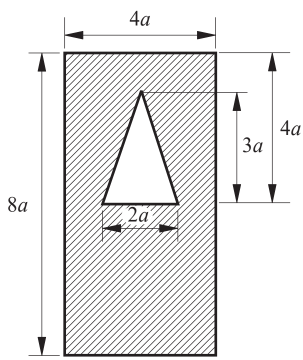
Chapter 5: Bending Moment Effects in Structures

Section 9: Unsolved Problems

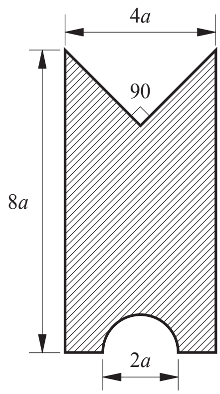
1- In the figures shown, calculate the coordinates of the centroid of the area.



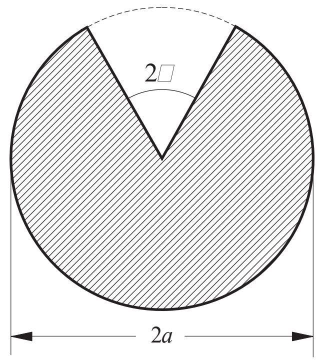
$x_c = 1.5a \quad , \quad y_c = 4a$



$x_c = 2a \quad , \quad y_c = 3.9a$

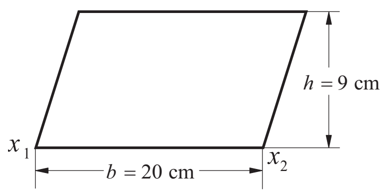


$x_c = 2a \quad , \quad y_c = 3.71a$



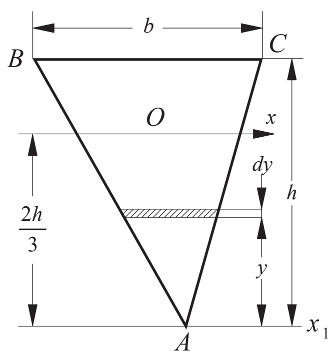
$$x_c = 0.5a \quad , \quad y_c = \frac{[\pi - \alpha - \frac{2}{3}\sin\alpha]}{\pi - \alpha}$$

2- Calculate the moment of inertia of the parallelogram shown in the figure below with respect to the x_1-axis.



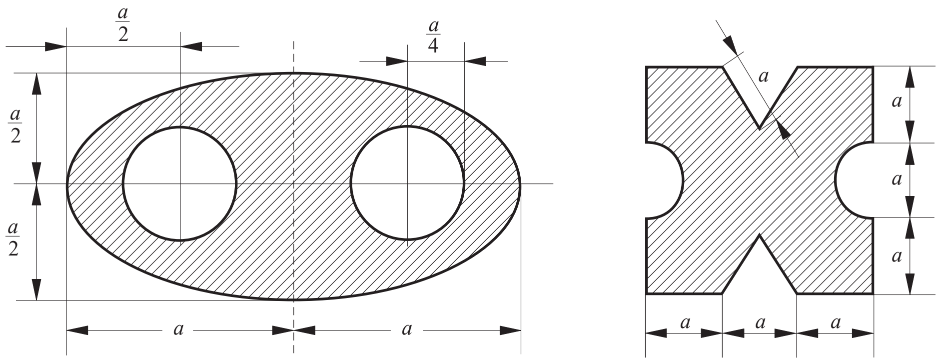
$$I_{x_1} = 4860 \ cm^4$$

3- Calculate the moment of inertia of the shown area with respect to the x_1-axis passing through point A. Also, determine the moment of inertia of the area about the centroidal axis x_0.

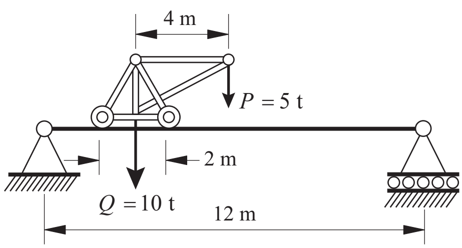


$$I_{x_1} = \frac{bh^3}{4} \quad , \quad I_x = \frac{bh^3}{36}$$

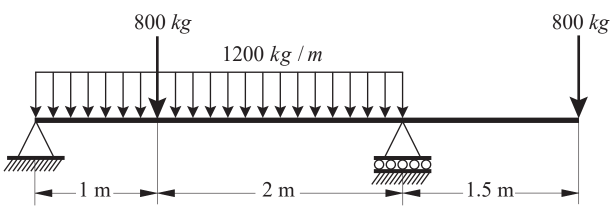
4- In the figures below, calculate the minimum section modulus.



5. For the most critical loading condition shown in the figure below, determine the required beam section modulus, assuming $\sigma = 1600\ kg/cm^2$.

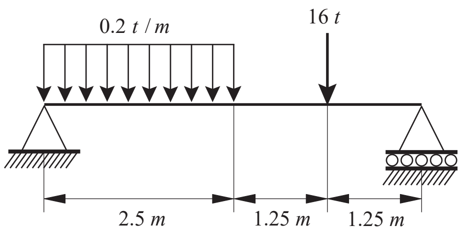


6- A wooden beam with a rectangular cross-section of width 15 cm and depth 30 cm is subjected to the loading shown below. Draw the bending moment and shear force diagrams, and calculate the maximum normal stress.



$\sigma_{\max} = 60\ kg/cm^2$

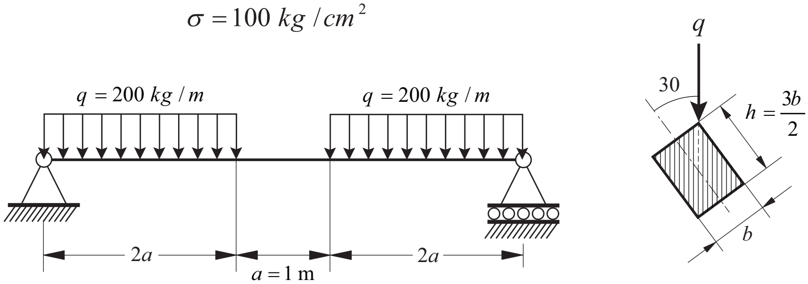
7- Draw the bending moment and shear force diagrams for the beam shown. If the allowable stress is $\sigma_w = 1100\ kg/cm^2$, determine the diameter of the beam.



$d = 11.3\ cm$

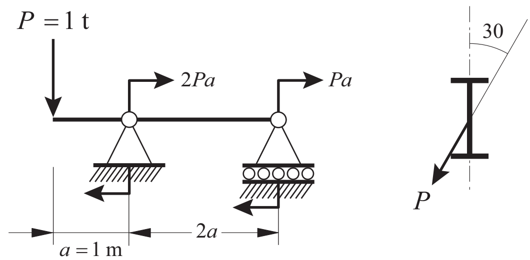
8- For the structures shown in the figure below, determine the cross-sectional dimensions.

$\sigma = 100\ kg/cm^2$



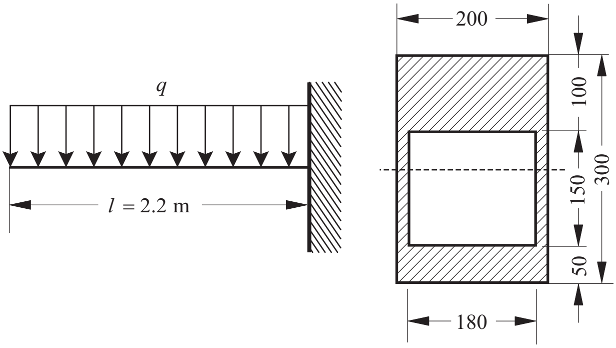
$b = 12\ cm$

$\sigma = 1400\ kg/cm^2$



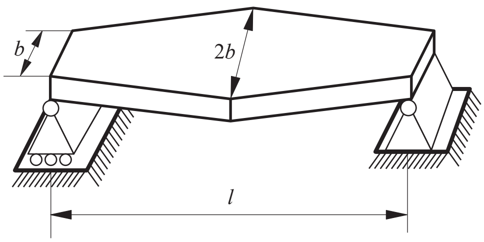
I27

9- Calculate the load-carrying capacity of the beam shown in the figure below. The allowable tensile stress is $120\ kg/cm^2$, and the allowable compressive stress is $500\ kg/cm^2$.

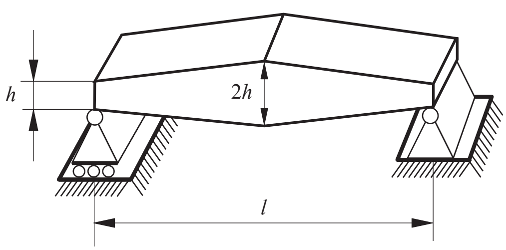


$q = 1.41\ t/m$

10- Determine the maximum normal stress due to the self-weight of the beams and identify the location of the critical cross-section.



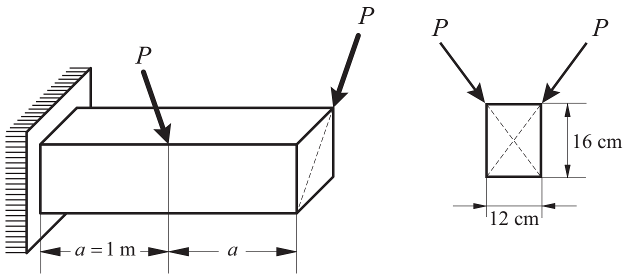
$$\sigma_{\max} = 1.061\frac{M_{\max}}{S} \quad , \quad x_c = 0.39l$$



$$\sigma_{\max} = 1.294\frac{M_{\max}}{S} \quad , \quad x_c = 0.27l$$

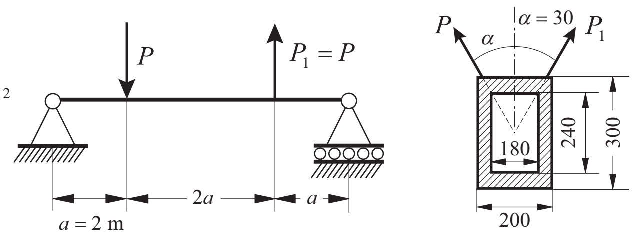
11- For the structures shown in the figure below, calculate the allowable load *P*.

$\sigma = 100\ kg/cm^2$



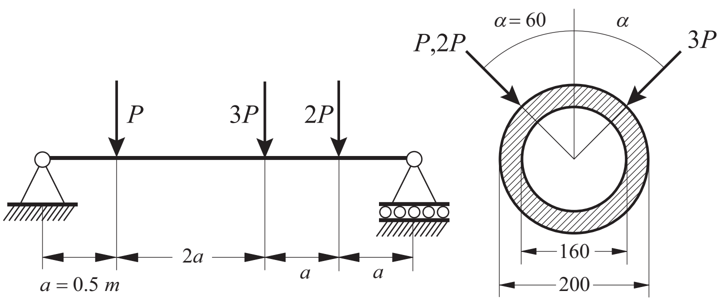
$P = 160\ kg$

$\sigma = 1600\ kg/cm^2$



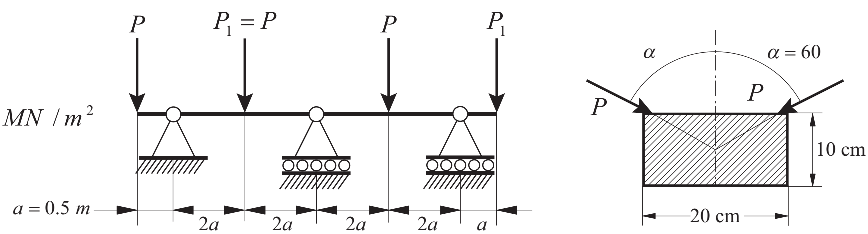
$P = 9220\ kg$

$\sigma = 160\ MN/m^2$

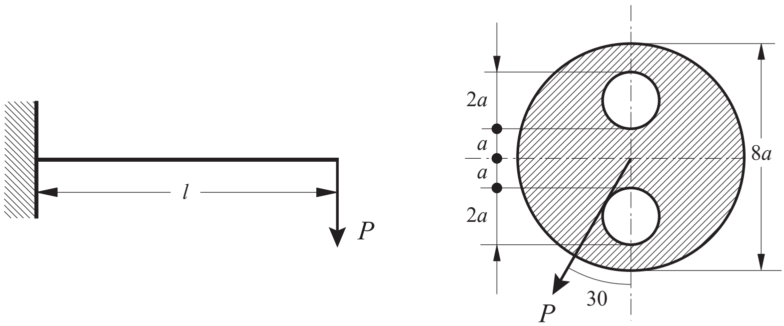


$P = 47.5\ KN$

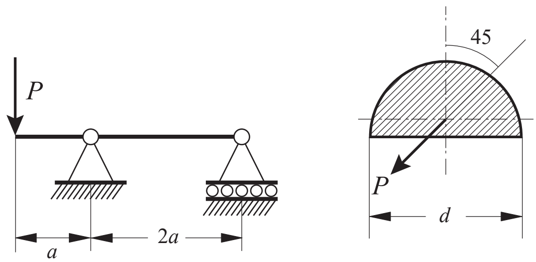
$\sigma = 10\ MN/m^2$



$P = 12.9\ KN$

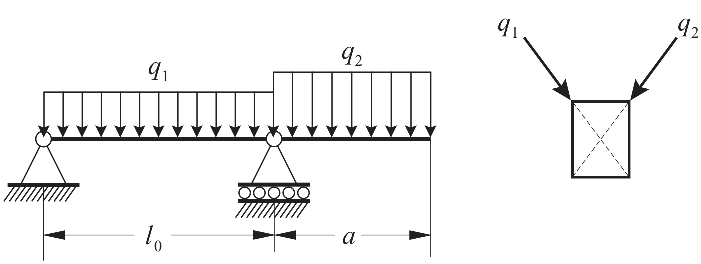


$$P = 44.9\frac{a^3\sigma}{l}$$

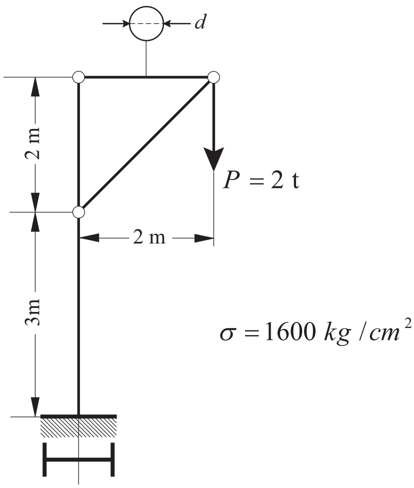


$$P = 0.0295\frac{d^3\sigma}{a}$$

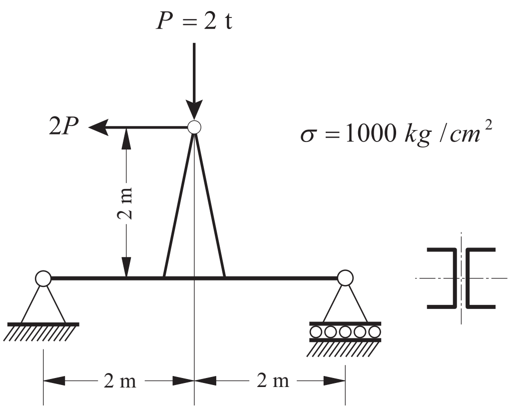
12- Based on the specifications shown in the figure below, prove that if $a \le l_0 / 2$ and $q_2 \le q_1$, the stress $\sigma_{\max}$ will be independent of the values of a and q_2.



13- For the structures shown in the figures below, determine the cross-sectional dimensions of the bars, beams, and members of the system.



I22 , $d = 1.26\ cm$



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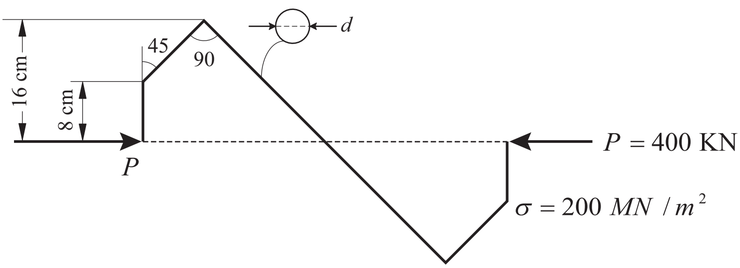
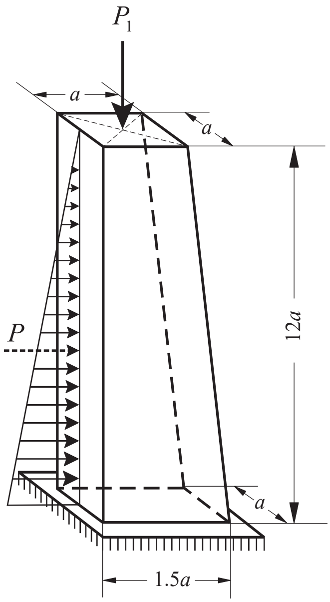


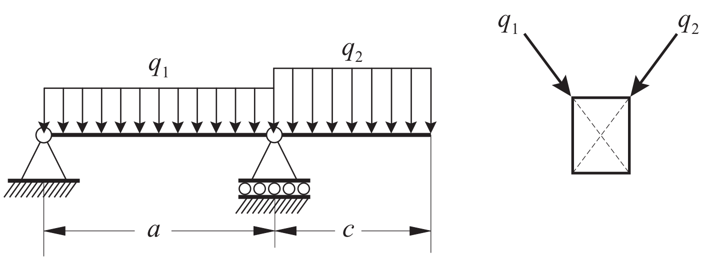
$d = 1.5\ cm$

14- Calculate the values of the force P and the stress $\sigma_{\min}$ such that $\sigma_{\max}$ at the upper right section becomes zero.



$$P_1 = 8P \quad , \quad \sigma_{\min} = -\frac{32P}{3a^2}$$

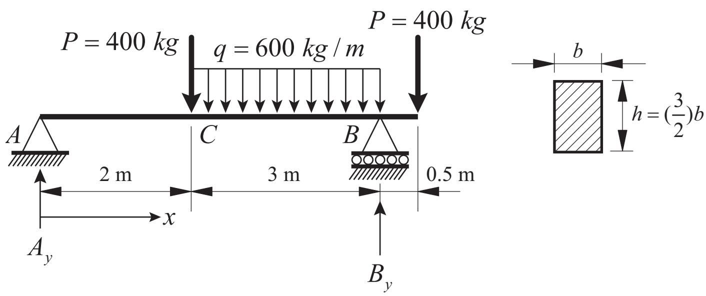
15- Based on the specifications shown in the figure below, prove that if $c \le a$ and $q_2 \le q_1$, the stress $\sigma_{\max}$ will be independent of q_2.



Chapter 6: Shear Stress in members under Bending Moments

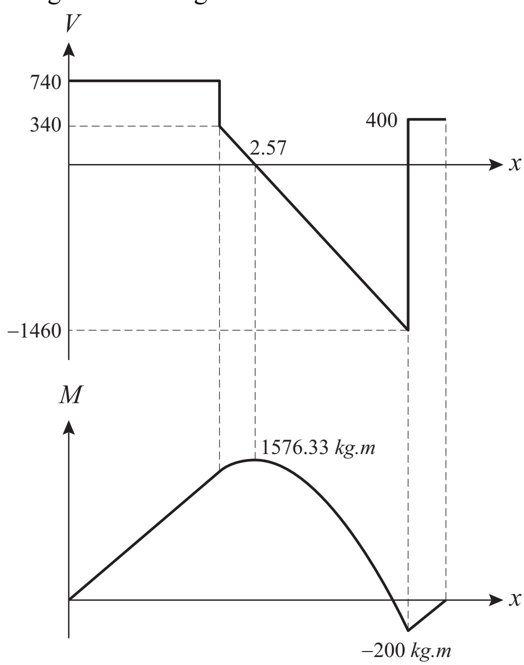
Section 1: Shear Stress in Beams

1- According to the figure, determine the dimensions of the rectangular beam, and then calculate the maximum shear stress in the beam. The allowable normal stress is $100\ kg/cm^2$.



$$\begin{cases} \sum M_B = 0 \ \rightarrow \ A_y = 740\ kg \uparrow \\ \sum F_y = 0 \ \rightarrow \ B_y = 1860\ kg \uparrow \end{cases}$$

The shear force and bending moment diagrams of the beam are as shown below:



$$M_{C-B} = 740x - 400(x-2) - 600\frac{(x-2)^2}{2}$$

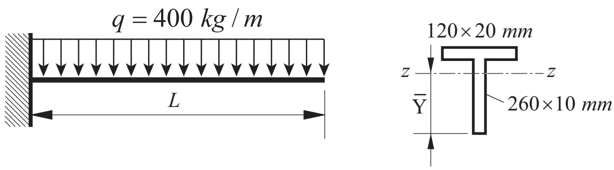
$$M'_{C-B} = 0 \ \rightarrow \ x = 2.57\ m \ \rightarrow \ M_{\max} = 1576.3\ kg.m$$

$$\sigma = \frac{M_{\max} y_{\max}}{I_z} \rightarrow 100 = \frac{(1576.33\times 10^2)(0.75b)}{\left[\frac{1}{12}(b)(\frac{3}{2}b)^3\right]} \rightarrow b = 16.14\ cm$$

$$h = \frac{3}{2}b \rightarrow h = 24.2\ cm$$

$$\tau_{\max} = 1.5\tau_{ave} = 1.5(\frac{V_{\max}}{A}) = 1.5\left[\frac{1460}{(24.2)(16.14)}\right] = 5.6\ kg/cm^2$$

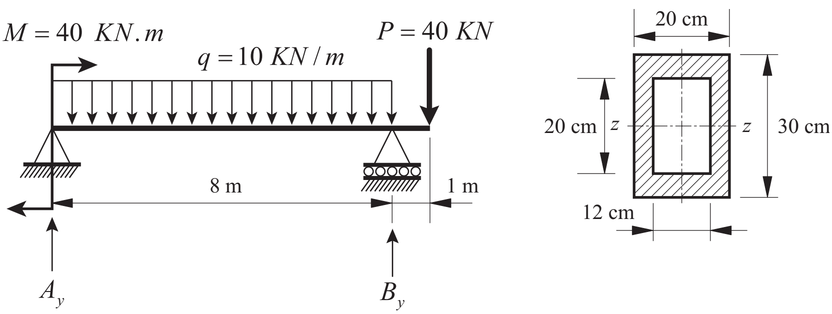
2- In the beam shown in the figure, calculate the maximum allowable span L. Then, determine the maximum shear stress in the beam. The allowable tensile normal stress is $400\ kg/cm^2$, and the allowable compressive normal stress is $1500\ kg/cm^2$.



$L \simeq 3\ m$

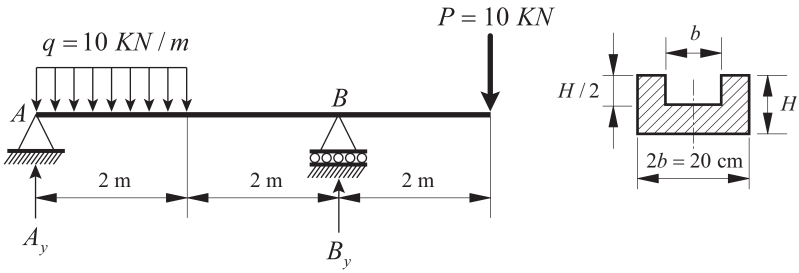
$\tau_{\max} = 61\ kg/cm^2$

3- In the beam shown in the figure, calculate the maximum shear stress developed.



$\tau = 2.79\ MN/m^2$

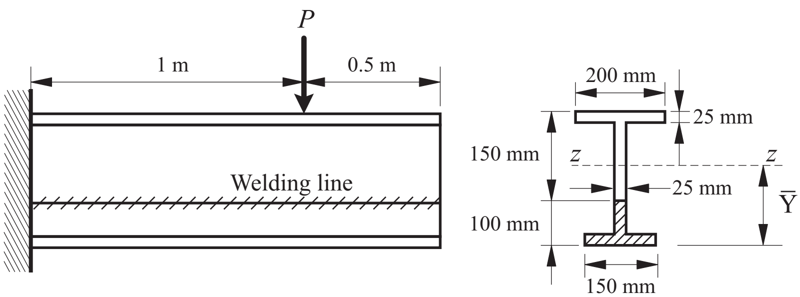
4- In the beam shown in the figure, calculate the cross-sectional height of the beam H based on the allowable normal stress eaqual to $\sigma_w = 160\ MN/m^2$. Then, determine the maximum shear stress developed in the beam.



$H = 8\ cm$

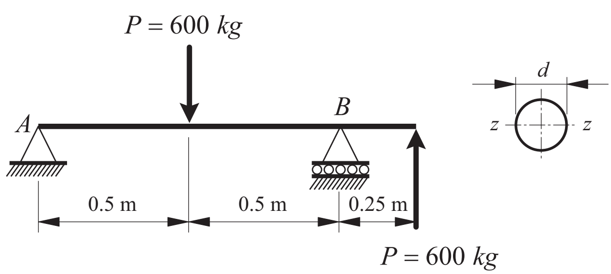
$\tau_{\max} = 0.189\ KN/cm^2$

5- The cantilever steel beam shown in the figure is made of two welded T shape sections. The allowable tensile and compressive stresses of the steel are 150 MN/m^2, and the allowable shear stress is 100 MN/m^2. Additionally, the shear strength of the weld is $q = 2\ MN/m$. Neglecting the weight of the beam, calculate the allowable load P.



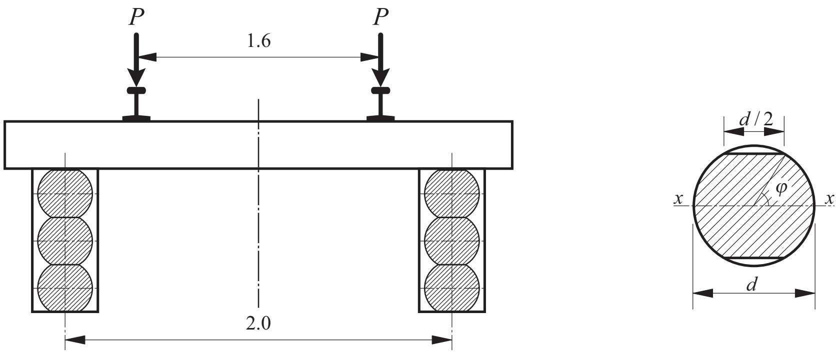
$P_{all} = 140.5\ KN$

6- In the beam shown in the figure, determine the diameter d of the beam using the allowable normal stress of 450 kg/cm^2. Then, calculate the maximum shear stress developed in the beam.

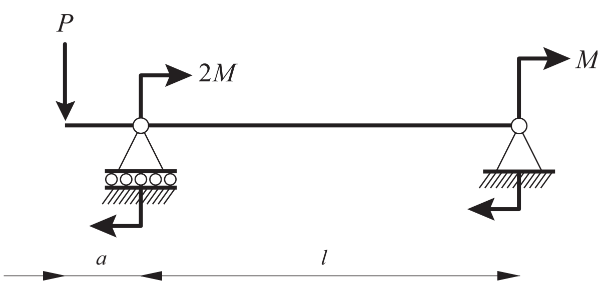


$d = 8\ cm$
$\tau \simeq 16\ kg/cm^2$

7- According to the figure, the transverse railroad beams are made from tree trunks with a diameter of $d = 25\ cm$, which have been cut at the top and bottom, and are subjected to a load $P = 6\ t$. Given that the allowable tensile and compressive stresses are $\sigma_w = 100\ kg/cm^2$, and the allowable shear stress is $\tau_w = 20\ kg/cm^2$, evaluate the adequacy of the beam dimensions.

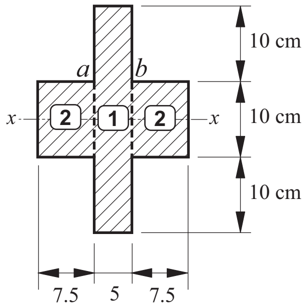


8- Assuming $P = 4\ t$, $M = 2\ t.m$, $a = 0.5\ m$, $l = 4\ m$, σ=1600 kg/cm^2 and τ=1000 kg/cm^2, determine the dimensions of the required I-shaped beam.



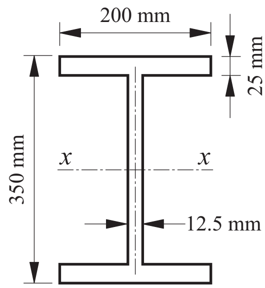
IPE 18

9- In the beam shown in the figure, if the total shear force on the section is $V = 22500\ kg$, calculate the shear stress along the line ab.



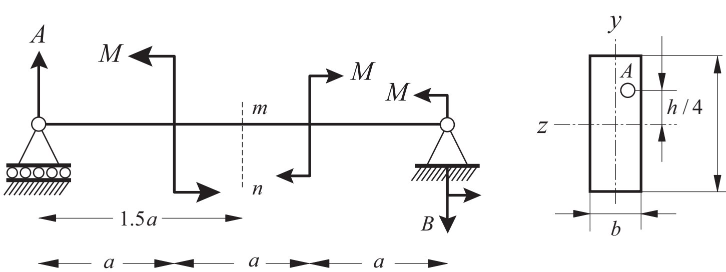
$\tau_{AB} = 180\ kg/cm^2$

10- In the beam shown in the figure, calculate the ratio of the maximum shear stress to the average shear stress in the web of the beam.



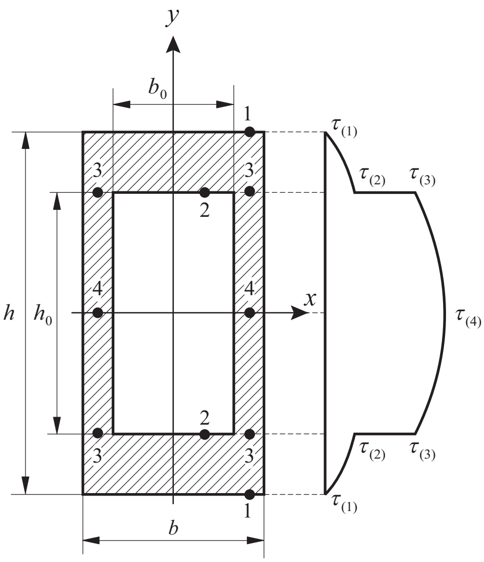
$$\frac{\tau_{max}}{\tau_{ave}} = 1.1375$$

11- Assuming $M_1 = 40\ KN.m$, $M_2 = 20\ KN.m$, $M_3 = 10\ KN.m$, $h = 12\ cm$, $b = 4\ cm$ and $a = 1\ m$, determine the values of σ_A and τ_A at section *m*-*n* of the beam shown in the figure below.



$\sigma_A = 130\ MN/m^2 \qquad \tau_A = 2.34\ MN/m^2$

12- For a beam with a box cross-section, assuming $b_0 = 4\ cm$, $b = 6\ cm$, $h_0 = 8\ cm$, $h = 12\ cm$ and $Q = 8\ t$, plot the shear stress distribution perpendicular to the neutral axis.



At point 1, $\tau_1 = 0$

At point 2: $\tau_2 \simeq 115.4\ kg/cm^2$

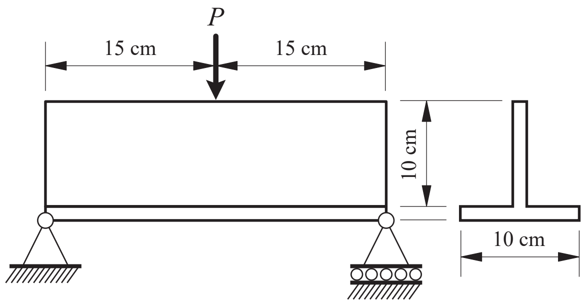
At point 3: $\tau_3 = 346.2\ kg/cm^2$

At point 4: $\tau_4 \simeq 438.5\ kg/cm^2$

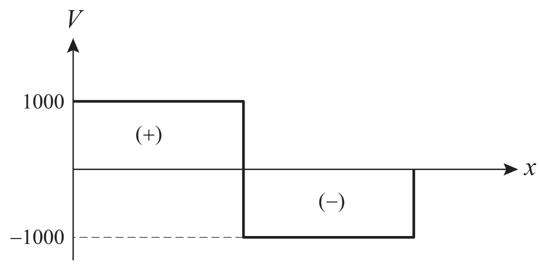
Chapter 6: Shear Stress in members under Bending Moments

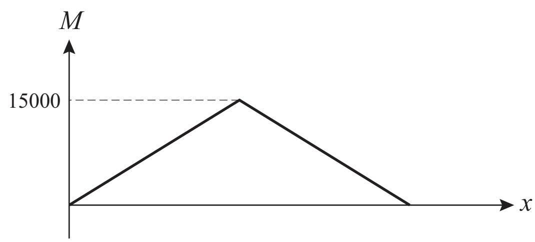
Section 2: Determination of Principal Stresses in Beams

1- As shown in the figure below, a load $P = 2000\ kg$ is applied at the midspan of a short cast-iron beam with a span of 30 cm. The beam has a T-shaped cross-section, with both the flange and web thickness equal to 5 cm. Determine the location of the maximum principal tensile stress in the beam and compute its magnitude.



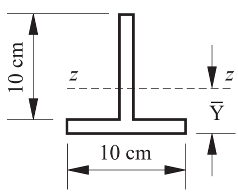
By drawing the bending moment and shear force diagrams, it is found that:





$M_{\max} = 1500\ kg$, $V_{\max} = 1000\ kg$

Calculation of the section's moment of inertia:



$$\bar{Y} \sum A_i = \sum (\bar{y}_i A_i)$$

$$\bar{Y} = \frac{[(5.5)(0.5\times10)+(0.25)(0.5\times10)]}{[(0.5\times10)+(0.5\times10)]} = 2.875\ cm$$

$$I_z = \left[\frac{1}{12}(10)(0.5^3)+(10)(0.5)(2.875-0.25)^2\right]+\left[\frac{1}{12}(0.5)(10^3)+(0.5)(10)(5.5-2.875)^2\right]$$

$$\rightarrow I_z = 110.68\ cm^4$$

$$M_z = 1000x$$

$$\sigma = \frac{M_x y_{max}}{I_z} \rightarrow \sigma = \frac{1000xy}{110.68} = 9.035xy$$

$$\tau = \frac{V_y Q_z}{I_z t_z} = \frac{1000Q_z}{(110.68)(0.5)} = 18.07Q_z$$

$$Q_z = (10)(0.5)(2.875-0.25)+(2.875-0.5-y)(0.5)(y+\frac{2.375-y}{2})$$

$$\rightarrow (Q_z)(13.125)+(2.375-y)(0.5)(\frac{y}{2}+1.19) = 13.125+0.59y+1.413-0.25y^2-0.595y$$

$$\rightarrow Q_z = -0.25y+0.005y+14.54 \rightarrow \tau = -4.52y^2-0.09y+262.74$$

According to Mohr's circle:

$$\sigma_{max} = \frac{\sigma}{2}+\sqrt{(\frac{\sigma}{2})^2+\tau^2} = 4.52xy+\sqrt{(4.25xy)^2+(-4.52y^2-0.09y+262.74)^2}$$

Since the moment at midspan ($x = 15\ cm$) is maximum, it follows that:

$$\sigma_{max} = \left[4596.8y^2+(-4.52y^2-0.09y+262.74)^2\right]^{1/2}+67.8y$$

$$\frac{d\sigma}{dy} = 0 \rightarrow 67.8+\frac{1}{2}\left[4596.8y^2+(-4.52y^2-0.09y+262.74)^2\right]^{1/2} = 0$$

$$\rightarrow \left[(2)(4596.8)y+2(-4.52y^2-0.09y+262.74)(-2\times4.52y-0.09)\right] = 0$$

$$\rightarrow 67.8+9193.7y+2(4.52y^2+0.09y-262.74)(9.04y+0.09) = 0$$

$$\rightarrow y = 0.005\ cm \rightarrow \begin{cases}\sigma = 0.08\ kg/cm^2 \\ \tau = 262.43\ kg/cm^2\end{cases} \rightarrow \sigma_{max} = 262.7$$

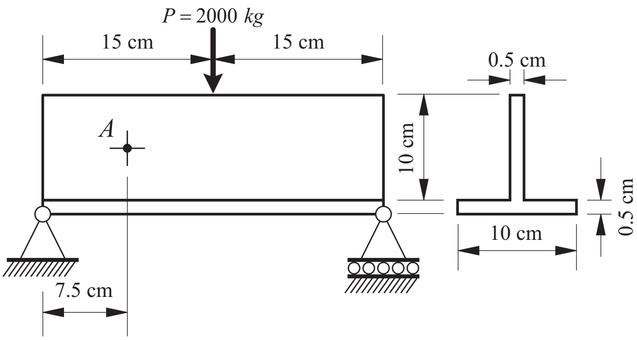
$$\sigma_x = \frac{(100)(15)(2.875)}{110.68} = 389.64\ kg/cm^2\ \text{(Extreme fiber)}$$

$$\tau = \frac{V_y Q_x}{I_x t_x} = \frac{(1000)[(5)(0.5)(2.875-0.25)]}{(110.68)(0.5)} = 118.59\ kg/cm^2$$

The principal stress is calculated from Mohr's circle:

$$\sigma_{max} = \frac{\sigma}{2}+\sqrt{(\frac{\sigma}{2})^2+\tau^2} = 422.9\ kg/cm^2$$

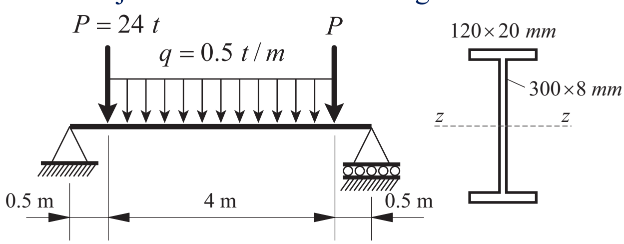
2- According to the figure, draw Mohr's circle for the stress state at point A, which is located 7.5 cm from the left support and at mid-height of the beam. Also, determine the magnitude and direction of the principal tensile stress at that point using Mohr's circle.



$\sigma_{\max} = 170\ kg/cm^2$

$\alpha = 71.26^\circ$

3- In the figure below, calculate the principal stresses developed in the most critical section of the beam at the junction between the flange and the web.

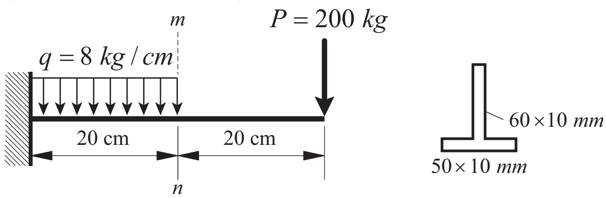


0

$\sigma_{\max} = 1744.4\ kg/cm^2$

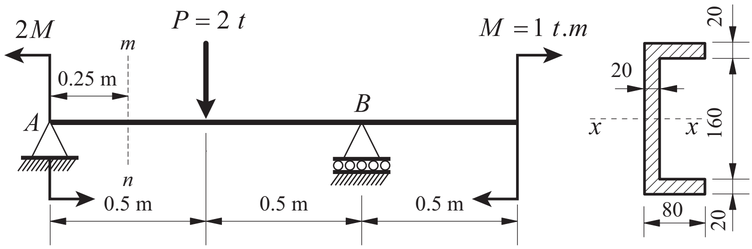
$\sigma_{\min} = -415\ kg/cm^2$

4- In the figure below, determine the principal stresses developed at section m-n, specifically at the junction of the flange and the web of the beam.



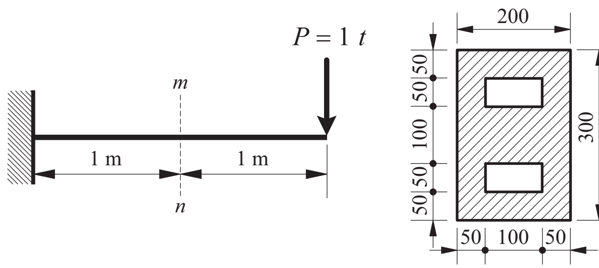
$\sigma_1 = -120.2\ kg/cm^2$

5- In the figure below, determine the principal stresses developed at section *m-n*, specifically at the junction of the flange and the web of the beam. The dimensions of the beam's cross-section are given in millimeters.



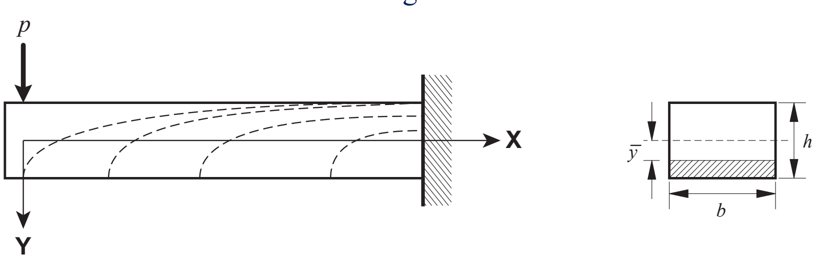
$$\sigma_{\max} = 370.44\ kg/cm^2$$
$$\sigma_{\min} = -5.19\ kg/cm^2$$

6- In the figure below, determine the principal stresses developed at section *m-n*, specifically at the junction of the flange and the web of the beam. The dimensions of the beam's cross-section are given in millimeters.



$$\sigma_1 = 25.922\ kg/cm^2$$
$$\sigma_2 = -0.393\ kg/cm^2$$

7- In the figure, derive the differential equation of the trajectories of the principal stresses for a cantilever beam with a rectangular cross-section.

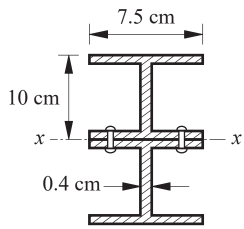


$$(\frac{h^2}{4} - y^2)(\frac{dy}{dx})^2 + 2xy(\frac{dy}{dx}) - (\frac{h^2}{4} - y^2) = 0$$

Chapter 6: Shear Stress in members under Bending Moments

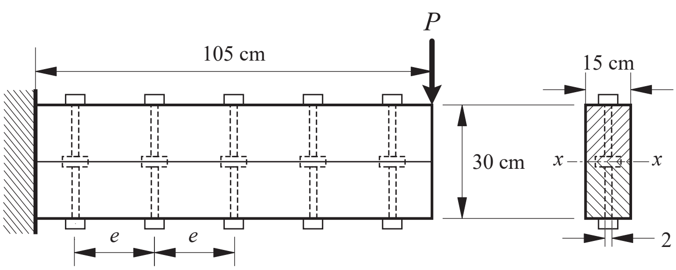
Section 3: Design of Shear Connections in Beams

1- According to the figure, a beam is constructed by connecting two steel beams with rivets of $2\,cm$ diameter. The center-to-center spacing of the rivets is $10\,cm$. Each I-shaped beam has a height of $10\,cm$, flange width of $7.5\,cm$, web thickness of $0.4\,cm$, a moment of inertia of $257.5\,cm^4$ (about the horizontal axis through the centroid), and a cross-sectional area of $14.6\,cm^2$. The span of the composite beam is $1.5\,m$, with simple supports at both ends. The beam is subjected to a uniformly distributed load that produces a maximum normal stress of $1120\,kg/cm^2$. Calculate the average shear stress developed in the rivets near the two ends of the beam.



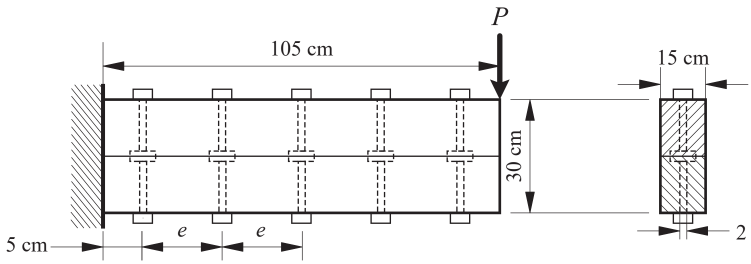
$\tau_{\max} = 347\ kg/cm^2$

2- According to the figure, a cantilever beam is made from two wooden beams, each with a dimension of $15 \times 15\ cm$. These beams are connected by bolts and connecting rings. The diameter of the bolt holes is $2\,cm$, and each connecting ring can safely transfer a shear force of $3000\,kg$. If the load applied at the free end of the beam is $P = 2500\ kg$, calculate the required spacing (e) between the bolts.



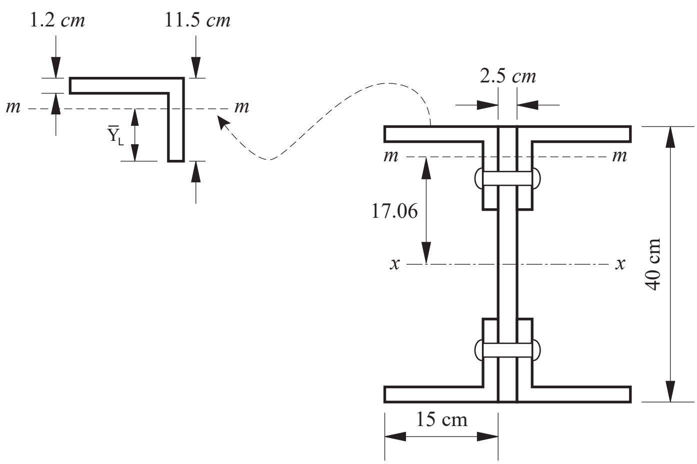
$\mathrm{e} = 24\ cm$

3- Calculate the maximum normal stress in the beam shown in the figure, assuming that the distance of the first bolt from the wall is $5\,cm$. Each connecting ring can safely transfer a shear force of $3000\,kg$.



$\sigma_{\max} = 140\ kg/cm^2$

4- A built-up beam is constructed by connecting four angle sections, each with dimensions $150 \times 115 \times 10\ mm$, to a $40 \times 2.5\ cm$ plate using rivets of $2.4\ cm$ diameter, as shown. The beam has a span of $6\ m$ with pinned supports at both ends. A concentrated load p is applied at mid span. The allowable normal stress for the beam is $\sigma_w = 1120\ kg/cm^2$, and the allowable average shear stress in the rivets is $\tau_w = 420\ kg/cm^2$. Determine the spacing between the rivets so that the maximum bending capacity of the beam is fully utilized.

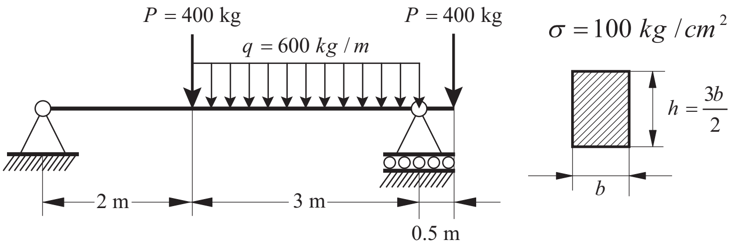


$e = 19.65\ cm$

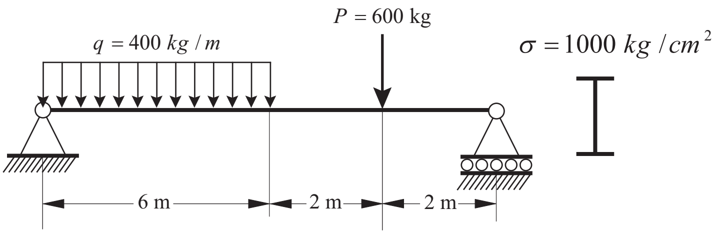
Chapter 6: Shear Stress in members under Bending Moments

Section 4: Unsolved Problems

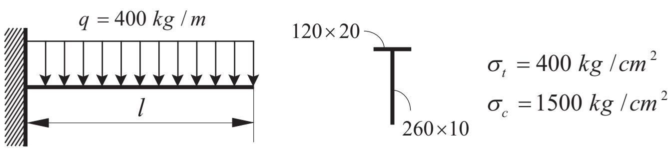
1- For the problems shown in the figure below, calculate the maximum shear stress $\tau_{\max}$. If necessary, determine the required cross-sectional dimensions or the allowable load based on the permissible normal stresses.



$h = 24.2\ cm$, $\tau_{\max} = 60.5\ kg/cm^2$

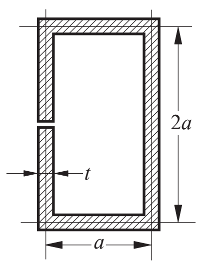


I27 , $\tau_{\max} = 125\ kg/cm^2$

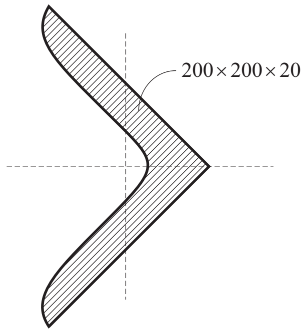


$l = 3\ m$, $\tau_{\max} = 60.5\ kg/cm^2$

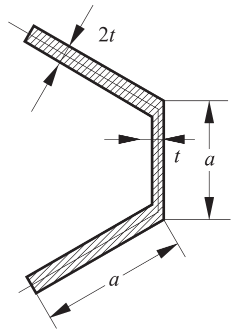
2- For the thin-walled cross-sections shown below, determine the distance d between the shear center and the centroid of the mass-section.



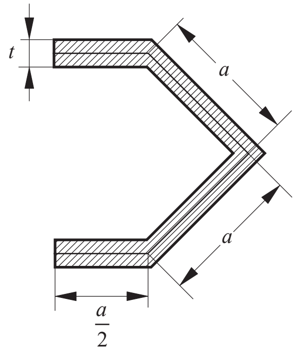
$d = 1.2a$



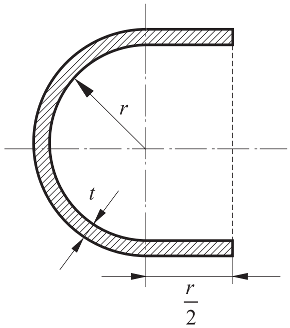
$d = 6.65\ cm$



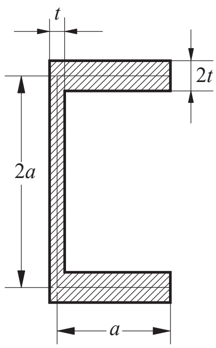
$d = 1.034a$



$d = 0.702a$

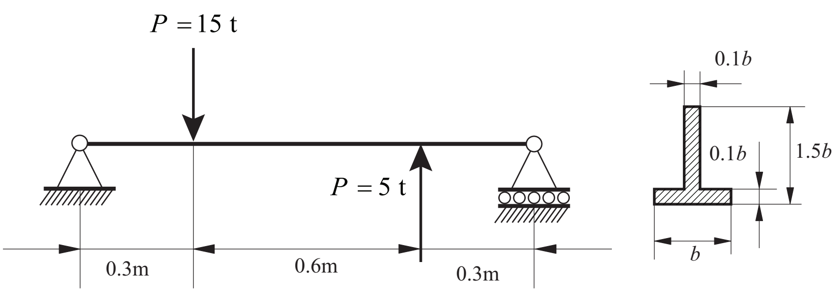


$d = -0.5827r$



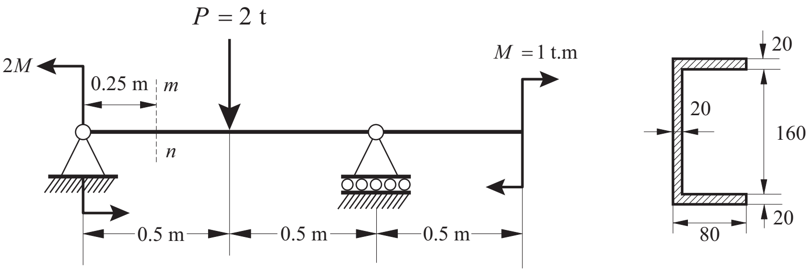
$d = -0.762a$

3- Determine the required cross-sectional dimensions for the beam shown in the figure below. The values of $\sigma_w = 1600\ kg/cm^2$ and $\tau_w = 1000\ kg/cm^2$ are given.

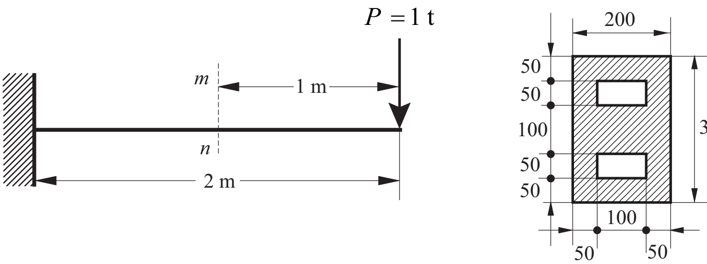


$b = 15.7\ cm$

4- Determine the principal stresses σ_1 and σ_2 at section m-n of the beams shown below. The values are $\sigma_w = 1600\ kg/cm^2$ and $\tau_w = 1000\ kg/cm^2$.

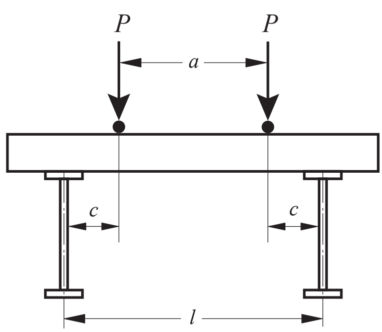


$\sigma_1 = 370\ kg/cm^2$, $\sigma_2 = -5\ kg/cm^2$



$\sigma_1 = 26\ kg/cm^2$, $\sigma_2 = -0.4\ kg/cm^2$

5- Determine the dimensions of the rectangular cross-section of a wooden beam for a bridge, given that the aspect ratio is $h/b = 4/3$. The allowable tensile and compressive stresses are $\sigma = 25\ kg/cm^2$, and the allowable shear stress is $\tau = 25\ kg/cm^2$. The span length is $l = 2\ m$. The spacing between the applied loads P is 1.6 m. The load applied to the beam due to the passage of the train is 9.8 t.

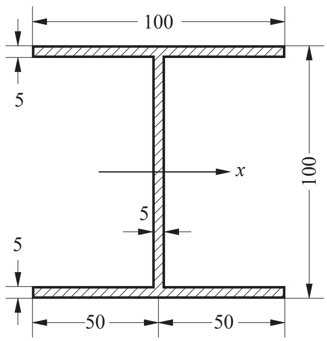


$21 \times 28\ cm$

6- A simply supported beam with a rectangular cross-section of $10 \times 20\ cm$ and a span length of $l = 4\ m$ is subjected to a uniformly distributed load of $q = 4\ t/m$. Calculate the maximum shear stress in the beam cross-section and at the midspan. In which plane does the shear stress $\tau_{\max}$ occur.

$\tau_{\max} = 600\ kg/cm^2$

7- A thin-walled I-shaped beam is subjected to bending in its vertical plane. Determine the principal stresses at the critical points of the web and the flange. The bending moment at the section is $800\ kg.m$, and the shear force Q is 3600 kg. The dimensions shown in the figure are in millimeters.



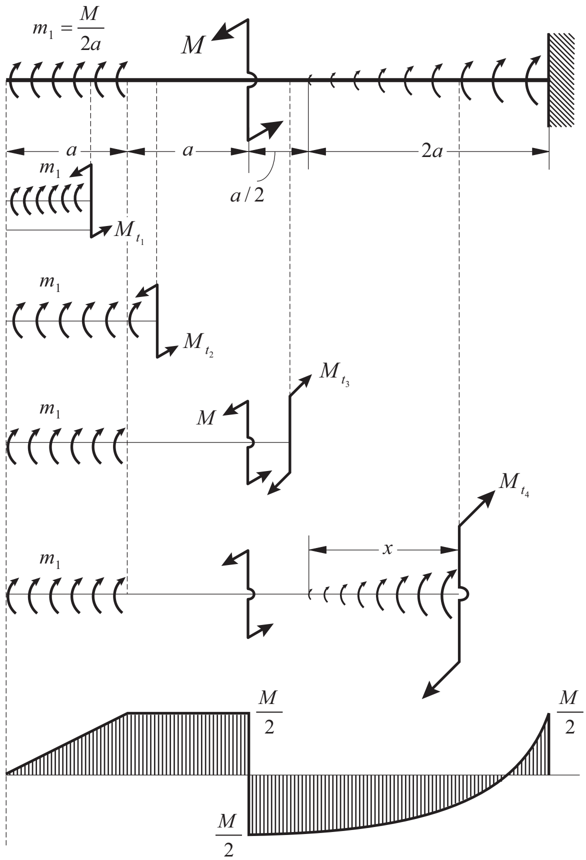
At flange $\sigma_1 = 1515\ kg/cm^2$ $\sigma_3 = -45\ kg/cm^2$

At web $\sigma_1 = 1580\ kg/cm^2$ $\sigma_3 = -250\ kg/cm^2$

Chapter 7: Torsion Effects

Section 1: Torsion of Members with Circular Sections

1- As shown in the figure, a torsional moment M is applied to a beam. Draw the torsion diagram M_t for it.



By entering the section at each part of the beam, the equilibrium equations can be written as follows:

$$M_{t_1} = m_1 x = \frac{M}{2a} x \quad , \quad M_{t_1, x=0} = 0 \quad , \quad M_{t_1, x=a} = \frac{M}{2}$$

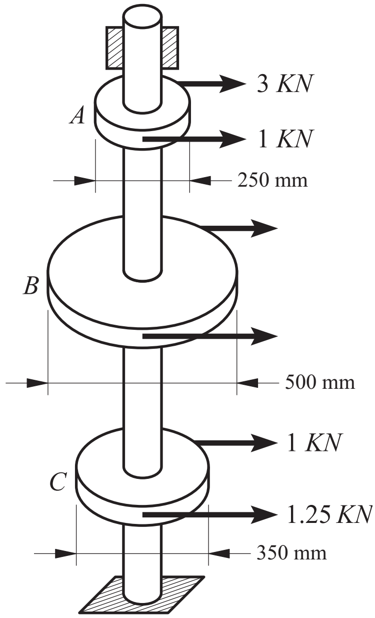
$$M_{t_2} = m_1 x = \frac{M}{2} \quad , \quad M_{t_3} = m_1 a - M = \frac{M}{2} - M = -\frac{M}{2}$$

$$M_{t_4} = m_1 a - M + \int_0^x \frac{2m_1}{2a} x dx = \frac{M}{2} - M + \frac{M}{2a^2} \int_0^x x dx = -\frac{M}{2} + \frac{Mx^2}{4a^2}$$

$$M_{t_4, x=0} = -\frac{M}{2} \qquad M_{t_4, x=2a} = \frac{M}{2}$$

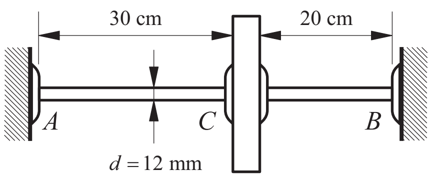
The diagram M_t is shown below the problem statement.

2- As shown in the figure, a shaft rotates with a uniform angular velocity. The known belt forces are indicated in the figure, and the pulleys are rigidly attached to the mechanical shaft. If the allowable shear stress is $\tau_a = 50\ MPa$, determine the required diameter of the mechanical shaft. Neglect the weight of the vertical shaft, the pulleys, and the bending of the shaft.



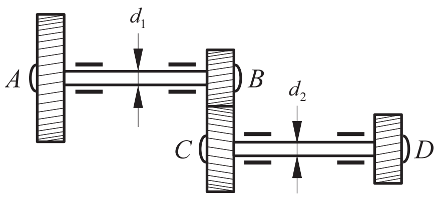
$d \simeq 29\ mm$

3- A steel mechanical shaft with a diameter of 12 mm is fixed at both ends A and B, and it carries a circular plate at point C. Assuming the allowable shear stress in the shaft is $\tau_a = 700\ kg/cm^2$ and that the circular plate is rigidly attached to the shaft, determine the maximum angle of twist that can be applied to the circular plate. Take G as $840000\ kg/cm^2$.



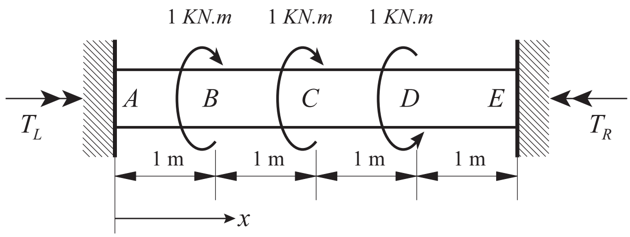
$\phi_{max} = 1.59^\circ \simeq 1^\circ, 35'$

4- In the system shown, power is transmitted from gear A to gear D. If the pitch diameters of gears B and C are in a ratio of 0.5, what is the appropriate ratio of the mechanical shaft diameters (d_1 / d_2) such that both shafts experience the same maximum shear stress?



$$\frac{d_1}{d_2} = 0.794$$

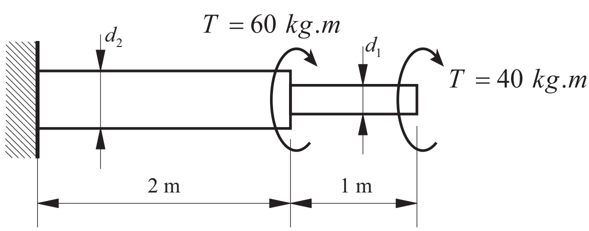
5- As shown in the figure, the bar has a circular cross-section that remains constant along its entire length. Determine the torsional reactions at both ends of the bar.



$T_L = 1\ KN.m$

$T_R = 0$

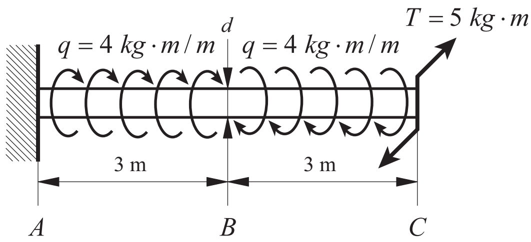
6- In the figure below, if the allowable shear stress is $800\ kg / cm^2$ and the shear modulus is $G = 8 \times 10^5\ kg / cm^2$, calculate the diameters d_1 and d_2 of the bar and the total angle of twist.



$d_{1BC} = 29.4\ mm$

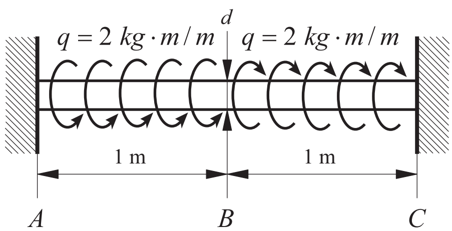
$d_{2AB} \simeq 40\ mm \quad \phi = 9.6^{\circ}$

7- In the figure below, if the allowable shear stress of the bar is $250\ kg/cm^2$ and the shear modulus is $G = 8 \times 10^5\ kg/cm^2$, calculate the diameter d of the bar and the total angle of twist.



$d \simeq 2.5\ cm$
$\phi = -1.27^\circ$

8- As shown in the figure below, the allowable shear stress of the bar is $250\ kg/cm^2$, its modulus of elasticity is $E = 2 \times 10^6\ kg/cm^2$, and its shear modulus is $G = 8 \times 10^5\ kg/cm^2$. Calculate the diameter d of the bar and the angle of rotation at section B.

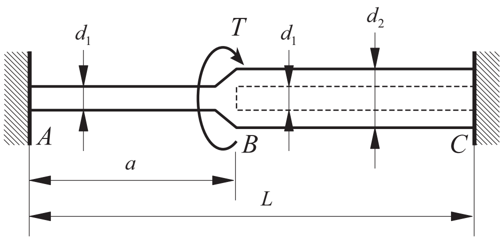


$d = 12.7\ mm$
$\phi_B = 0$

9- As shown in the figure below, two equal torsional moments of $T = 1000\ kg.m$ are applied to a circular bar fixed at both ends. If the allowable shear stress is $600\ kg/cm^2$, calculate the diameter of the bar.

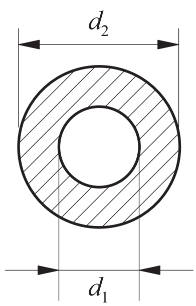
$d = 8.27\ cm$

10- As shown in the figure below, bar ABC is fixed at both ends and subjected to a torsional moment T at section B. From A to B, the bar has a solid circular cross-section with diameter d_1, and from B to C, it has a hollow circular cross-section with outer diameter d_2 and inner diameter d_1. Determine the value of a/L such that the torsional reactions at points A and C are equal in magnitude.



$$\frac{a}{L} = (\frac{d_1}{d_2})^4$$

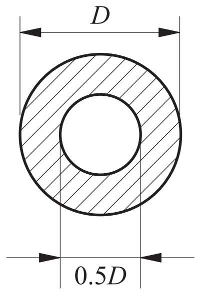
11- The outer and inner diameters of a hollow steel tube are d_2 and d_1, respectively, and the tube is used as a torsion gauge. For every $100\ kg.cm$ of applied torsional moment, the bar must twist by an amount equal to 1° over a $30\ cm$ length, and the resulting shear stress must not exceed the allowable value of $420\ kg/cm^2$. Assuming $G = 84\times10^4\ kg/cm^2$, calculate the required values of d_1 and d_2.



$d_2 = 1.719\ cm$

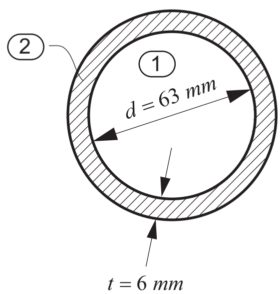
$d_1 = 1.606\ cm$

12- The outer and inner diameters of a steel mechanical shaft are D and $D/2$, respectively. If the shaft is required to transmit 200 hp at 105 rpm, calculate the appropriate value of D. The allowable shear stress is $420\ kg/cm^2$.



$D = 12.08\ cm$

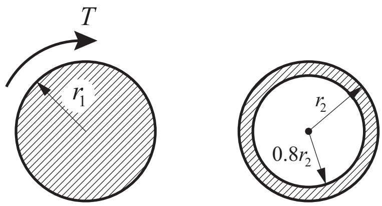
13- A brass bar with a diameter of $d = 63\ mm$ is placed inside a steel sleeve with a wall thickness of 6 *mm*, such that it is fully bonded to the sleeve and reinforces it. The shear modulus of steel and brass are $G_2 = 84 \times 10^4\ kg/cm^2$ and $G_1 = 35 \times 10^4\ kg/cm^2$, respectively, and the allowable shear stresses for steel and brass are $\tau_{a2} = 840\ kg/cm^2$ and $\tau_{a1} = 525\ kg/cm^2$, respectively. Determine the allowable torsional moment that the entire assembly can sustain. Also calculate the ratio of the torsional moment obtained above to the torsional moment that the brass bar alone can sustain.



$T = 493.7\ kg.m$

$$\frac{T}{T_1} = 3.4$$

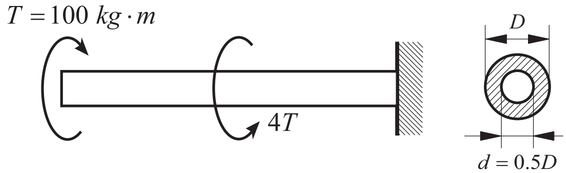
14- A hollow circular mechanical shaft and a solid circular mechanical shaft made of the same material must be designed to transmit a torsional moment T such that the maximum shear stress developed in both shafts is the same. If the inner radius of the hollow shaft is 0.8 of its outer radius, determine:
(a) the ratio of the outer radius of the hollow shaft to the radius of the solid shaft
(b) the ratio of the weight of the hollow shaft to the weight of the solid shaft



$$\frac{r_2}{r_1} = 1.192$$

$$\frac{W_2}{W_1} = 0.51$$

15- In the figure below, the allowable shear stress of the bar is $\tau_a = 500\ kg/cm^2$, and the angle of twist per unit length must not exceed $2^\circ/m$. Assuming a shear modulus of $800000\ kg/cm^2$, calculate the outer diameter (D) of the bar.



$D = 6.88\ cm$

16- A steel mechanical shaft with a diameter of 6 *mm* rotates at 1000 *rpm*. If the allowable shear stress is $350\ kg/cm^2$, determine the maximum power that this shaft can transmit.

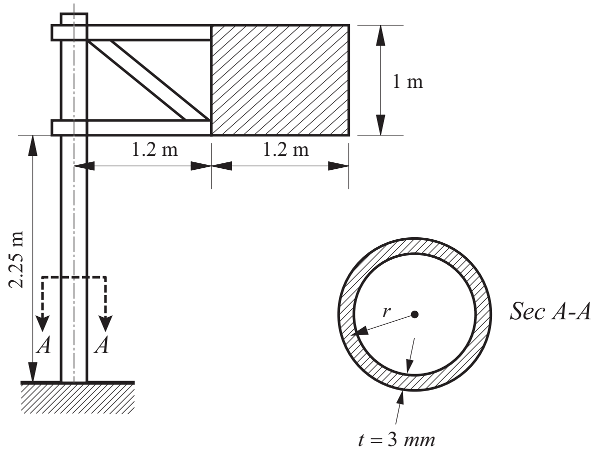
$H_m \simeq 2.1\ hp$

17- If the allowable shear stress for a steel mechanical shaft is $420\ kg/cm^2$, determine the required diameter to transmit 200 *hp* at 105 *rpm*.
$d = 11.8\ cm$

Chapter 7: Torsion Effects

Section 2: Torsion of Members with Thin -Walled Sections

1- As shown in the figure, a steel tube is used as the support for a highway traffic sign. The maximum wind pressure acting on the sign (the hatched area in the figure) is $200\ kg/m^2$. The angle of rotation of the tube at the level just below the sign must not exceed 6°, and the maximum shear stress due to torsion in the tube's cross-section must not exceed $350\ kg/cm^2$. If the wall thickness of the tube is $t = 3\ mm$ and $G = 84\times10^4\ kg/cm^2$, calculate its mean diameter.



$$q = 200\ kg/m^2 = 0.02\ kg/cm^2$$

$$\phi_{\max} = (6^\circ)(\frac{\pi}{180}) = 0.1047\ rad$$

The torsion developed at the support is given by:
$$T = qAx = (0.02)(120\times100)(120+60) = 43200\ kg.cm$$

$$\phi = \frac{TL}{GJ} = \frac{(43200)(225+50)}{(81\times10^4)\left[2\pi r^3(0.3)\right]} = 0.1047\ rad$$

$$\rightarrow \frac{180+r}{r^3} = 2.423 \ \rightarrow\ r = 4.24\ cm \rightarrow \bar{d}_1 = 2r = 8.4\ cm$$

On the other hand, since $\tau_a = 350\ kg/cm^2$, it follows that:

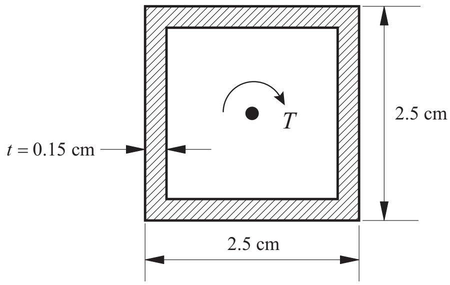
$$\tau_a = \frac{T}{2A_m t} = \frac{T}{2(\frac{\pi}{4}\bar{d}_2^2)t}$$

$$\rightarrow \bar{d}_2 = \sqrt{(\frac{4}{\pi})(\frac{T}{2\tau_a t})} = \sqrt{(\frac{4}{\pi})\left[\frac{43200}{(2)(350)(0.3)}\right]} = 16.18\ cm$$

$$d = \max\left\{\bar{d}_1, \bar{d}_2\right\} = 16.20\ cm$$

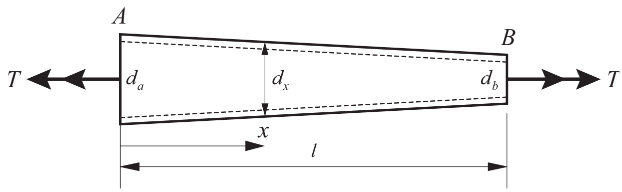
2- As shown in the figure, assuming a shear modulus of aluminum

$G = 28\times10^4 \ kg/cm^2$, calculate the angle of twist per unit length of the aluminum tube under a torsional moment of $695 \ kg.cm$.



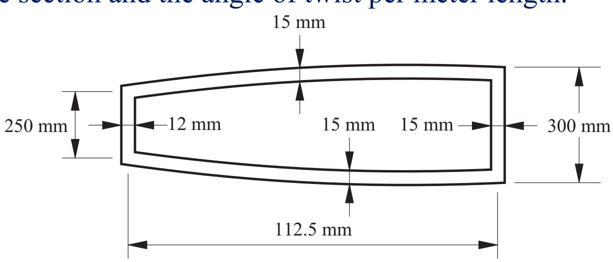
$\theta = 0.00128 \ rad/cm$

3- As shown in the figure, a long thin-walled tube with a circular cross-section of varying diameter is subjected to a torsional moment T at its ends. The wall thickness of the tube is constant and equal to t. The mean diameters of the cross-sections at ends A and B are d_a and d_b, respectively. Calculate the angle of twist ϕ of the tube.



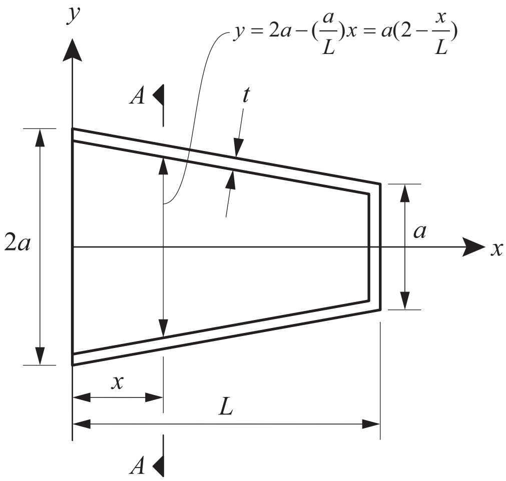
$$\phi = \left(\frac{2Tl}{G\pi t d_a^3}\right)\left(\frac{d_a}{d_b}\right)\left(\frac{d_a}{d_b}+1\right)$$

4- As shown in the figure, the thin-walled cross-section below represents a part of an airplane wing. The wall thicknesses of all four sides are uniform, and the mean height of the section is 285 mm. Assuming an allowable shear stress of 65 MPa and a shear modulus of $G = 3\times10^{10} \ N/m^2$, calculate the maximum torsional moment that can be transmitted by the section and the angle of twist per meter length.



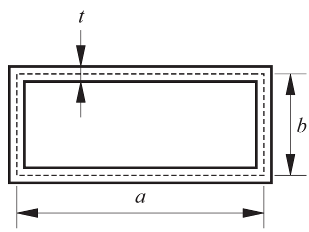
$\theta = 0.008 \; rad/m$

5- The cross-section of the tube shown below is square. If the side length of the middle square at end A is $2a$, at end B is a, and the wall thickness of the tube is constant and equal to t, derive the relation for the twist between the two ends of the tube.



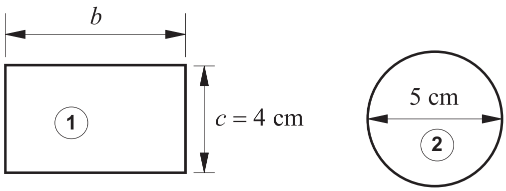
$$\phi = \frac{3TL}{8Gta^3}$$

6- As shown in the figure, a thin-walled tube with constant wall thickness t and a rectangular cross-section is subjected to a torsional moment T. If the cross-sectional area of the tube is constant, how does the shear stress τ in the tube vary with the ratio $\beta = a/b$? How does the torsional constant J change with the factor β?



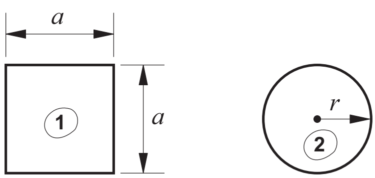
$$J = k_2 \frac{\beta^2}{(1+\beta)^4}$$

7- The width of the cross-section of a prismatic rectangular bar is 4 cm. Determine the height of the section so that, under a given torsional moment, the maximum shear stress developed in it is equal to the maximum shear stress developed in a circular bar with a diameter of 5 cm.



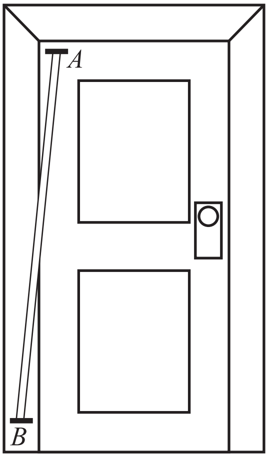
$b = 6.52\ cm$

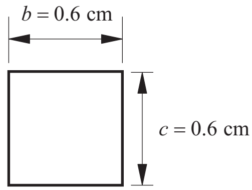
8- A circular cross-section bar and a square cross-section bar are subjected to the same torsional moment. Determine:
(a) If the cross-sectional area of both bars is the same, in which bar is the shear stress higher.
(b) If the maximum shear stress developed in both bars is the same, calculate the ratio of their weights.



$\dfrac{W_1}{W_2} = 1.23$

9- As shown in the figure, one end of a spring, in the form of a square cross-section steel bar, is attached to the door at point A, and the other end is connected to the doorway at point B. The length of the bar is $L = 2\ m$, the width of the cross-section is 6 mm, and the distance of the handle from the hinge is $d = 1\ m$. Calculate the force P required to open the door by 90° (by pulling the handle). Also, calculate the angle α by which the bar must be twisted so that the maximum shear stress during door opening does not exceed $5000\ kg/cm^2$.

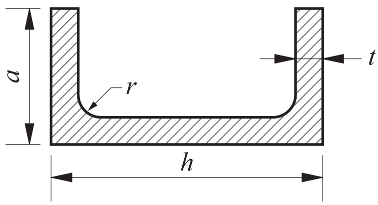




$P = 2.325\ kg$

$\theta \simeq 87^{\circ}$

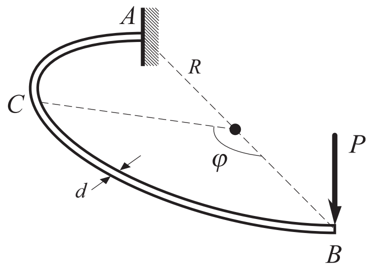
10- The aluminum channel section shown below has dimensions $a = 2.5\ cm$, $h = 7.5\ cm$, and $t = 1.5\ cm$. The radius of the two concave corners are $r = 0.75\ mm$. If the allowable shear stress is $560\ kg/cm^2$, calculate the permissible torsional moment for the channel section. Also, calculate the angle of twist per unit length due to the applied torsional moment. The shear modulus is $G = 28\times10^4\ kg/cm^2$.



$T_{\max} = 25.62\ kg.cm$

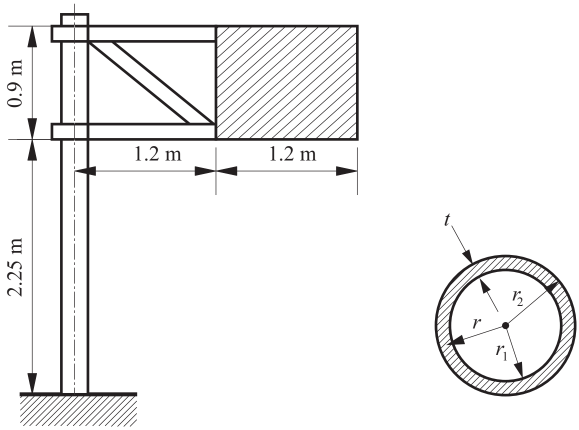
$\theta_{\max} = 0.00667\ rad/cm$

11- A semicircular bar of radius R and diameter d is fixed at the support and subjected to a load P at the free end B, perpendicular to the plane of the semicircle. In this case, an arbitrary section, such as at C, is subjected to combined bending and torsion. If d is small relative to R so that the theory of straight beam bending can be applied, calculate the angle ϕ corresponding to the point where the principal stress σ_1 reaches its maximum value.



$\phi = 120^{\circ}$

12- As shown in the figure, a circular steel tube is used as the support for a highway sign. The maximum wind pressure on the hatched area is $250\ kg/cm^2$. If the ratio of the outer diameter to the inner diameter of the tube is 1.12 and the allowable shear stress is τ_a=600 kg/cm^2, calculate the required outer diameter d of the tube.



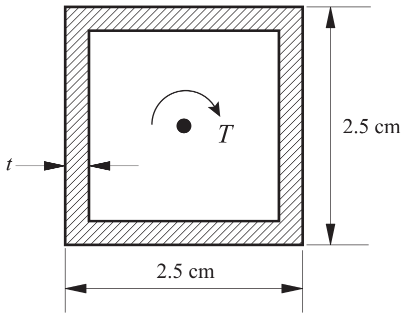
$d_2 \simeq 10\ cm$

13- A torsional moment T equal to 62.5 $t.m$ is applied to a thin-walled tube with a nominal diameter of $d = 12.5\ cm$. Determine the wall thickness of the tube so that the shear stress τ does not exceed $800\ kg/cm^2$. If the shear modulus $G = 8\times10^5\ kg/cm^2$ is given, calculate the angle of twist per unit length of the tube.

$t = 0.318\ cm$

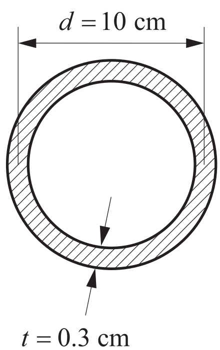
$\phi = 0.016\ rad$

14- An aluminum tube with a square cross-section, having an outer side length of $a = 2.5\ cm$, must withstand a torsional moment of $T = 695\ kg.cm$. Assuming an allowable shear stress of $\tau_a = 420\ kg/cm^2$, calculate the required wall thickness of the tube.



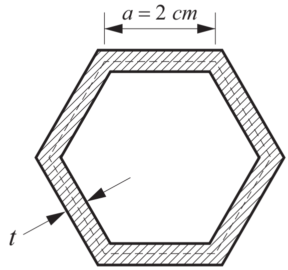
$t = 0.15\ cm$

15- A circular hollow mechanical shaft with a mean diameter of 10 cm and a wall thickness of 3 mm must transmit 50 hp. Assuming that the allowable shear stress does not exceed $\tau_a = 420\ kg/cm^2$, calculate the required rotational speed of the shaft in revolutions per minute (rpm).



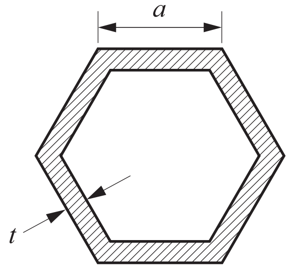
$n = 181\ rpm$

16- A thin-walled steel tube 1.8 m long has a hexagonal cross-section with a mean line length of $L_m = 6a = 12\ cm$. The wall thickness of the tube is $t = 1.5\ mm$. If the allowable shear stress is $\tau_w = 560\ kg/cm^2$, determine the permissible torsional moment T that can be applied to the tube. Also, calculate the angle of twist between the two ends of the tube.



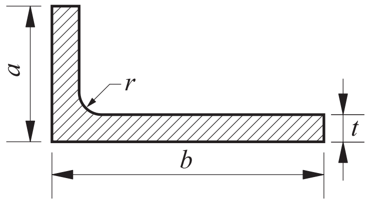
$\theta = 3^\circ\ 58'$

17- A thin-walled tube of length L has a regular n-sided polygonal cross-section. The mean line length is $l_m = na$, and the wall thickness of the tube is t. If a uniform torsional moment T is applied to the tube, calculate the resulting angle of twist.



$$\phi = \frac{4TL}{Gtna^3 \cot^2(\dfrac{\pi}{n})}$$

18- The steel angle section shown below has dimensions $a = 10\ cm$, $b = 22\ cm$, and $t = 12\ mm$. The radius of the concave corner is $r = 12\ mm$. If the allowable shear stress is $840\ kg/cm^2$, calculate the permissible torsional moment for the angle section. Also, calculate the angle of twist per unit length due to the applied torsional moment. The shear modulus is $G = 84 \times 10^4\ kg/cm^2$.



$T_{\max} = 7096\ kg.cm$
$\theta = 0.00047\ rad / cm$

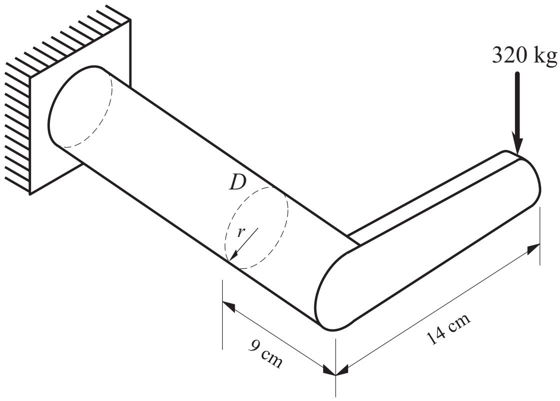
19- A prismatic bar with a square cross-section of $2.5 \times 2.5\ cm$ is subjected to torsional moments $T = 2500\ kg.cm$ at both ends. Calculate the maximum shear stress developed in the bar. Assuming $G = 84 \times 10^4\ kg / cm^2$, calculate the angle of twist per unit length of the bar under the applied torsional moments.

$\tau_a = 769.2\ kg.cm$
$\theta = 5.4 \times 10^{-4}\ rad / cm$

Chapter 7: Torsion Effects

Section 3: Members Under Torsion and Bending Moment

1- As shown in the figure, the diameter of the circular part of the structure is 50 *cm*. Calculate the principal stresses and the maximum shear stress developed at point *D*.

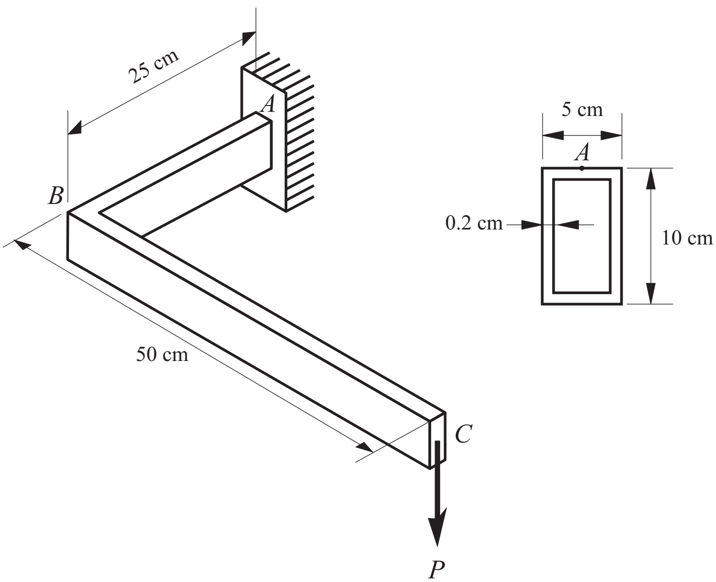


$\tau_{\max} = 217\ kg/cm^2$

$\sigma_1 = 334.3\ kg/cm^2$

$\sigma_2 = -100\ kg/cm^2$

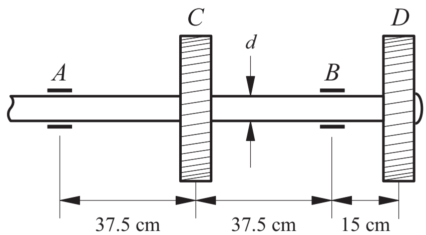
2- As shown in the figure, bar *ABC* lies in a horizontal plane and carries a load $P = 162\ kg$. Calculate the principal stresses developed at point *A*, located at the support on the top of the bar.



$\sigma_{\max} = 580\ kg/cm^2$

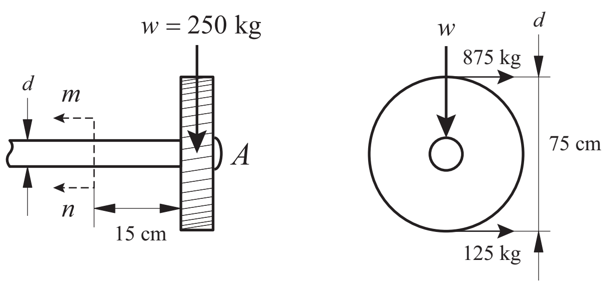
$\sigma_{\min} = -319\ kg/cm^2$

3- As shown in the figure, a mechanical shaft with diameter d is used to transmit 50 *hp* at 300 *rpm* from pulley C to pulley D. The required power is applied via a horizontal belt at C and transmitted out via a vertical belt at D. Each pulley has a diameter of 75 *cm*, and the ratio of the tight-side belt tension to the slack-side tension is $P_1/P_2 = 3$. Assuming an allowable shear stress of $\tau_w = 420\ kg/cm^2$, calculate the required diameter d of the shaft.



$d_{design} = 6\ cm$

4- As shown in the figure, a mechanical shaft with a diameter of 6 *cm*, supported on bearings, carries a pulley with a diameter of 75 *cm* and a weight of 250 *kg* at both ends. If the horizontal tensile forces in the belt are as shown, calculate the principal tensile stress developed in section *m-n*.



$\sigma_{\max} = 1121.3\ kg/cm^2$

Chapter 7: Torsion Effects

Section 4: Unsolved Problems

1- A solid bar with a diameter of 10 cm and a length of 6 m is rotated by 4°. If the value of G is $8\times10^5\ kg/cm^2$, calculate the maximum shear stress resulting from the rotation.

$\tau = 466\ kg/cm^2$

2- An approximate formula for calculating the shaft diameter is given by $d = \sqrt[3]{2.5\ (N\ /n)}$. In this relation, d is the shaft diameter, N is the transmitted power in horsepower, and n is the rotational speed in revolutions per minute. Determine the allowable shear stress on which this formula is based.

$\tau = 600\ kg/cm^2$

3- Calculate the maximum torsional moment applied to a steel bar with a diameter of 10 mm so that the shear stress does not exceed $1500\ kg/cm^2$. If the angle of twist is 90°, determine the minimum length of the bar. Assume $G = 8\times10^5\ kg/cm^2$.

$T = 295\ kg.cm \qquad l = 418\ cm$

4- A bar with a diameter of 90 mm is capable of transmitting 90 hp. If the allowable shear stress is $600\ kg/cm^2$, calculate the maximum rotational speed of the bar.

$n \geq 75\ rev/\min$

5- The shaft connecting a turbine and a generator has an outer diameter of 40 cm and an inner diameter of 22.5 cm. The rotational speed of this shaft is 120 rpm. If the transmitted power is 10000 hp, calculate the maximum shear stresses.

$\tau = 530\ kg/cm^2$

6- A hollow shaft has replaced a solid shaft with a diameter of 40 cm. The inner diameter of the hollow shaft is 60% of its outer diameter. If the allowable shear stress is the same for both shafts, calculate the outer diameter of the hollow shaft. Also, compare the weight of the solid shaft to that of the hollow shaft.

$d = 42\ cm \quad , \quad \frac{W_{solid}}{W_{hollow}} = 1.41$

7- If a hollow shaft transmits 9600 hp at a rotational speed of 110 rpm, and its inner diameter is 0.6 times the outer diameter, calculate the outer diameter of the shaft. The allowable shear stress is $560\ kg/cm^2$.

$d = 40.2\ cm$

8- A hollow shaft with a length of 1.8 m is subjected to a torsional moment of $0.6\ t.m$. Determine the outer and inner diameters of the shaft so that the angle of twist does not exceed 2°. The allowable shear stress is $700\ kg/cm^2$.

$d_1 = 90.4\ mm \qquad d_2 = 72.4\ mm$

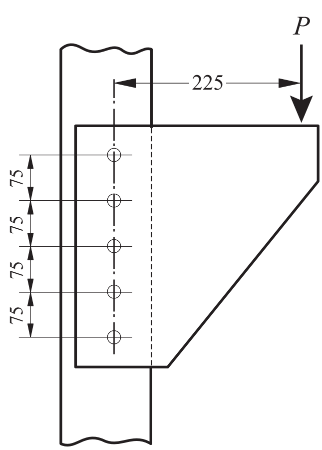
9- The width of a rectangular cross-section bar is 4 cm. Determine the height of this rectangle so that the maximum shear stress developed in this bar is the same as in a circular bar with a diameter of 5 cm, with both bars subjected to the same torsional moment.

$h = 6.52\ cm$

10- A rectangular bar is subjected to a torsional moment of $60\ kg.m$. If the width of the cross-section is 20 mm, calculate its height. The bar is made of steel with a tensile yield stress of $3500\ kg/cm^2$. The shear yield stress of the bar is 0.6 of its tensile yield stress, and the allowable shear stress is taken as 2/3 of the shear yield stress.

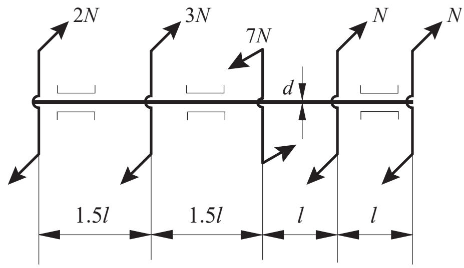
$h = 4.28\ cm$

11- The bolted connection shown in the figure below supports a load P of $3.5\ t$. The diameter of the bolts is 20 mm. Calculate the shear stress in the most critical bolt. The shear of the bolts is one-way.

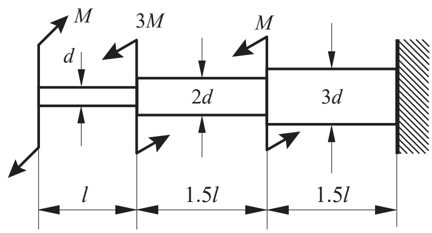


$\tau = 704\ kg/cm^2$

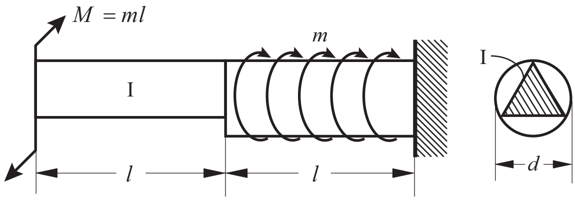
12- For the shafts shown in the figure below, draw the torque-twist (T-ϕ) diagram and determine the maximum shear stress $\tau_{\max}$.



$$\tau_{\max} = \frac{80N}{\pi d^3}$$

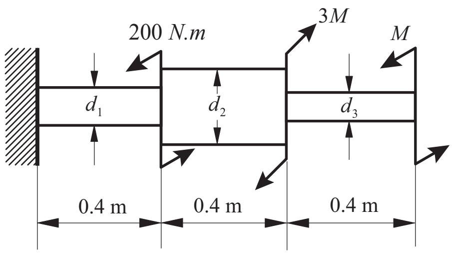


$$\tau_{\max} = \frac{16M}{\pi d^3}$$



$$\tau_{\max} = \frac{10ml}{d^3}$$

13- In the figure below, calculate the dimensions of the shaft's cross-section and the total angle of twist. The specifications of the shaft are shown on the figure.

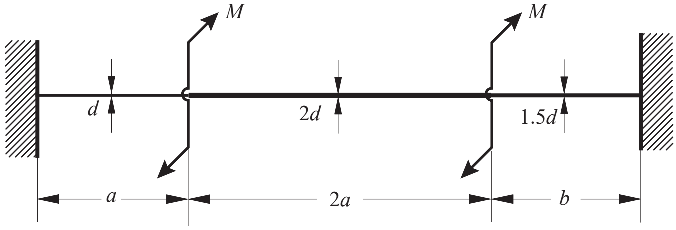


$\tau = 40\ MN/m^2$

$G = 80 \times 10^4\ MN/m^2$

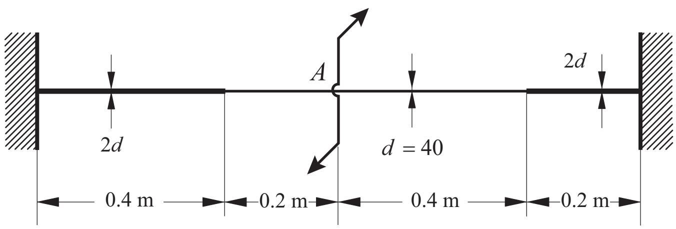
$d_1 = d_3 = 2.94\ cm$, $d_2 = 3.72\ cm$, $\phi = 1.05 \times 10^{-2}\ rad$

14- In the figure below, assuming $\tau_{a,\max} = \tau_{b,\max}$, calculate the length b.



$b = 1.28a$

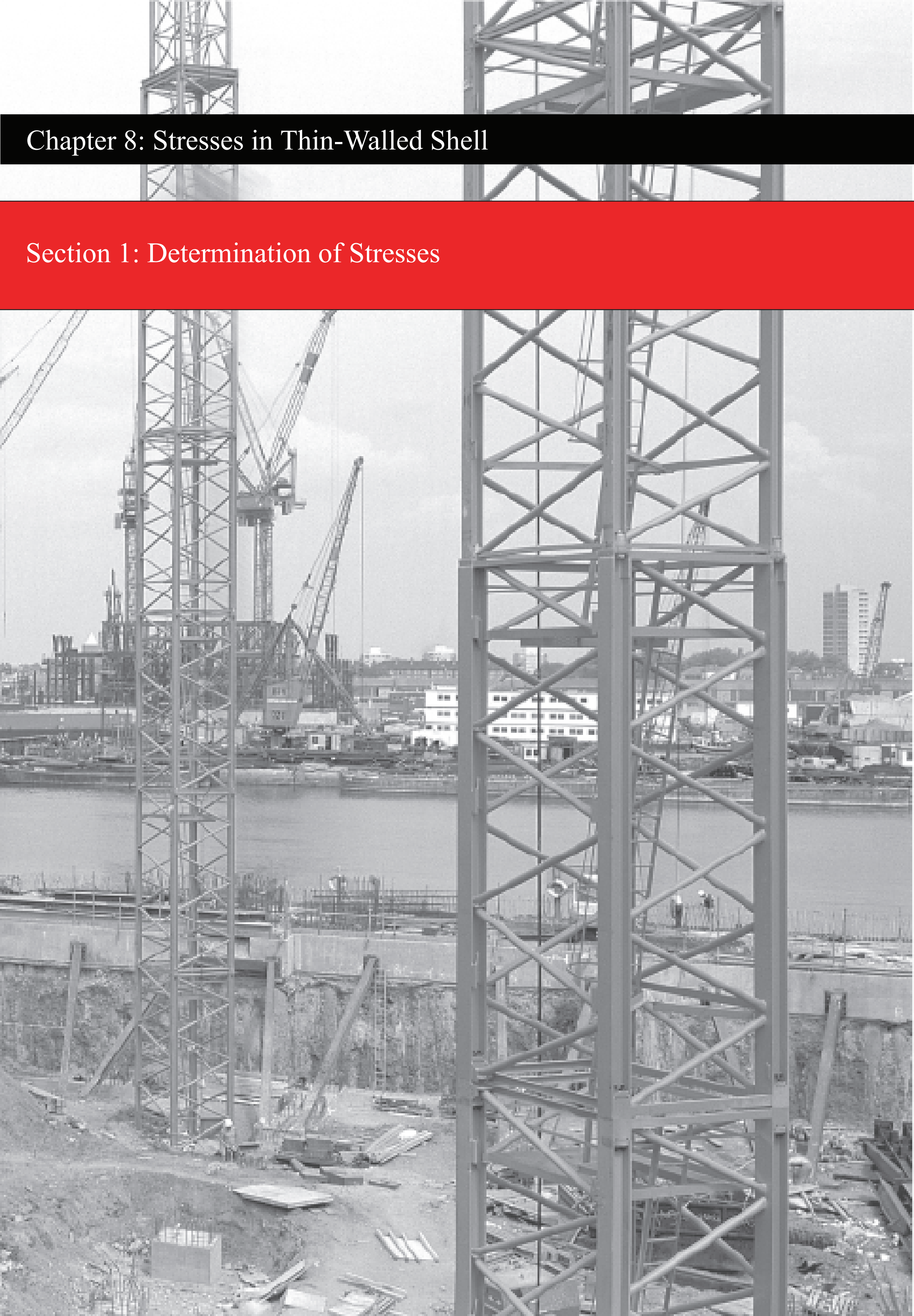
15- In the figure below, assuming $\tau = 20\ MN/m^2$ and $G = 8 \times 10^5\ kgf/cm^2$, calculate the values T_A and ϕ_A.



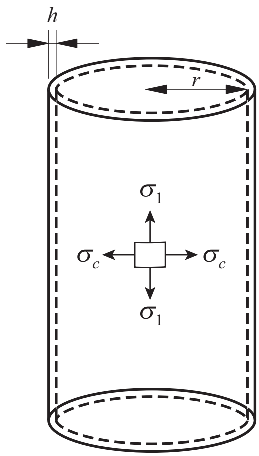
$T_A = -390\ N.m$, $\phi_A = 2.81 \times 10^{-3}\ rad$

Chapter 8: Stresses in Thin-Walled Shell

Section 1: Determination of Stresses



1- As shown in the figure below, both ends of a cylinder subjected to a uniform internal pressure of p are closed by circular plates. The cylinder has a radius of r, a thickness of h, a modulus of elasticity of E, and a Poisson's ratio of v. Assuming that the radial expansion of the cylinder is uniform along its entire length, derive an expression for the volumetric strain (volume change per unit volume) of a thin-walled cylindrical shell.



The derivative of the cylinder's volume $V = \pi r^2 L$ with respect to the two variables r and L is obtained to determine the relationship for the volumetric change.

$$dV = (2\pi r dr)L + (\pi r^2)dL$$

Dividing dV by V gives the volumetric strain (volume change per unit volume).

$$\frac{dV}{V} = \frac{(2\pi r dr)L + (\pi r^2)dL}{\pi r^2 L} \quad \rightarrow \quad \frac{dV}{V} = 2\frac{dr}{r} + \frac{dL}{L}$$

Since the deformations in the longitudinal and radial directions are small, the ratio of the change in length to the original length can be assumed equal to the strain. In other words, the radial strain is $dr / r = \varepsilon_d$ and the longitudinal strain is $dL / L = \varepsilon_L$.

$$\frac{dV}{V} = 2\varepsilon_d + \varepsilon_L$$

Since the ratio of circumference to diameter remains constant, the radial strain is equal to the hoop strain.

$$\text{Circumference / diameter} = \frac{2\pi D(1+\varepsilon_c)}{D(1+\varepsilon_d)} = 2\pi \quad \rightarrow \quad \varepsilon_c = \varepsilon_d$$

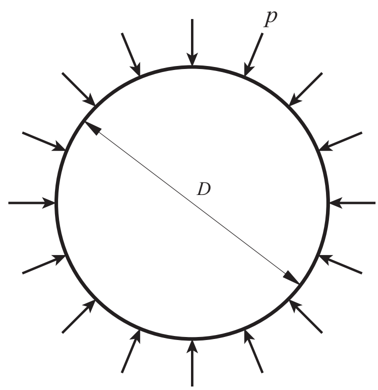
$$\sigma_c = \frac{pr}{h} \quad , \quad \sigma_1 = \frac{pr}{2h}$$

$$\begin{cases} \varepsilon_d = \varepsilon_c = \dfrac{1}{E}(\sigma_c - v\,\sigma_1) = \dfrac{1}{E}(\dfrac{pr}{h} - \dfrac{vpr}{2h}) = \dfrac{pr}{hE}(1-0.5v) \\ \varepsilon_L = \dfrac{1}{E}(\sigma_1 - v\,\sigma_c) = \dfrac{1}{E}(\dfrac{pr}{2h} - v\,\dfrac{pr}{h}) = \dfrac{pr}{hE}(0.5 - v) \end{cases}$$

$$\frac{dV}{V} = 2\varepsilon_d + \varepsilon_L = 2\frac{pr}{hE}(1-0.5v) + \frac{pr}{hE}(0.5-v)$$

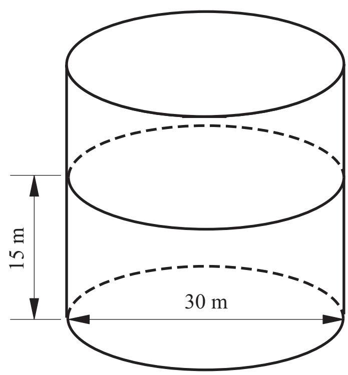
$$\rightarrow \frac{dV}{V} = \frac{pr}{hE}(2.5-2v)$$

2- If the allowable tensile stress of cast iron is $200\ kg/cm^2$, determine the wall thickness of a cast iron water pipe with a diameter of $100\ cm$ and submerged at a depth of 120 meters.



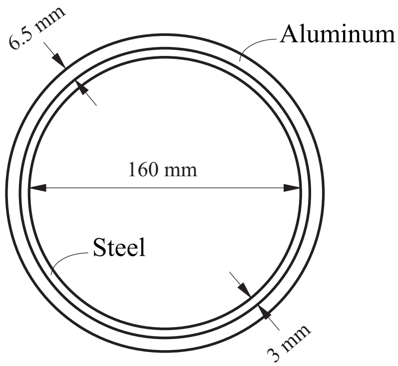
$t = 30\ mm$

3- A vertical cylindrical gasoline storage tank has a diameter of 30 m. The tank is filled with gasoline up to a height of 15 m, with a specific gravity of 0.74. If the yield stress of the steel plate used to construct the tank is $250\ MPa$ and a factor of safety of 2.5 is sufficient, determine the required thickness of the cylindrical wall at the bottom. Neglect the effects of local bending.



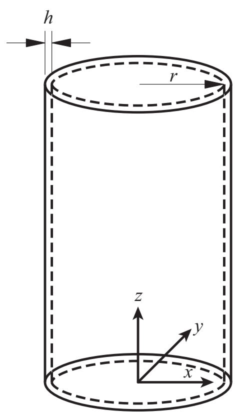
$t = 16.33\ mm$

4- A steel cylinder has an internal diameter of 160 mm and a wall thickness of 3 mm. The steel cylinder is lined with an 6.5 mm thick aluminum layer. If the internal pressure is $33\ kg/cm^2$, determine the stress induced in the steel cylinder wall. Neglect the stresses along the cylinder's length. Also, assume that $E_{st} = 2\times10^6$ and $E_{Al} = 0.7\times10^6$.



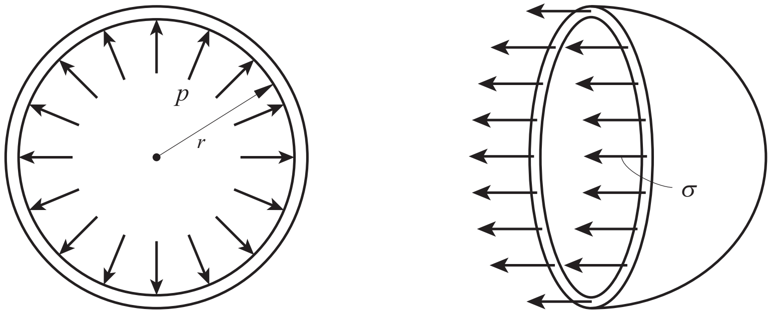
$\sigma_{st} = 508.4\ kg/cm^2$

5- As shown in the figure below, both ends of a thin-walled cylindrical tank with radius r and wall thickness h are closed. A strain gauge is installed on the tank's outer surface. When the tank is subjected to an internal pressure p, the strain gauge measures a strain of ε_0 in the direction parallel to the cylinder axis. Express the internal pressure of the tank in terms of the known parameters.



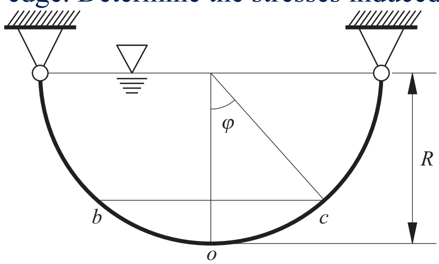
$$p = \frac{2hE\varepsilon_0}{(1-2v)r}$$

6- In the figure below, r is the shell radius, h its thickness, E the modulus of elasticity, and v Poisson's ratio. Determine the radial extension of the shell when it is subjected to an internal pressure p.



$$\Delta r = \frac{pr^2}{2hE}(1-\nu)$$

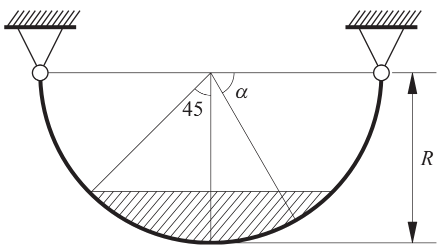
7- As shown in the figure below, a hemispherical tank with radius R and wall thickness h is filled with a liquid of specific weight γ. The shell is supported along the entire length of its upper edge. Determine the stresses induced in the shell.



$$\sigma_\phi = \frac{\gamma R^2(1-\cos^3\phi)}{3h\sin^2\phi}$$

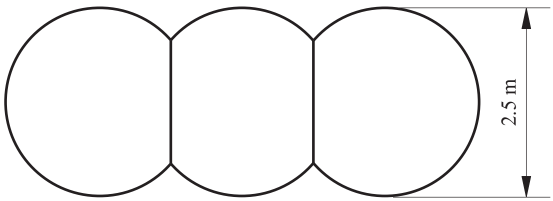
$$\sigma_\theta = \frac{\gamma R^2}{3h}(3\cos\phi - \frac{1-\cos^3\phi}{\sin^2\phi})$$

8- As shown in the figure below, a hemispherical tank with radius R and wall thickness h is filled with a liquid of specific weight γ up to a certain height. The shell is supported along the entire length of its upper edge. Determine the stresses induced in the shell.



$\alpha < 45$, $\sigma_\theta = -\sigma_\phi = -(\frac{\gamma R^2}{h})(\frac{0.0387}{\cos^2\alpha})$

9- As shown in the figure below, a marine transport vehicle composed of three spherical shells joined together is designed to assist submarines in emergencies. The shells are made of steel with an outer diameter of 2.5 *m* and a wall thickness of 20 *mm*. The yield stress of the steel is 900 *MPa*, and the specific weight of seawater is $10^4\ N/m^3$. Neglecting stress concentration effects at the joints and the possibility of buckling due to hydrostatic pressure, determine the hoop stress induced in the spherical shells at a sea depth of 1000 *m*.



$\sigma_c = 312.5\ MPa$

10- A spherical steel tank with a wall thickness of 15 *mm* is used to store gas under pressure. The tank has a diameter of 25 *m*, and the yield stress of the steel is 250 *MPa*. If the factor of safety is 2.5, determine the maximum allowable internal pressure in each of the following cases:

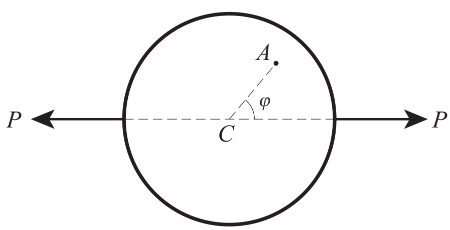
a) The strength of the steel plate joints is equal to the strength of the steel itself.

b) The strength of the steel plate joints is 75% of the steel's strength.

$P_{all} = 0.24\ MPa$

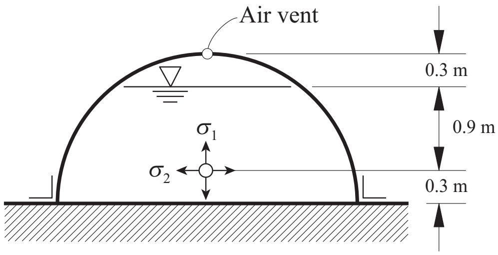
$P_{all} = 0.18\ MPa$

11- As shown in the figure below, a spherical shell with a mean radius *r* and wall thickness *t* is considered. The shell is subjected to tensile forces *p* along one of its diameters. Determine the principal membrane stresses σ_1 and σ_2 at point *A*, which is located at a specified angle ϕ.



$$\sigma_2 = -\frac{p}{2\pi rt}$$

12- As shown in the figure below, a spherical shell with a mean radius 1.5 m and wall thickness $t = 0.025\ cm$ is fixed to a horizontal slab. The shell is filled with water of specific weight $\gamma = 1\ g/cm^3$ up to a height of 1.2 m. To equalize the air pressure inside and outside the shell, a small air vent is provided at the top. Determine the principal membrane stresses σ_1 and σ_2 for an element located at a height of 0.3 m.



$\sigma_1 = 69\ kg/cm^2 \qquad \sigma_2 = 471\ kg/cm^2$

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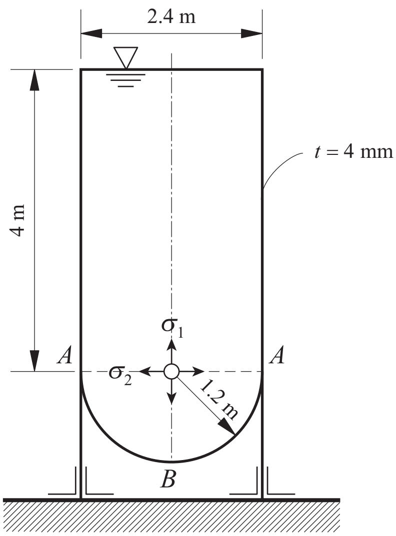
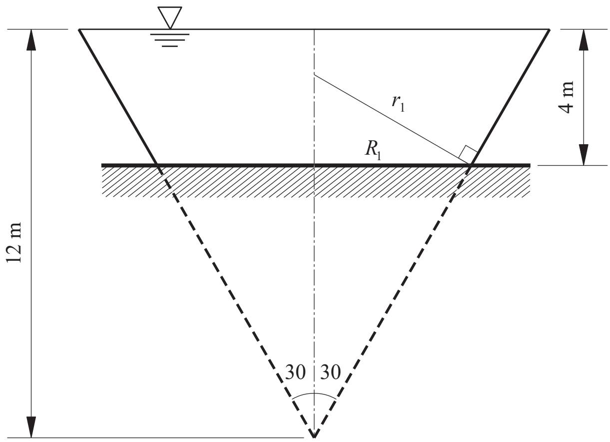
13- As shown in the figure below, a cylindrical tank with a diameter 2.4 m and wall thickness 4 mm, whose bottom is hemispherical, is filled with water up to a height 4 m. Determine the maximum normal stresses in the walls of the cylindrical and spherical sections of the tank.



$\sigma_2 = 48\ kg/cm^2 \qquad \sigma_1 = 72\ kg/cm^2$

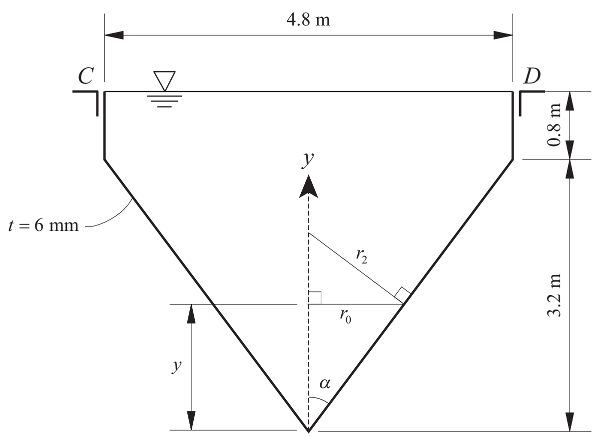
14- As shown in the figure below, a liquid tank in the form of a truncated cone with wall thickness 8 *mm* is supported at its smaller end. The tank is filled with a liquid of specific weight $0.9\ t/m^3$. Determine the maximum meridional and hoop stresses induced in the wall of the tank.



$\sigma_\theta = 240\ kg/cm^2$

$\sigma_\phi = 70\ kg/cm^2$

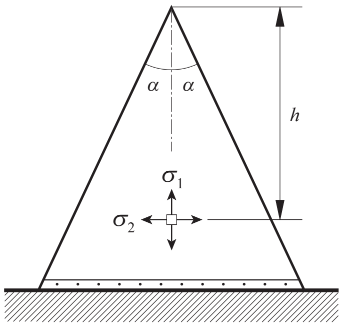
15- A liquid tank consists of two parts: a) an upper cylindrical section with diameter 4.8 *m* and height 80 *cm*, and b) a lower conical section with height 3.2 *m* and wall thickness 6 *mm*. The tank is suspended from the cylindrical section *CD* and is filled with oil of specific weight $0.72\ t/m^3$. Determine the maximum meridional and hoop stresses induced in the wall of the conical section.



$\sigma_\theta = 45\ kg/cm^2$

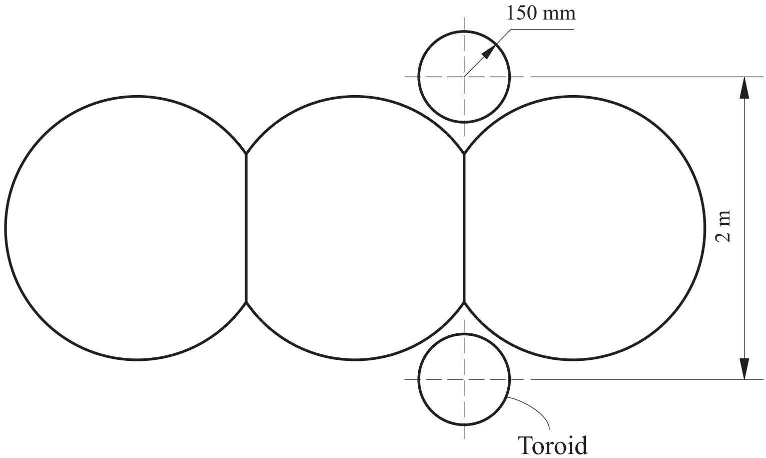
$\sigma_{\phi,\max} = 33.75\ kg/cm^2$

16- As shown in the figure below, a thin-walled cone is supported on a horizontal base and is subjected to internal gas pressure p. If the wall thickness of the cone is t and the weight of the cone is neglected, determine the principal stresses σ_1 and σ_2 at the point indicated in the figure.



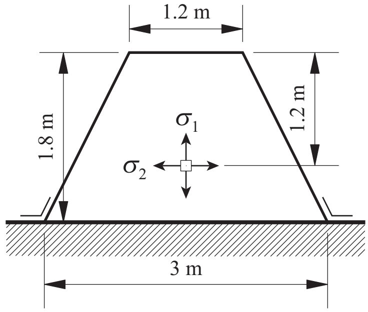
$$\sigma_1 = \frac{ph\tan\alpha}{2t\cos\alpha} = \frac{1}{2}\sigma_2$$

17- As shown in the figure below, the submarine has a compressed air tank in the form of a toroid, which is enclosed by two spherical shells. The toroidal tank is made of steel with a specified ultimate tensile strength 1000 MPa. Using a factor of safety of 3 against the ultimate strength, determine the required wall thickness of the toroid to withstand the internal pressure 30 MPa.



$h_{allowable} = 14.7\ mm$

18- As shown in the figure below, a truncated conical tank with a wall thickness of 0.03 *cm* is filled with water of specific weight $\gamma = 1\ g/cm^3$. Determine the membrane stresses σ_1 and σ_2 for the element *A* indicated in the figure.



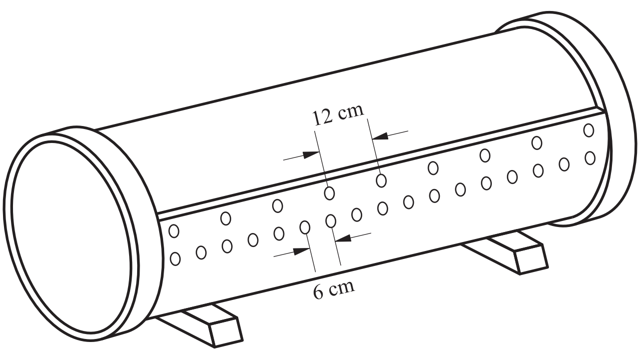
$\sigma_\theta = 536.7\ kg/cm^2$

$\sigma_\phi = 100\ kg/cm^2$

19- A conical tank with a wall thickness of 4 *mm* is suspended circumferentially along section *AB* and filled with water up to a height of 5 *m*. If the apex angle of the cone is $2\alpha = 40$, determine the maximum normal stress.

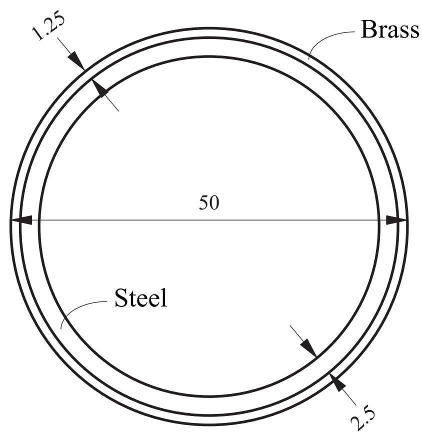
$\sigma = 45.4\ kg/cm^2$

20- As shown in the figure below, a steel sheet tank with a diameter 1.5 *m* and wall thickness 12 *mm* has two rows of riveted joints. Each rivet has a diameter 25 *mm*, and the spacing between rivets is 12 *cm* in the first row and 6 *cm* in the second row. If the allowable tensile stress is $\sigma_w = 1120\ kg/cm^2$ and the allowable shear stress is $\tau_w = 840\ kg/cm^2$, determine the maximum allowable internal pressure *p*.



$P_{\max} = 10.45\ kg/cm^2$

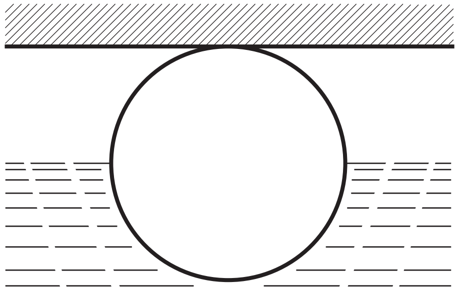
21- At a temperature of $216^\circ\ C$, a brass ring and a steel ring are placed exactly in each other, both initially stress-free. The width of the rings, perpendicular to the plane of the figure, is $2.5\ cm$. The modulus of elasticity and the coefficient of thermal expansion for steel are $E_s = 2\times10^6\ kg/cm^2$ and $\alpha_s = 12\times10^{-6}\ C^{-1}$, respectively, and for brass are $E_b = 9\times10^5\ kg/cm^2$ and $\alpha_b = 19\times10^{-6}\ C^{-1}$, respectively. Each ring has a mean radius of $r = 25\ cm$. If the temperature decreases to $21^\circ\ C$, determine the radial pressure q between the two rings and the hoop stress σ_b in the brass ring due to the temperature change.



$q = 125.3\ kg/cm^2$

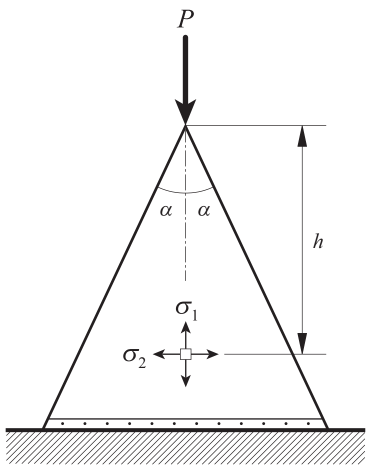
$\sigma_B = 1002.8\ kg/cm^2$

22- As shown in the figure below, a spherical shell with a mean radius R and wall thickness t is held submerged in water by a fixed roof support. In this situation, half of the spherical shell is submerged. If the specific weight of water is γ and the weight of the shell is neglected, determine the principal stresses σ_1 and σ_2 in the shell at the water surface.



$$\sigma_2 = -\sigma_1 = \frac{-\gamma R^2}{3t}$$

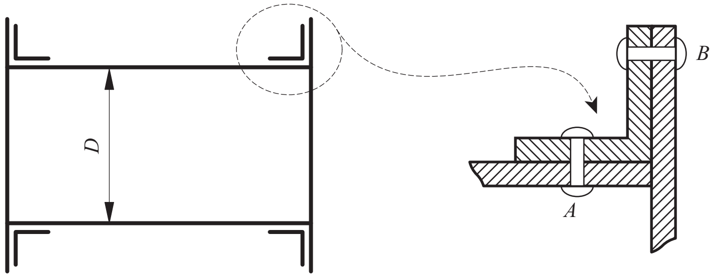
23- As shown in the figure below, if the wall thickness of the cone is t and a concentrated vertical force p is applied downward at its apex, determine the membrane stresses σ_1 and σ_2 at a height h below the apex of the cone. Assume that the cone is not subjected to internal pressure.



$$\sigma_1 = \frac{-P}{2\pi h t \sin\alpha}$$

$$\sigma_2 = 0$$

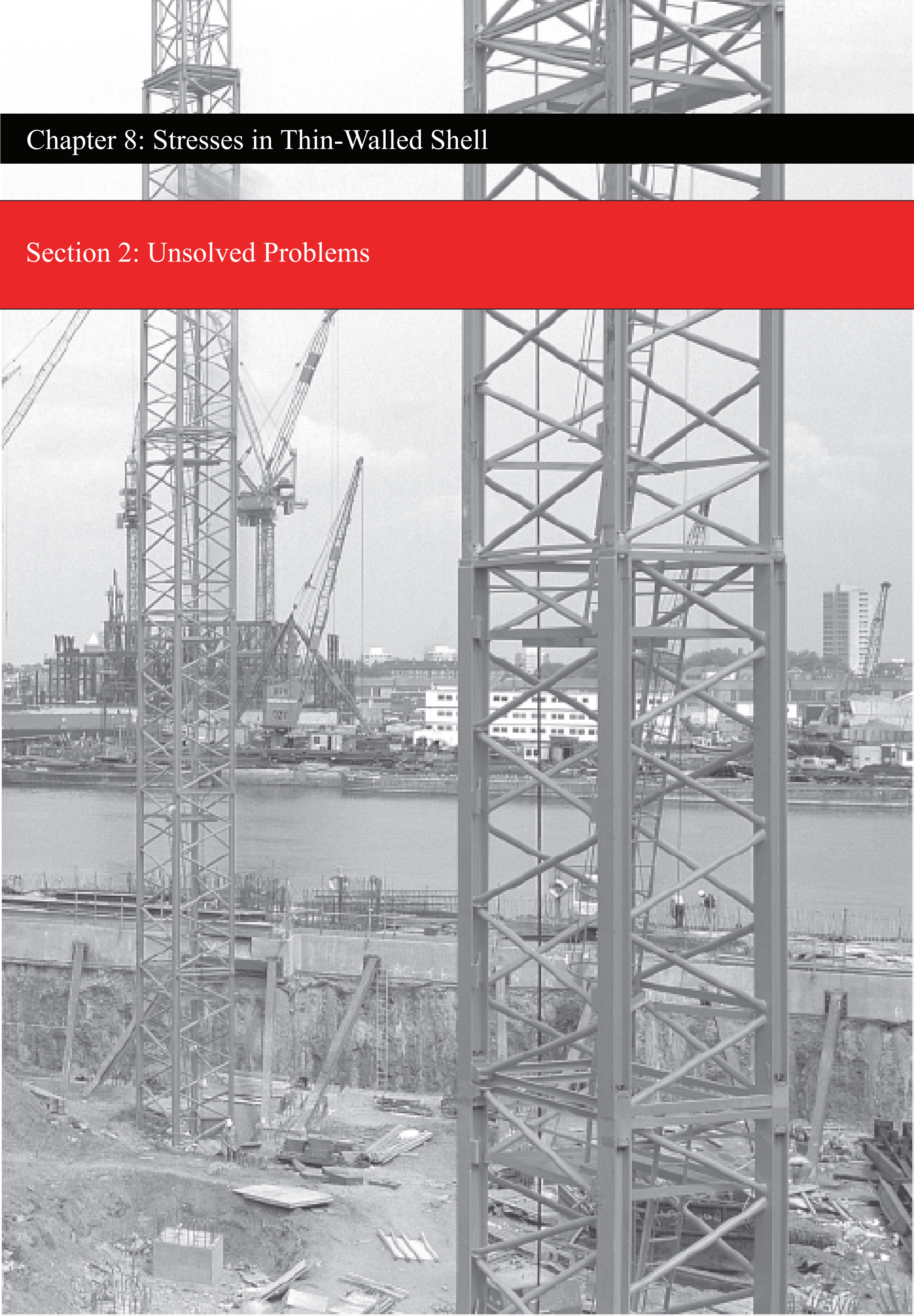
24- As shown below, the end plates of a steam boiler are connected to the boiler shell by rivets and angle sections. The boiler diameter is 100 cm, the shell and angle thicknesses are both 10 mm, and the internal pressure is 10 atm. The allowable shear stress is $\tau_a = 700\ kg/cm^2$, and the allowable bearing stress is $f_a = 1600\ kg/cm^2$. If the rivet diameter is 20 mm, determine the number of rivets required to connect the boiler shell to the angle sections.



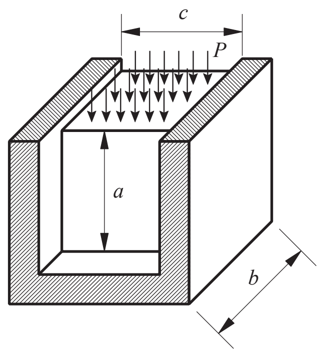
$n = 36$

Chapter 8: Stresses in Thin-Walled Shell

Section 2: Unsolved Problems



1- In the figure below, calculate the values of $\sigma_{1,2,3}$, Δ_a and Δ_b. The values of E and μ are assumed.

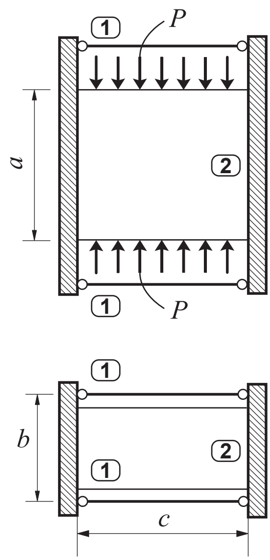


$\sigma_1 = 0$, $\sigma_2 = -\mu p$, $\sigma_3 = -p$

$$\Delta_a = -\frac{1-\mu^2}{E} pa$$

$$\Delta_b = \frac{\mu(1+\mu)}{E} pb$$

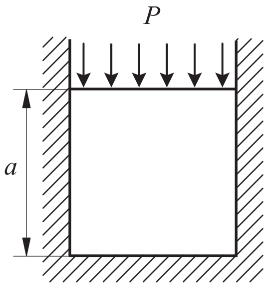
2- In the figure below, determine the value of σ_1 for bar 1, and the values of $\sigma_{1,2,3}$ and $\Delta V / V$ for the cylinder 2. The values of E_1, A_1, E_2 and μ_2 are assumed.

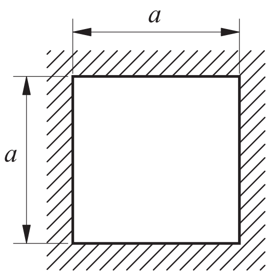


$\sigma_1 = 0$

$$\sigma_1 = \frac{-4\mu_2 pE_1A_1}{(E_2ab+4E_1A_1)} \quad , \quad \sigma_2 = \sigma_3 = -p \quad , \quad \frac{\Delta V}{V} = -\frac{(1-2\mu_2)p(4\mu_2E_1A_1+E_2ab+4E_1A_1)}{E_2(E_2ab+4E_1A_1)}$$

3- In the figure below, calculate the values of $\sigma_{1,2,3}$ and Δ_a. The values of E and μ are assumed.





$$\sigma_1 = \sigma_2 = \frac{-\mu p}{1-\mu} \quad , \quad \sigma_3 = -p \quad , \quad \Delta_a = -\frac{(\mu+2\mu^2-1)pa}{E(1-\mu)}$$

4- A cylindrical steam boiler with a diameter of $d = 2\ m$ and a wall thickness of $t = 10\ mm$ is subjected to an internal pressure of $q = 10\ atm$. The allowable stress of the material is $900\ kg/cm^2$. Evaluate the strength of the boiler shell.

$\sigma = 866\ kg/cm^2 < 900\ kg/cm^2$

5- A steel water pipe with a diameter of 60 cm is submerged to a depth of 120 m below the water surface. If the allowable stress of the pipe material is $800\ kg/cm^2$ and a wall corrosion allowance of 2.5 mm is anticipated, determine the required thickness of the pipe.

$t = 7\ mm$

References

[1] Belyayev, N. M, "Problems in Strength of Material", Pergamon Press, 1966.

[2] Mirolyubov, N, et all, "An Aid to Solving Problems in Strength of Materials" Mir Publisher, 1974.

[3] Timoshenko, S.P, and Gere, J.M.,"Mechanics of Materials", D. Van Nostrand Co., 1972.

[4] Nash, W.A., "Strength of Materials", 2nd ed., McGraw Hill Book Company, 1977.