Chapter 7

Analysis and Design of a Structure with Lead-Rubber Bearings (LRBs)

7.1 Introduction

During an earthquake, two main factors contribute to damage in structural and non-structural members. The first factor is the inter-story lateral displacement (drift), and the second is the absolute floor acceleration. Therefore, two design philosophies are discussed to reduce the vulnerability of structural and non-structural components. In the first approach, stiff structures are desirable because they reduce inter-story lateral displacement; however, they generate high floor accelerations. Conversely, flexible structures absorb less energy during an earthquake and have lower floor accelerations, but they experience large inter-story lateral displacements that severely damage components sensitive to lateral drift. Consequently, a method must be sought that reduces both inter-story lateral displacement and floor accelerations. Seismic isolators embody this concept, as they significantly reduce both floor accelerations and inter-story lateral displacements. In fact, in a base isolated structure, rather than changing the seismic capacity of the structure, the seismic demand is reduced through isolation.

By adding seismic isolators, three vibration modes are added to the other structural modes. These three modes relate to the displacements of the isolator itself, meaning the structure above it is considered a rigid body in these three modes. The period of these modes is greater than the periods of the structural vibration modes; thus, the first three vibration modes of the assembly consist of the isolator and the building above it. Because the first mode period of the isolator and superstructure assembly is much larger than the first mode period of the fixed-base structure, the acceleration applied to the base of the structure and the input earthquake energy will be much lower due to this period increase. However, because of this increase in period, the total displacement of the assembly increases. This increase in displacement manifests as a **rigid body motion** at the isolation level rather than inter-story drift. This large displacement is concentrated at the base of the building and is entirely controllable. Additionally, certain types of seismic isolators possess damping properties and add damping to the system. Figure 7.1 shows the effect of increasing the period and damping resulting from the use of isolators.

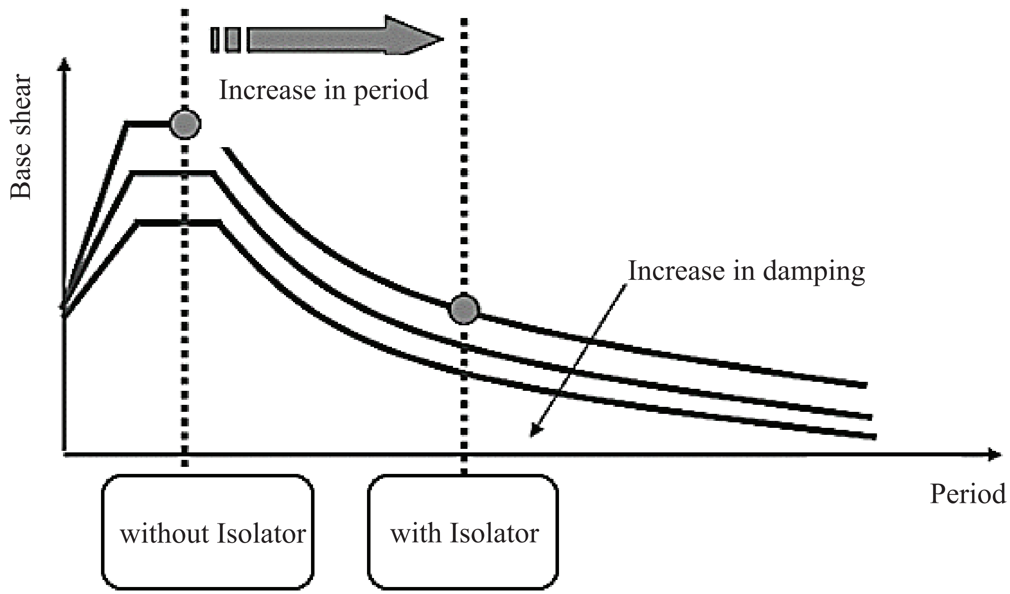

Figure 7.1 Effect of using isolators on the earthquake design spectrum

As observed in Figure 7.2, when using isolators, the design of the structure and isolator assembly is divided into three main parts: the design of the superstructure above the isolator, the design of the substructure below the isolator, and the design of the isolator itself.

7.2 Types of Seismic Isolators

Seismic isolation systems have a very wide variety. These systems are divided into different categories based on the type of material and performance, including the following:

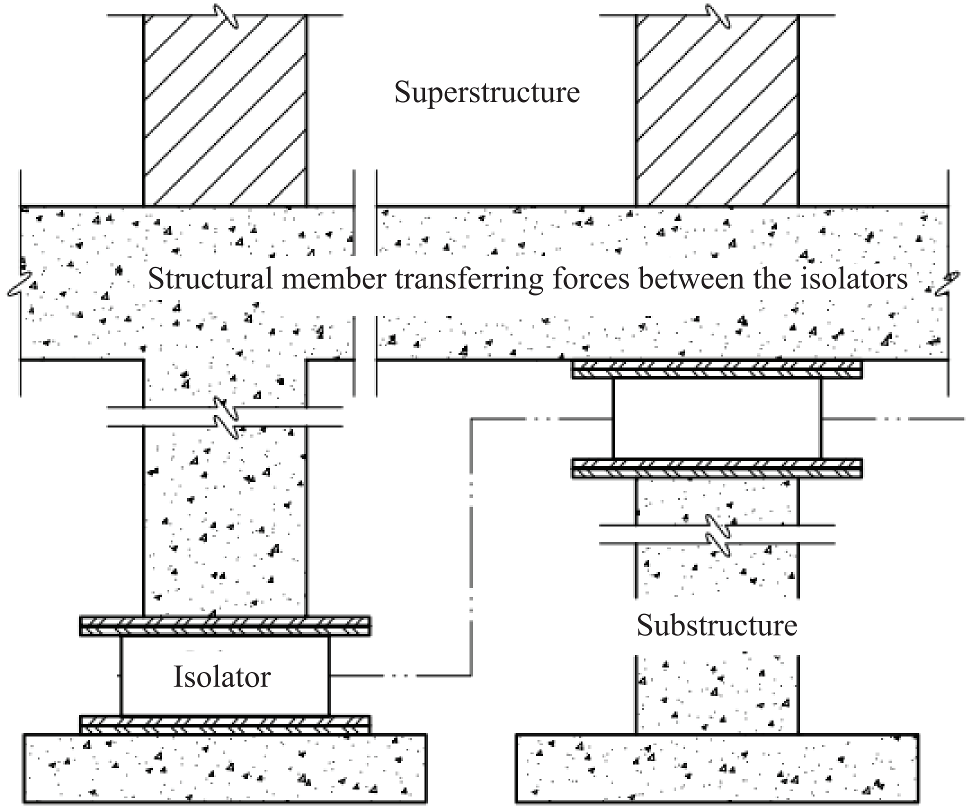

Figure 7.2 Structural classification in the case of using isolators

Natural and Synthetic Rubber Systems with Low Damping (LDR)

The LDR isolator is made of a type of rubber with low inherent damping. Figure 7.3 shows an example of it. This isolator has two steel end plates and many thin steel plates. These thin steel plates are used to prevent lateral bulging of the rubber and to provide high vertical stiffness. The bonding of the steel plates to the rubber is possible through both cold and hot processes. These plates have no effect on the horizontal stiffness of the isolator, which is provided by the shear modulus of the elastomer. The behavior of this isolator in shear is completely linear up to a shear strain of about 100%, and it has damping in the range of 2 to 3 percent.

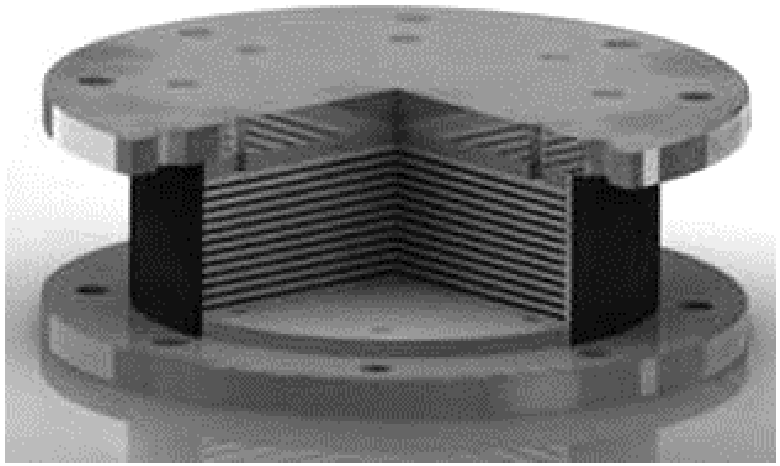

Figure 7.3 Low damping rubber bearings (LDR)

Lead Rubber Bearing (LRB) System

To increase the effective damping of elastomeric isolators made of low-damping rubber, one or more lead cores are used in their center. The lead core, which is placed inside the bearing hole by pressure or injection, yields and physically deforms under earthquake-induced shears at low stresses of approximately 8 to 10 MPa at normal temperatures. In this state, the lead acts as an elastic-plastic material or like recrystallized hot-rolled steel; thus, it exhibits bilinear behavior by forming hysteresis loops under consecutive cycles, increasing the damping from 3% in low-damping rubber to over 10%. The lead core, together with the rubber, provides the initial stiffness for service loads and dissipates the input earthquake energy. In most cases, the secondary or plastic stiffness is one-tenth to one-sixth of the initial or elastic stiffness. The reason for choosing lead is that this material does not experience fatigue during consecutive loading cycles and therefore possesses stable performance in energy dissipation. Figures 7.4 and 7.5 show different parts of this isolator.

High Damping Natural Rubber (HDR) Systems

This isolator was developed in 1982 by the Malaysian Rubber Producers' Research Association (MRPRA) in the United Kingdom. The objective of its production was to create a natural rubber compound with sufficient inherent damping to eliminate the need for supplementary energy dissipation systems. This damping is increased to a level between 10% and 20% at 100% shear strain by adding extra blocks of carbon, oil, resin, or other additives. Figure 7.6 shows an example of it.

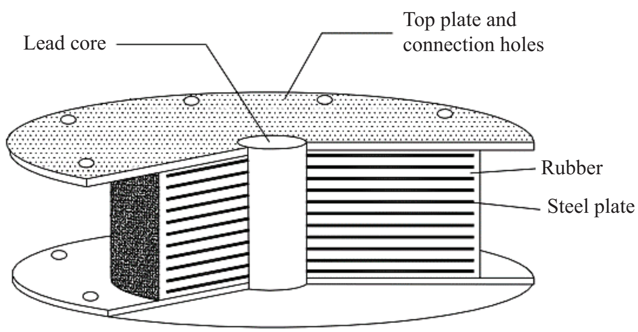

Figure 7.5 LRB isolator

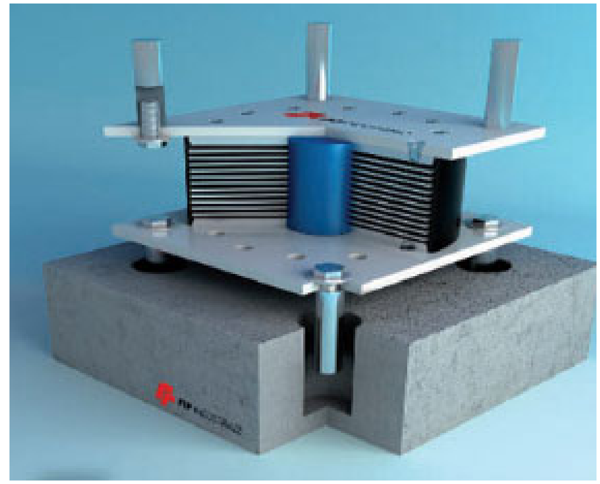

Figure 7.4 Sample of a lead core elastomeric seismic isolator

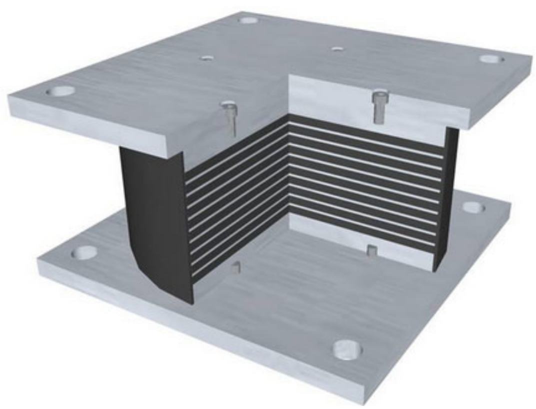

Figure 7.6 High damping rubber bearings (HDR)

The properties of high-damping rubber, such as shear modulus and equivalent damping, are highly dependent on the shear strain amplitude. In this type of rubber, as the shear strain increases up to approximately 150% to 200%, the shear modulus and inherent damping of the rubber initially decrease, and after this strain range, they increase again. This high initial stiffness ensures that the isolators are sufficiently stiff against service loads, such as wind loads. In addition to the dependence of the properties of this type of rubber on the shear strain amplitude, these properties are also dependent on the number of loading cycles due to the scragging phenomenon. In the first few cycles of loading with high strain amplitude, the molecular structure of the rubber undergoes changes, leading to a reduction in its stiffness and inherent damping. After the high-damping isolator has undergone the scragging phenomenon, the properties of the rubber remain nearly constant in subsequent cycles.

EERC Hybrid System

This hybrid system, combining elastomeric and sliding systems, was developed and tested on a shake table at the EERC research center. In this system, the internal columns are placed on sliding elements made of stainless steel with a Teflon coating, while the external columns are placed on low-damping natural rubber isolators. The elastomeric isolators control the restoring capability and structural torsion, while the sliding isolators provide the necessary damping.

TASS Hybrid System

The TASS system was developed by the TAISEI Corporation in Japan. In this system, the entire vertical load is supported by stainless steel elements with a Teflon coating. Additionally, neoprene isolators that do not carry vertical loads are used to provide the restoring force.

Friction Pendulum System (FPS)

This type of frictional isolator features an articulated slider that slides on a spherical steel surface. The part of the upper slider surface in contact with the spherical surface is covered with low-friction materials. The slider is coated with a material called polytetrafluoroethylene (PTFE), which is capable of sliding on the curved surface. This type of isolator will be discussed in detail in Chapter 8 of this book.

GERB Systems

The GERB system can protect structures against seismic vibrations in both vertical and horizontal directions. Figure 7.7 shows an example of it. This system utilizes large helical steel springs that are flexible in both horizontal and vertical directions. The frequency in the vertical direction is 3 to 5 times the frequency in the horizontal direction. The steel springs are completely without damping; therefore, this system is always used in conjunction with GERB viscous dampers.

The two main and most widely used groups of seismic isolators are elastomeric (rubber) isolators and frictional isolators. For this reason, this book addresses the design of these two groups. The design of High Damping Rubber (HDR) isolators and Lead Rubber Bearings (LRB) is similar; therefore, this chapter focuses on the design of the LRB isolator. The design of Friction Pendulum isolators will be addressed separately in Chapter 8.

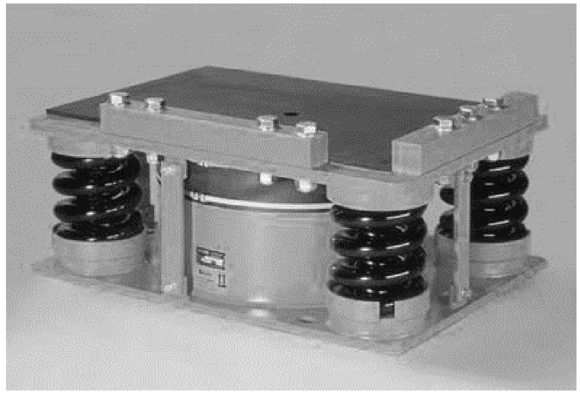

Figure 7.7 GERB isolators

7.3 Dampers for Controlling Isolator Deformations

As mentioned, due to the increase in the system period, the base displacement in isolated systems is high. Therefore, to increase damping at the base for improving seismic performance, dampers are used alongside isolators at the base level. This is done to prevent the instability of isolators susceptible to failure under large lateral deformations, to comply with architectural considerations, including the permitted isolation gap (isolation pit), and to meet executive limitations regarding the capacity of flexible utility pipes to withstand displacements. An example of the application of these systems is observed in Figure 7.8.

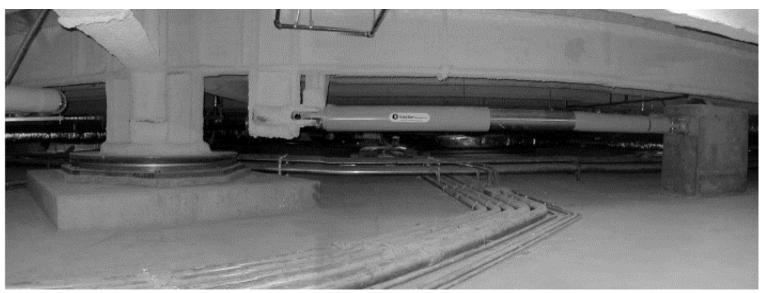

Figure 7.8 Viscous damper together with isolator for displacement control (New de Young Museum - San Francisco)

To control the displacement of isolators under service loads, an additional stiffening system can also be used at the base of the structure. These stiffening systems act elastically under service loading, such as wind loads, and yield under earthquake loading. An example of this system is shown in Figure 7.9, which consists of a vertical rod used in conjunction with rubber isolators.

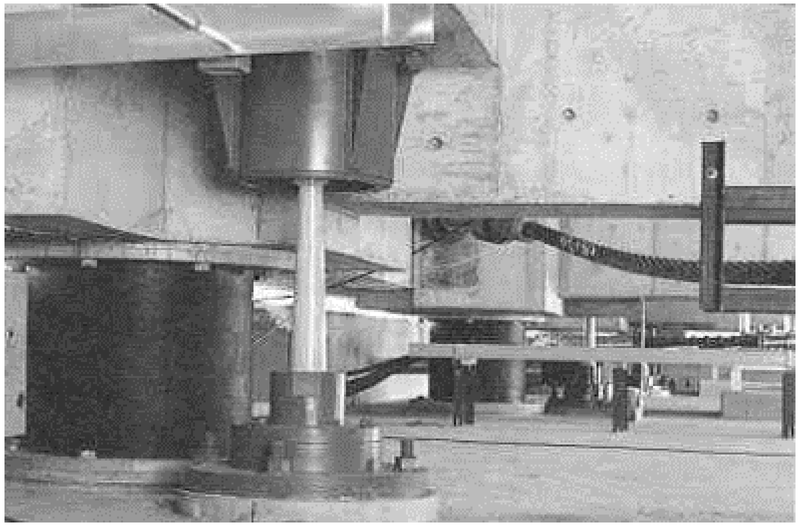

Figure 7.9 Stiffening system including vertical rod for displacement control under service loads

7.4 Design of a structures with seismic isolators

The first guideline containing design requirements for isolated buildings was published by SEAONC in 1986. This guideline was added as an appendix to the UBC 1991. In subsequent years and in newer versions of the IBC, the seismic regulations for buildings with seismic isolators were referred to the ASCE 7 standard. In This book, the building with isolators will be designed using Chapter 17 of this standard. The building with isolators is designed based on the Equivalent Lateral Force (ELF) procedure. To use this method according to ASCE 7-10, the following conditions must be satisfied:

1. The structure must be located in a region where $S_1 < 0.6g$.
2. The structure must be situated on soil types A, B, C, or D. This clause exists because the dominant period of earthquakes occurring on soft soils is often long, which causes the response of long-period structures, such as those with isolators, to be amplified. Therefore, according to the code recommendation, the design of buildings with isolators in these areas should be avoided. The soil type classification based on ASCE 7-10 and shear wave velocity is provided in Table 7.1.

Table 7.1 Soil classification based on ASCE 7-10

Site Class	$\overline{v}_s$	$\overline{N}$ or $\overline{N}_{ch}$	$\overline{s}_u$
A. Hard rock	>5,000 ft/s	NA	NA
B. Rock	2,500 to 5,000 ft/s	NA	NA
C. Very dense soil and soft rock	1,200 to 2,500 ft/s	>50	>2,000 psf
D. Stiff soil	600 to 1,200 ft/s	15 to 50	1,000 to 2,000 psf
E. Soft clay soil	<600 ft/s	<15	<1,000 psf
	Any profile with more than 10 ft of soil having the following characteristics: —Plasticity index $PI > 20$, —Moisture content $w \geq 40\%$, —Undrained shear strength $\overline{s}_u < 500$ psf		
F. Soils requiring site response analysis in accordance with Section 21.1	See Section 20.3.1		

3. The height of the structure is less than 19.8 meters (65 feet).
4. The effective period of the isolated structure at the maximum displacement *TM* is less than 3 seconds.
5. The effective period of the isolated structure at the design displacement *TD* is greater than 3 times the elastic period of the fixed-base structure, which is obtained from Equation 12.8-7 of the ASCE 7-10 standard.
6. The building does not have any irregularities.
7. The isolation system satisfies the following conditions:
   7.1 The effective stiffness of the isolation system at the design displacement is greater than one-third of the effective stiffness at 20% of the design displacement.
   7.2 The isolation system is capable of providing a restoring force in accordance with Section 17.2.4.4 of the standard.
   7.3 The isolation system does not limit the maximum considered earthquake displacement to less than the total maximum displacement.

In general, the design of seismic isolators is based on several underlying principles:

1. They must be sufficiently flexible to increase the period of the assembly (except in very soft soils).
2. They must possess sufficient energy dissipation capacity to prevent large displacements at the base.
3. Under service loads, they should not exhibit a functional difference compared to a similar fixed-base building.
4. They must be capable of supporting the weight of the superstructure.

7.5 Design of a Structure with LRB Seismic Isolators

In this chapter, a 5-story building whose architectural plan and story levels are considered the same as the first 5 stories of the 10- story building discussed in previous chapters is designed with LRB seismic isolators. The reason for this is that the 10- story structure, due to the high period of its first mode, does not require an isolator, and using an isolator in such a structure would be uneconomical. In the 5- story structure under cons ideration, 24 LRB isolators will be used under each of the 24 columns. Given that the building has 5 stories, the chosen design method is the static procedure. The empirical period of this steel structure, which has a moment-resisting frame system, is calculated from the following relationship. It is worth noting that the roof height in the aforementioned structure is 19.12 *m*, which is less than 19.8 *m*.

$$T = C_t h^x = 0.0724 \times 19.12^{0.8} = 0.76 \text{ sec}$$

First, an effective period and damping must be considered for the building with isolators. In this design, the effective period is taken as three times the period obtained from the code relationship (0.76 seconds), equaling 2.28 seconds, and the effective damping is assumed to be 15 percent. The assumed damping must be re-evaluated after the initial design and testing of the isolator, and the design must be controlled and revised based on the test results.

7.5.1 Design Displacement and Maximum Displacement

The displacement of the isolator at the DBE (Design Basis Earthquake) level is called the design displacement, and the displacement at the MCE (Maximum Considered Earthquake) level is called the maximum displacement. These two displacements are obtained from Equations 7.1 and 7.2 based on the initial assumed period and damping.

$$D_D = \frac{g}{4\pi^2} \times \frac{S_{D1} \times T_D}{B_D} \tag{7.1}$$

$$D_M = \frac{g}{4\pi^2} \times \frac{S_{M1} \times T_M}{B_M} \tag{7.2}$$

Based on Chapter 2, $S_S = 1.6g$ and $S_1 = 0.5g$ are the seismic parameters of the site which are corrected using the site effect coefficients F_a and F_v based on the tables and relationships mentioned in chapter 2, and then multiplied by 2/3 to reach the DBE level.

$$S_{M1} = F_V \times S_1 = 0.8 \times 0.5 = 0.4$$

$$S_{D1} = \frac{2}{3} S_{M1} = 0.27$$

$$\beta = 15\% \rightarrow \text{interpolation from table 17.5-1} \rightarrow B_D = 1.35$$

$$D_D = \frac{9.8}{4\pi^2} \times \frac{0.27 \times 2.27}{1.35} = 0.11\ m$$

$$D_M = \frac{9.8}{4\pi^2} \times \frac{0.4 \times 2.27}{1.35} = 0.17\ m$$

7.5.2 Minimum and Maximum Total Displacement

According to the code, the effects of inherent and accidental torsion must be considered to calculate the total displacement of each isolator. Given the symmetrical plan of the building and the uniform distribution of isolators under each column, the center of mass of the building and the center of stiffness of the isolators coincide. Therefore, only the effect of accidental torsion e, which the code specifies as 5%, must be considered. The value of y is the distance from the isolator being calculated to the center of mass in the direction perpendicular to the seismic load. Since the corner isolators of the building are typically subjected to the maximum displacement due to torsion, this value is taken as half of the building dimension in the direction perpendicular to the earthquake. The building dimensions are 15 meters by 25 meters. Because the 25-meter dimension is larger, it is used in the calculation of e and y.

$$D_{TD} = D_D \left[1 + y \left(\frac{12e}{b^2 + d^2} \right) \right] \tag{7.3}$$

$$D_{TM} = D_M \left[1 + y \left(\frac{12e}{b^2 + d^2} \right) \right] \tag{7.4}$$

$$D_{TD} = 0.11 \times \left[1 + 1250 \left(\frac{12 \times 0.05 \times 2500}{2500^2 + 1500^2} \right) \right] = 0.11 \times 1.22 = 0.138\ m$$

$$D_{TM} = 0.17 \times 1.22 = 0.207\ m$$

According to the code, the values calculated above must not be less than 1.1 times D_D and D_M, which in this case is 1.22 times. The different types of isolator displacements are shown in Figure 7.10.

In many cases, dampers are used at the base along with the isolators to reduce excessive displacements. These dampers increase the damping at the base of the structure. Dampers such as viscous dampers only increase the base damping, but other types of dampers affect the stiffness of the isolators in addition to the damping, which must be considered during design and modeling.

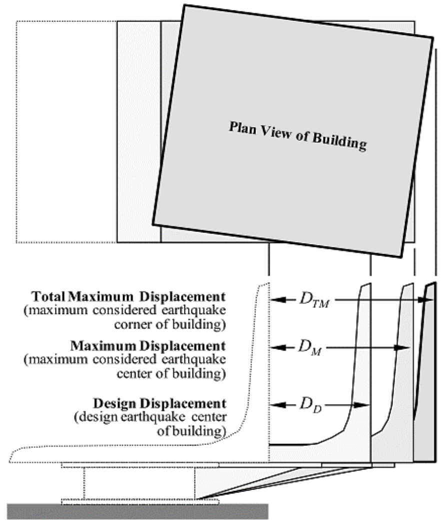

Figure 7.10 Design, maximum, and total displacement

7.5.3 Minimum and Maximum Effective Stiffness

At this stage, the minimum and maximum effective stiffness of the isolator must be determined. For this purpose, the seismic weight of the structure is required. However, since the structure has not yet been designed, the seismic weight of the structure is considered for the first 5 stories according to Table 7.2, similar to the calculations in the viscous dampers chapter. It is obvious that after the design stage of the superstructure, this assumed weight must be revised. Since this revision only pertains to the weight of the structural elements, there will be no significant change in the results.

Table 7.2 Seismic weight of building

Story	Seismic weight (*kg*)
1	409926.1
2	396569.4
3	396156.8
4	395744.2
5	395744.2
Sum	1994141

However, a point that must be noted is that in isolated systems, the ground floor of the structure is also part of the superstructure system and is added to the seismic weight of the structure. Therefore, the ground floor is constructed like other stories, meaning it possesses a framing system and a structural floor. In this book, it is assumed that the specifications of the ground floor slabs and beams are the same as other stories. In reality, however, due to the higher dead and live loads of the ground floor, this level typically features beams and slabs with larger dimensions. Consequently, the total seismic weight is equal to $W = 1994141 + 409926.1 = 2404067.1\ kg$. The minimum effective stiffness is calculated from Equation 7.5.

$$K_{Dmin} = \frac{4\pi^2}{g} \times \frac{W}{T_D^2} \tag{7.5}$$

Therefore, the following result is obtained:

$$K_{Dmin} = \frac{4\pi^2}{9.8} \times \frac{2404067.1}{2.27^2} = 1558878\ kg / m$$

Therefore, the stiffness of each isolator is equal to:

$$k_{Dmin} = \frac{1558878}{24} = 64953.3\ kg / m$$

The values of k_{Dmin} and $k_{D\max}$ are unknown to the designer during the initial design phase of the isolator. Therefore, the initial design phase begins by assuming these values as k_{eff} . After the initial design of the isolator, the isolators are tested, and the maximum and minimum stiffness values of the isolator, which influence the determination of the effective period of the structure, as well as the assumed damping ratio, are revised based on the test results. Figure 7.11 shows Hysteretic curve of the isolator.

In the calculation of the effective stiffness of the isolator from sample testing, its value changes from one cycle to another, and based on this, the concepts of maximum and minimum effective stiffness have been established. According to the code, during three cycles at the design displacement, the value of the maximum effective stiffness must not be more than 30% greater than the value of the minimum effective stiffness. Accordingly, the value of the maximum effective stiffness is considered to be 1.3 times the minimum effective stiffness.

$$K_{Dmax} = 1.3\ K_{Dmin} = 2026542\ kg / m$$

7.5.4 Design Lateral Force for the Seismic Isolation System and the Components Below It (Substructure)

The design forces for the structural system above the isolator (superstructure) as well as below the isolator (substructure) are determined based on the obtained design displacement and the stiffness of the isolators. Accordingly, the system below the isolator is designed for the maximum stiffness of the isolator or, in other words, the maximum probable force that the isolator encounters at the design level.

$$V_b = K_{Dmax} \times D_D \tag{7.6}$$

$$V_b = 2026542 \times 0.11 = 228404\ kg$$

7.5.5 Lateral Force for Stability Control and Ultimate Capacity of the Isolation System

The lateral force for stability control and the ultimate capacity of the isolation system is checked based on the displacement of the isolators at the MCE level. This force is obtained from Equation 7.7 considering the maximum stiffness and maximum displacement.

$$V_{MCE} = K_{Dmax} \times D_M \tag{7.7}$$

$$V_{MCE} = 2026542 \times 0.169 = 342606\ kg$$

7.5.6 Design Lateral Force for the Structure above the Isolation System (Superstructure)

A reduced response modification coefficient is used for the design of the 5-story building above the isolator. The reason the response modification coefficient used in the design of an isolated structure is much lower than that of a non-isolated structure is the issue of period increase, which causes both a greater contribution of higher modes to the structural design force and a reduction in the effect of hysteric damping on response reduction. The base shear value is obtained from Equation 7.8.

$$V_s = \frac{V_b}{R_I} \tag{7.8}$$

$$1 < R_I = \frac{3}{8} R < 2 \quad (7.9)$$

Given the response modification coefficient of 8 for a special steel moment frame:

$$R_I = \frac{3}{8} \times 8 = 3 \ \rightarrow \ R_I = 2$$

$$V_s = \frac{228404}{2} = 114202\ kg$$

According to the ASCE 7-10 code, the value of the base shear V_s must not be less than the following three values:

1- The lateral force required for the design of a fixed-base building, considering a seismic weight and period equal to the seismic weight and period of the isolated building.

$S_{MS} = F_a \times S_S = 0.8 \times 1.6 = 1.28$

$S_{DS} = \frac{2}{3} S_{MS} = 0.85$

$C_S = \frac{S_{DS} \times I}{R} = \frac{0.85 \times 1}{8} = 0.106$

According to the code, it is not necessary for this base shear coefficient to be greater than the value obtained from the following relations:

$C_S = \frac{S_{D1} \times I}{T_D \times R} = \frac{0.27 \times 1}{2.27 \times 8} = 0.015$

The obtained value must not be less than the base shear coefficient calculated from one of the following two relations. Given that $S_1 = 0.5g$, the following result is obtained:

$C_S = 0.044 S_{DS} I > 0.01 \qquad S_1 < 0.6g$

$C_S = \frac{0.5\, S_1\, I}{R} \qquad S_1 > 0.6g$

$C_S = 0.044 \times 0.85 \times 1 = 0.0374 > 0.01$

Therefore, the code base shear will be equal to:

$V_s = 1994141 \times 0.0374 = 74580.9\ kg$

2- Design Wind Force: Due to the high stiffness of the isolators, it is assumed that the earthquake load is the controlling factor in this design.

3- Lateral Force Required to Fully Activate the Isolator: This force is obtained from Equation 7.10. In this equation, K_e is equal to the elastic stiffness of the isolators and D_y is equal to the yield displacement of the isolator. These parameters are obtained in the final stages of the initial design. Therefore, this clause must be checked at the end of the design.

$$V_a = 1.5\, K_e\, D_Y \tag{7.10}$$

The total value of this force for 24 isolators in this project is equal to:

$V_a = 1.5 \times 428274 \times 0.00647 = 66499.3\ kg$

Therefore, a shear force of 114.2 *ton* is the governing base shear for the design of the building.

7.5.7 Distribution of Lateral Force in the Structure above the Isolator and Its Design

Now, the base shear obtained from the previous stage must be distributed over the height of the structure. For this purpose, the force distribution relation according to Chapter 17 of the ASCE 7-10 code is used, which is shown in Equation 7.11. The distributed lateral force values at each story are summarized in Table 7.3.

$$F_i = \frac{W_i h_i}{\sum W_i h_i} V_s \tag{7.11}$$

Table 7.3 Distribution of ELF lateral forces along the building height

Story	W_i (kg)	h_i (m)	$W_i h_i$ $(kg.m)$	F_i (kg)
1	108627.5	4.57	496427.675	8793.643087
2	108627.5	8.22	892918.05	15817.01229
3	108627.5	11.87	1289408.425	22840.3815
4	108627.5	15.52	1685898.8	29863.7507
5	108627.5	19.17	2082389.175	36887.11991
Sum				114202

These forces are entered into the software as User Loads, accessible via **Define > Load Pattern**, as the seismic loads in the X and Y directions. After including these seismic loads in the design load combinations, the building is designed under the forces resulting from these combinations, and the relative displacement of the story centers of mass is also checked under these forces. As mentioned earlier, since the dimensions have been finalized, the change in seismic weight resulting from them must also be checked against the initial assumed weight. Upon checking this weight, it was observed that the seismic weight had a very good approximation. The design results are shown in Figure 7.12.

Based on Section 17.5.6 of the ASCE 7-10 code, to control the relative displacement of isolated structures, the C_d coefficient must be taken as equal to R_I. Therefore, the value of C_d in this project will be equal to 2. On the other hand, this clause requires the designer to ensure that the relative displacement of the isolated structure does not exceed 0.015*h*, where *h* is the story height. These calculations are presented in Table 7.4

As is observed, the relative story displacement is less than the allowable limit, and therefore, the structure meets the design requirements.

7.6 Step-by-Step Design of the LRB Isolator

After reviewing and designing the structure above the isolators, the design of the isolators themselves is addressed. The step-by-step process of this design is examined in the following sections.

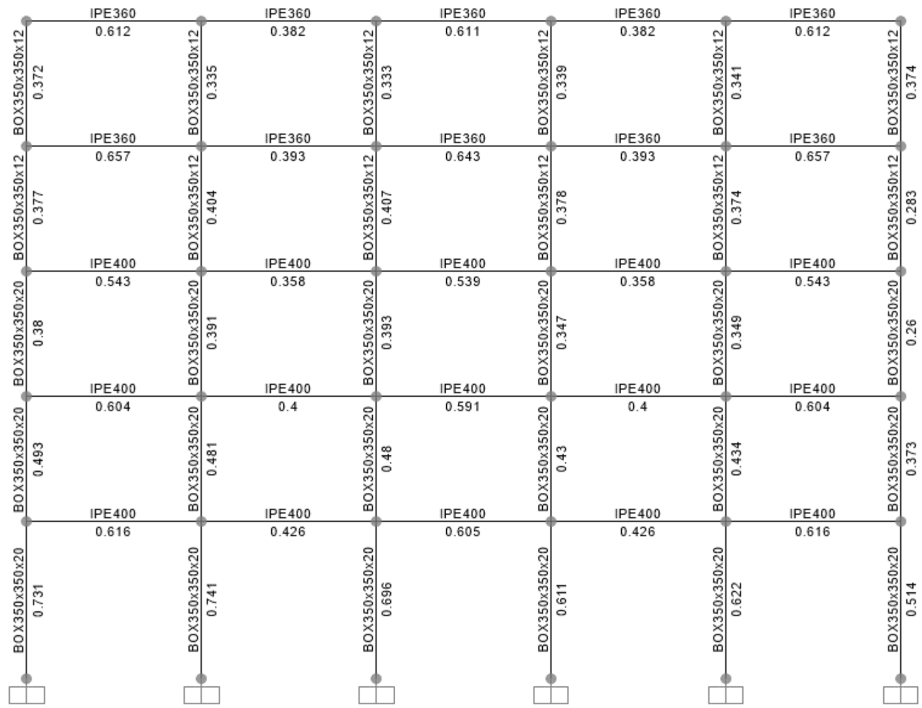

Figure 7.12 Beam and column cross-sections together with member strength ratios

Table 7.4 Control of relative displacement (*m*)

Story	Story mass center displacement due to seismic load in the X direction	Story mass center displacement due to seismic load in the Y direction	Relative story displacement under seismic load in the X direction	Relative story displacement under seismic load in the Y direction	Story height	$0.015h / C_d$
1	0.0046	0.0061	0.0046	0.0061	4.57	0.03428
2	0.0091	0.0117	0.0045	0.0056	3.65	0.02738
3	0.0127	0.0162	0.0036	0.0045	3.65	0.02738
4	0.0152	0.0194	0.0025	0.0032	3.65	0.02738
5	0.0161	0.02	0.0009	0.0006	3.65	0.02738

7.6.1 Selection of Desired Rubber Properties

Table 7.5 shows the specifications of several rubber samples published by the Bridgestone Company in 1990. Specifications including Young's modulus, the shear modulus of the elastomer, and other properties of the tested cases are shown. Rubbers are categorized based on the IRHD value, which is the unit for measuring rubber hardness.

Table 7.5 Characteristics of various types of rubber

Rubber Hardness IRHD ±2	Young's Modulus E (N/cm²)	Shear Modulus G (N/cm²)	Modified Factor k
30	92	30	0.93
35	118	37	0.89
40	150	45	0.85
45	180	54	0.8
50	220	64	0.73
55	325	81	0.64
60	445	106	0.57
65	585	137	0.54
70	735	173	0.53
75	940	222	0.52

IRHD-50 rubber is used in this project. The properties of this rubber are presented in Table 7.6.

Table 7.6 Properties of the rubber used

E (N/cm^2)	220
G (N/cm^2)	64
k (modification factor)	0.73

7.6.2 Calculation of the Total Height of Rubber Layers

Natural rubber is a viscoelastic material that possesses the ability to reach shear strains of up to 300% without causing rupture on its surface. Usually, for design purposes, a shear strain of less than 100% is considered. In this section, based on reliable references, a shear strain of 50% is assumed as the design shear strain of the rubber. Based on Equation 7.12, the height of the rubber is also determined such that it reaches a maximum of 50% at the design level.

Maximum shear strain: $\gamma_{max} = 50\%$

$$t_r = \frac{D_D}{\gamma_{max}} \tag{7.12}$$

Based on the design displacement, the following result is obtained:

$$t_r = \frac{0.11}{0.5} = 0.22\ m$$

7.6.3 Design of the Lead Core (Lead Plug)

The lead core is designed to provide the required initial stiffness in the support and to dissipate vibrational energy during relatively strong and severe earthquakes through the yielding process of the lead. In the design of the lead core, by considering the pre-yield stiffness, post-yield stiffness, and the bilinear force-displacement behavior, the following relations are established between the characteristic strength Q_d and the amount of dissipated energy W_d or the effective damping ratio ξ_{eff}. The energy dissipated in each cycle is equal to:

$$W_D = 2\pi\, K_{eff}\, D^2\, \beta_{eff} \tag{7.13}$$

The area of the hysteric loop at the displacement level D is equal to:

$$W_D = 4Q_d\left(D - D_y\right) \tag{7.14}$$

By neglecting D_y in comparison to D, the yield strength of the lead Q_d is equal to:

$$Q_d \cong \frac{W_D}{4D}\quad A_P = \frac{Q_d}{f_{py}}, \tag{7.15}$$

Using the above relations, the cross-sectional area of the lead core is calculated as follows:

$$W_D = 2\pi \times 1558878 \times 0.11^2 \times 0.15 = 777.6\ kg.m$$

$$Q_d \cong \frac{777.6}{4 \times 0.11} = 1724.83\ kg$$

$$A_P = \frac{1724.83}{882000} = 0.00196 = 19.6\ cm^2$$

f_{py} is the shear yield strength of the lead core, which is considered to be $8.82\ MPa$. Therefore, the core diameter is taken as $6\ cm$.

$$A_P = 28.27\ \ cm^2$$

$$Q_d = 0.002827 \times 882000 = 2493.72\ \ kg$$

7.6.4 Calculation of Shape Factor S

The effective parameter in the stiffness and vertical load-bearing capacity of rubber isolators is called the shape factor, which is defined as the ratio of the load-bearing area of the isolator to its perimeter area. This is illustrated in Figure 7.13. For lead core isolators, the area of the central hole is not considered as a load-bearing area, and this area must be subtracted from the total area. The vertical compressive modulus of rubber, E_c is a function of the shape factor S and the rubber material properties E and is equal to:

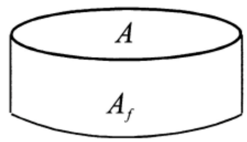

Figure 7.13 Schematic definition of shape factor

$$E_c = E \times \left(1 + 2kS^2\right) \tag{7.16}$$

The ratio of vertical stiffness to shear stiffness is equal to:

$$\frac{K_v}{K_h} = \frac{\frac{E_c.A}{t_r}}{\frac{G.A}{t_r}} = \frac{E_c}{G} = \frac{E \times \left(1 + 2.k.S^2\right)}{G} \geq 400 \tag{7.17}$$

t_r is height of the rubber and With the substitution of the properties of IRHD-50 rubber, the following result is obtained:

$$\frac{220 \times \left(1 + 2 \times 0.73\ S^2\right)}{64} \geq 400$$

$$S = \frac{A}{A_f} \geq 8.9 \quad \& \quad S > 10 \quad \rightarrow \quad S = 15$$

The vertical compressive modulus of rubber depends on the shape factor and the material properties of the rubber, and is expressed as:

$$E_c = E\left(1 + 2k\ S^2\right) = 22 \times \left(1 + 2 \times 0.73 \times 15^2\right) = 7249\ \ kg/cm^2$$

7.6.5 Effective Cross-Sectional Area Based on Allowable Axial Stress

The exact value of the allowable axial stress must be determined based on testing or the manufacturer's certified report. However, empirically, this value is recommended to be 6.9 to $8\ N/mm^2$. Based on the vertical gravity force of the column and the allowable compressive stress, the isolator area A_0 is calculated:

$$\sigma_c = \frac{P_{DL+LL}}{A_0} \le 80\ kg/cm^2 \tag{7.18}$$

After the analysis of the structure, the base of the critical column in terms of vertical load-bearing, which is the middle column of the structure, is selected, and then through the **Show Tables** option and selecting **joint reaction**, the column force is read in the gravity load combination, which here is 1.2 DL + 1.2 WL + 1.6 LL. This force is shown in Figure 7.14 and is given by:

$$P = 212540\ \ kg$$

$$A_0 = \frac{212540}{80} = 0.2656\ \ m^2$$

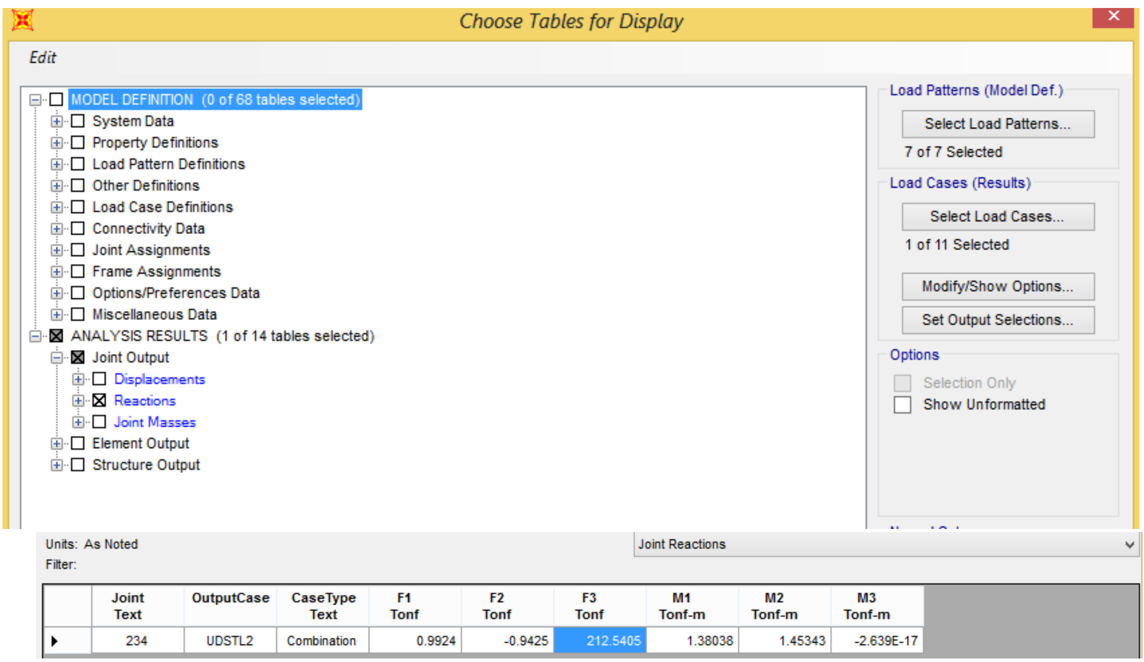

Figure 7.14 Column base vertical force output

7.6.6 Effective Cross-Sectional Area Based on Shear Strain from Vertical Force

The area A_1 is calculated based on the relative shear deformation limit under vertical load as follows. ε_b is the maximum tensile strain of the rubber at rupture, which is considered to be 500% according to the recommendations of references. On the other hand, a safety factor of 3 is applied to this strain.

$$\gamma_c \big|_{DL+LL} = 6S \frac{P_{DL+LL}}{E_c A_1} \leq \frac{\varepsilon_b}{3} \quad (7.19)$$

$$6 \times 15 \times \frac{212540}{72490000 \times A_1} \leq \frac{500\%}{3} \quad \rightarrow \quad A_1 = 0.1583\ m^2$$

7.6.7 Elastic Stiffness of the Isolator

As observed in Figure 7.15, the behavior of lead core isolators is bilinear, in which K_e is the elastic stiffness or unloading stiffness and K_d is the post-yield stiffness. Since the lead core possesses no stiffness after yielding, the post-yield stiffness results solely from the rubber stiffness K_r.

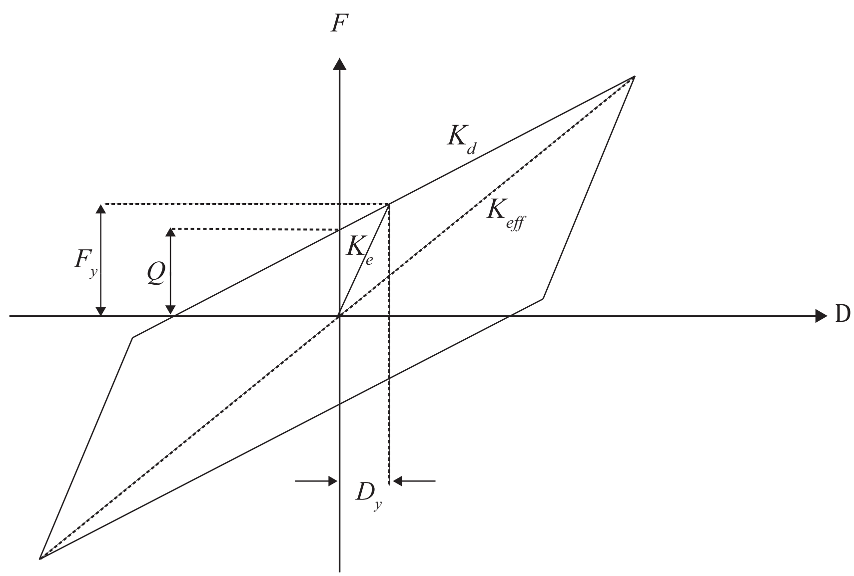

Figure 7.15 Nonlinear behavior of the lead rubber bearing (LRB)

In LRB isolators, unlike High Damping Rubber (HDR) isolators, the post-yield stiffness is modified using the empirical Equation 7.21.

Isolator HDR $$K_d = K_r \tag{7.20}$$

LRB isolator $$K_d = K_r \left(1 + 12\frac{A_P}{A_0}\right) \tag{7.21}$$

Stiffness of the lead core:

$$K_{pd} = \frac{Q_d}{D} \tag{7.22}$$

Based on the design displacement, the stiffness of the lead core is obtained:

$$K_{pd} = \frac{Q_d}{D} = \frac{2493.72}{0.11} = 22125.87 \ kg / m$$

$$K_d = K_{eff} - K_{pd} = 64953.3 - 22125.87 = 42827.39 \ kg / m$$

$$42827.39 = K_r \left(1 + 12 \times \frac{0.002827}{0.265675}\right)$$

$$K_r = 37977.454 \ kg / m$$

7.6.8 Reduced Cross-Sectional Area from Isolator Shear Strain

Based on the stiffness provided by the rubber and the height and shear modulus of the rubber, the required cross-sectional area is obtained from Equation 7.23:

$$A_{sf} = \frac{K_r \ .t_r}{G} \tag{7.23}$$

Therefore, the following result is obtained:

$$A_{sf} = \frac{37977.454 \times 0.22}{64000} = 0.1337 \ m^2$$

$$d = 0.4126 \ m$$

Now, the effective cross-sectional area A_2 is obtained as the reduced cross-sectional area. The reduced cross-sectional area refers to the remaining vertical cross-sectional area resulting from the displacement of the isolator. This cross-sectional area is obtained based on the shape of the isolator from Equations 7.24 and 7.25. This is illustrated in Figure 7.16.

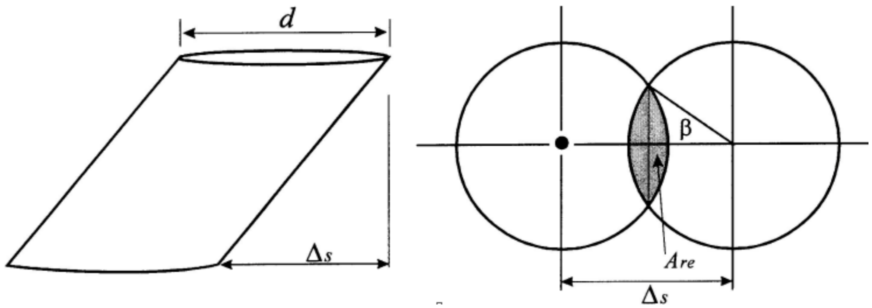

Figure 7.16 Reduced cross-section of the circular isolator

For the square isolator $A_{re} = L(B - \Delta_s)$ (7.24)

For the circular isolator $A_{re} = \frac{d^2}{4}(\beta - \text{Sin}\beta)$ (7.25)

In the above relations, L is the dimension perpendicular to the direction of displacement, B is the dimension parallel to the displacement, and Δ_s is the horizontal displacement of the isolator. The parameter β is also obtained from Equation 7.26. In this relation, the isolator diameter must always be greater than the isolator displacement; otherwise, the reduced cross-sectional area will be zero, which indicates an incorrect design process.

$$\beta = 2\,\text{Cos}^{-1}\left(\frac{\Delta_s}{d}\right) \quad (7.26)$$

$$\beta = 2\,\text{Cos}^{-1}\left(\frac{0.11}{0.412}\right) = 2.58$$

$$A_{re} = \frac{0.412^2}{2}(2.58 - \text{Sin}(2.58)) = 0.0878 \ m^2$$

7.6.9 Final Area

At this stage, the final area of the isolator is obtained. The final area is determined based on the maximum area obtained previously.

$$A = \max\left(A_0, A_1, A_2\right) = \max\left(0.2656, 0.1583, 0.0878\right)$$

$$A = 0.2656 \ m^2$$

$$A_{total} = A_{rubber} + A_{lead\ core}$$

$$\text{A}_{total} = 0.2656 + 0.0028 = 0.2685 \ m^2$$

The selected diameter is $d = 0.6\ m$, and as a result of this diameter, the isolator area will be $A = 0.2827\ m^2$. Now, based on these values, the reduced area must be recalculated

$$\beta = 2\,\mathrm{Cos}^{-1}\left(\frac{0.11}{0.6}\right) = 2.76$$

$$A_{re} = \frac{0.6^2}{2}\left(2.76 - \mathrm{Sin}(2.76)\right) = 0.2155\ m^2$$

7.6.10 Number and Thickness of Rubber Layers

Based on the dimensions of the isolator cross-section and also the shape factor obtained for the isolator, the thickness of each rubber layer is calculated according to Equations 7-27 and 7-28.

For the square isolator $S = \dfrac{BL}{2(L+B)t}$ (7.27)

For the circular isolator $S = \dfrac{\frac{\pi d^2}{4}}{\pi d t} = \dfrac{d}{4t}$ (7.28)

Since the shape factor was obtained as 15, the following result is obtained:

$$S = \frac{d}{4t} \rightarrow t = \frac{0.6}{4 \times 15} = 0.01m$$

With the total thickness of the layers and the thickness of each rubber layer, the number of rubber layers used in the isolator can be calculated:

$$N = \frac{t_r}{t} = \frac{0.22}{0.01} = 22$$

7.6.11 Thickness of Steel Plates

As previously mentioned, steel plates are used between the rubber layers to impose shear strain. The thickness of these plates is obtained based on the axial force of the column and the properties of the steel used.

$$t_s \geq \frac{2\left(t_i + t_{i+1}\right) P_{DL+LL}}{A_{re} F_s} \geq 2\ mm \tag{7.29}$$

Assuming the use of ST37 steel for the plates, the allowable stress is:

$$F_s = 0.6F_y \quad \rightarrow for\ ST\,37 \rightarrow\ F_s = 0.6\times 2400 = 1440\ \ kg/cm^2$$

$$t_s > \frac{2\times(0.01+0.01)\times 221540}{0.2155\times 14400000} = 0.0027\ m \rightarrow t_s = 3mm$$

Now, the total thickness of each isolator unit can be calculated. The total height of the isolator, considering two 2.5 cm plates at the top and bottom, is equal to:

$$h = 2\times 2.5 + (N-1)\times t_s + t_r$$

$$h = 2\times 2.5 + (22-1)\times 0.3 + 22.5$$

$$h \cong 35\ cm$$

7.6.12 Control of Shear Strain

With the finalization of the isolator area, the shear strain limit under vertical load is re-controlled. ε_b is the maximum tensile strain of the rubber at the time of failure, which is considered to be 500%.

$$\gamma_c\big|_{DL+LL} = 6S\,\varepsilon_c = 6S\,\frac{P_{DL+LL}}{E_c.A} \le \frac{\varepsilon_b}{3} \quad (7.30)$$

$$\gamma_c\big|_{DL+LL} = 6\times 15\times \frac{212540}{72490000\times 0.2827} = 0.93 \le \frac{5}{3} = 1.67 \quad \text{OK}$$

7.6.13 Control of Isolator Stability

The buckling of the isolator gains greater importance in cases such as the isolation of relatively light structures. The average stress created in the isolator must not exceed the critical stress.

$$\sigma_c = \frac{P}{A} \le \sigma_{cr} = \frac{GSL}{2.5t_r} \tag{7.31}$$

It should be noted that the two following relations were also used by Naeim and Kelly in 1999:

For the rectangular isolator $$\sigma_c = \frac{P}{A} \le \sigma_{cr} = \frac{\pi GSL}{2\sqrt{2}t_r} \tag{7.32}$$

For the circular isolator $$\sigma_c = \frac{P}{A} \le \sigma_{cr} = \frac{\pi GSL}{\sqrt{6}t_r} \tag{7.33}$$

For circular isolators, L is equal to the diameter of the circle, and for rectangular isolators, it is equal to the smaller side.

$$\sigma_c = \frac{221540}{0.2827} = 75.17\ kg/cm^2 \leq \frac{64000 \times 15 \times 0.6}{2.5 \times 0.22} = 102.2\ kg/cm^2$$

7.6.14 Control of Lead Core Aspect Ratio

The ratio of height to diameter for LRB isolators must be within the range determined by Equation 7.34. H_p is the effective height of the core and d_p is the core diameter.

$$1.5 \leq \frac{H_p}{d_p} \leq 5 \tag{7.34}$$

$$1.5 \leq \frac{0.22}{0.06} = 3.7 \leq 5 \qquad \text{OK}$$

7.6.15 Control of Shear Strain for Earthquake Load

Under seismic conditions and seismic load combinations, the shear strain of the isolator must satisfy Equation 7.35.

$$\gamma_{sc} + \gamma_{eq} + \gamma_{sr} \leq 0.75\ \varepsilon_b \tag{7.35}$$

The parameters of the above relation are obtained as follows:

$$\gamma_{sc} = 6S\,\frac{P_{DL+LL+EQ}}{E_c A_{re}} \tag{7.36}$$

$$\gamma_{eq} = \frac{D}{t_r} \tag{7.37}$$

$$\gamma_{sr} = \frac{B^2\theta}{2t \times t_r} \qquad \text{and} \qquad \theta = \frac{12De}{b^2 + d^2} \qquad (7.38$$

γ_{sc} is the shear strain under compression, which is obtained from the vertical force resulting from the load combination including the maximum earthquake. γ_{eq} is the shear strain under earthquake and γ_{sr} is the shear strain under the rotation of the isolator. θ is the rotation angle of the isolator caused by the earthquake, e is the eccentricity plus the accidental eccentricity of 5%, and d and b are the dimensions of the structure with a rectangular plan. Therefore:

$$\gamma_{eq} = \frac{0.11}{0.22} = 0.5$$

$$\gamma_{sr} = \frac{0.6 \times 0.6 \times 0.00199}{2 \times 0.01 \times 0.22} = 0.158$$

$$\theta = \frac{12 \times 0.11 \times 0.05 \times 25}{25^2 + 15^2} = 0.00199$$

To calculate $P_{DL+LL+EQ_{MCE}}$, first the earthquake load must be multiplied by a **scale factor** of 1.5 through the **Load Case** section to reach the MCE (Maximum Considered Earthquake) level. Then, similar to the calculation of P_{DL+LL}, the most critical vertical support reaction is read from the load combinations including the earthquake. By doing this, the vertical reaction force will be:

$$P_{DL+LL+EQ} = 185650 \ kg$$

$$\gamma_{sc} = 6 \times 0.15 \times \frac{185650}{7249000 \times 0.2155} = 1.069$$

$$0.5 + 0.158 + 1.069 = 1.72 \le 0.75 \times 5 = 3.75 \qquad \text{OK}$$

7.6.16 Control of Isolator Rollout

The lateral displacement of the isolator must also be controlled to prevent the occurrence of rotational instability, or so-called Rollout. In fact, the isolator must be stable in the state shown in Figure 7.17. Equation 7.39 is obtained based on a moment equilibrium at the base of the isolator. It is worth mentioning that this control is performed for the MCE (Maximum Considered Earthquake) level.

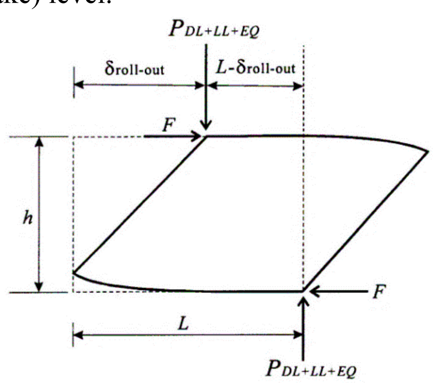

Figure 7.17 Stability limit state of the isolator under seismic load

For LRB $$D_D \le \delta_{roll-out} = \frac{P_{DL+LL+EQ} \times L}{P_{DL+LL+EQ} + K_d \times h} \tag{7.39}$$

For HDR $$D_D \le \delta_{roll-out} = \frac{P_{DL+LL+EQ} \times L}{P_{DL+LL+EQ} + K_{eff} \times h}$$

$$D_D = 0.11\ m < \delta_{roll-out} = \frac{185650 \times 0.6}{185650 + 42827.39 \times 0.35} = 0.556\ m \quad \text{OK}$$

7.7 Isolator Modeling in SAP2000

To model the isolator in SAP2000, the **Rubber isolator** link is used. For the nonlinear modeling of the LRB isolator, the 5 following parameters must be provided to the software. The first case is the effective stiffness of the isolator, which was previously introduced as the minimum stiffness of the isolator.

$$K_{eff} = 64953.3\ kg/m$$

The damping ratio of the isolator is also designed by the following method and based on the assumed damping.

$$C_{eff} = \frac{4\pi}{g \times T_D} \times W\ \beta = \frac{4\pi}{9.81 \times 2.27} \times 2404067.1 \times 0.15 = 202549.2\ \ kg.sec/m$$

Initial elastic stiffness must now be obtained. The post-yield stiffness for each isolator is easily calculated. However, the initial elastic stiffness can vary. Based on the research of Naeim and Kelly, the ratio of these two stiffnesses for lead rubber bearings is considered to be approximately 10. Therefore, the following result is obtained:

$$K_e \approx 10\ K_d$$

$$K_e = 428273.9\ kg/m$$

The yield displacement of the isolator is actually the point where the lead core yields. This displacement is calculated by the following method:

$$D_y = \frac{Q}{K_e - K_d} = \frac{2493.72}{10 \times 42827.39 - 42827.39} = 0.0065\ m$$

The yield force of the isolator is also equal to:

$$F_y = Q + K_d \times D_y = 2493.72 + 42827.39 \times 0.0065 = 2770.8\ \ kg$$

To introduce the LRB isolator in the software, after selecting the **Link/Support Properties** option, select the **Rubber Isolator** type from the **Link/Support Type** drop-down menu. In the **Directional Properties** section, as shown in Figure 7.18, the **Nonlinear** checkboxes for the **U2** and **U3** directions, which correspond to the two horizontal translational movements of the isolator, must be activated. Now, after clicking on **Modify/Show for U**, the parameters obtained above for the two mentioned directions must be entered as shown in Figure 7.19. The **U1** direction corresponds to the vertical stiffness of the isolator, which is considered as **Fixed** in this section. In the modeling in **Perform 3D** section, the method for obtaining the vertical stiffness of the lead core isolator is explained.

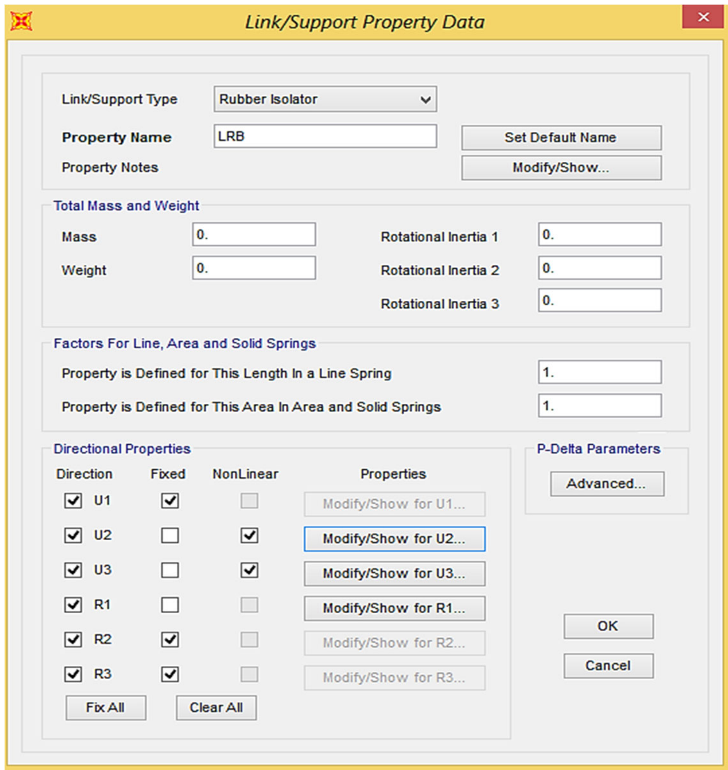

Figure 7.18 Definition of the LRB isolator in the software

To draw the isolator, proceed as follows: first, select the nodes at the base of the columns and then, using the **Restraints** command from the **Assign > Joint** menu, set them to free (uncheck all restraint boxes). Then, after selecting the **Draw One Joint Link** option from the **Draw** menu, click on the bottom nodes of the first-floor columns to draw the desired isolator. By doing this, the isolator link is drawn between the ground and the bottom node of the column. The model of the base-isolated structure is shown in Figures 7.20 and 7.21.

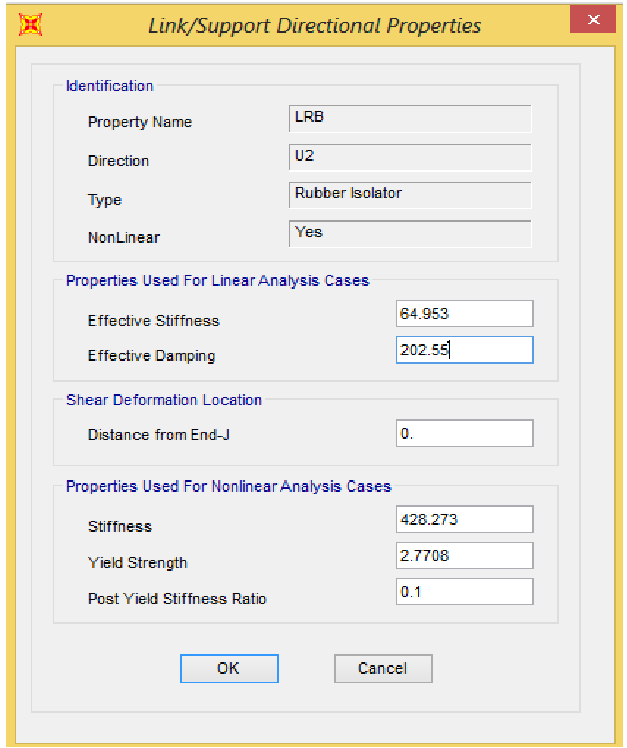

Figure 7.19 Modeling parameters of the LRB isolator in the software

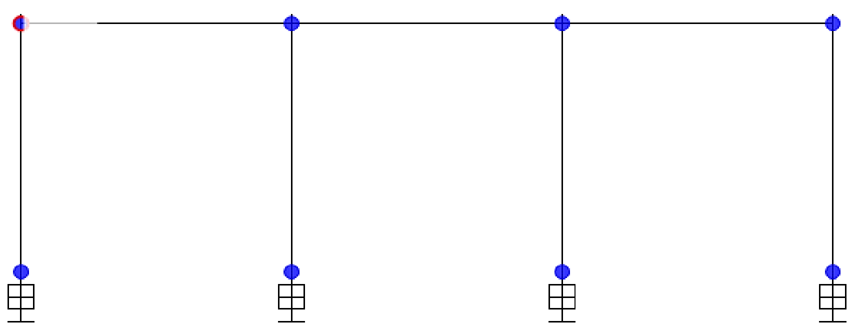

Figure 7.20 Geometric model of the isolator in the software

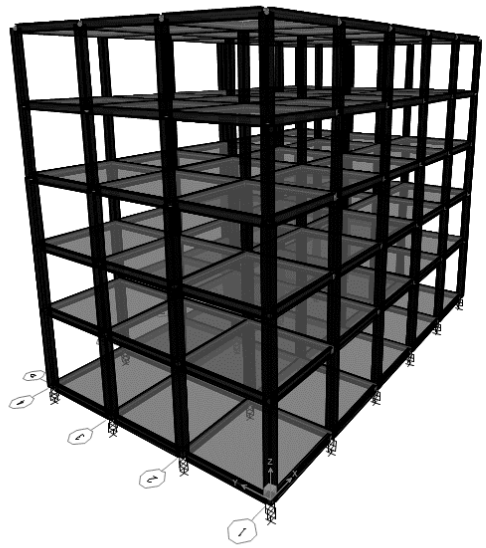

Figure 7.21 Three-dimensional view of the base-isolated structure

In Modal Nonlinear Dynamic Analysis (FNA), it must be noted that since the first three modes of the structure and isolator assembly correspond to the isolator, the modal damping ratio in the **Modal Damping** section of the FNA load case definition should be set to zero for these modes. This is because the damping of these three modes is modeled directly using nonlinear links. This is illustrated in Figure 7.22.

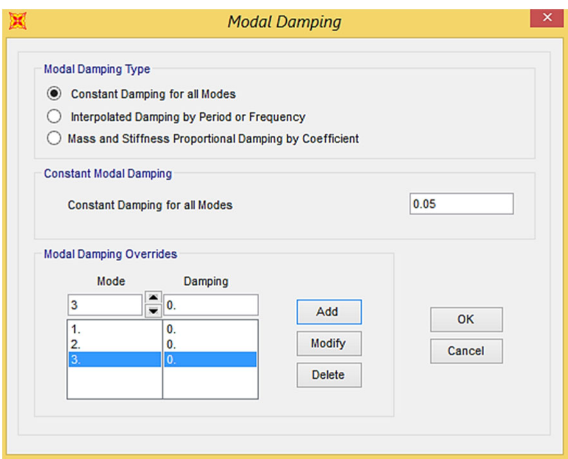

Figure 7.22 Zero damping in the first three vibration modes

7.8 Time History Analysis of the Structure

To investigate the behavior of the structure along with the isolator and the performance of the lead rubber bearing in SAP2000 software, Modal Nonlinear Time History Analysis (FNA) is used. In this type of analysis, only the nonlinear effects resulting from the links are observed. As mentioned earlier, Nonlinear Direct-Integration Time History Analysis, which also includes the nonlinear effects of the structure, can also be used. If this type of analysis is used, energy curves cannot be viewed in the **Plot Function** output. On the other hand, the structure along with the isolator is expected not to enter the nonlinear phase significantly and to remain elastic.

7.9 Output of Isolator Results in SAP2000

It should be noted that for full familiarity with how to obtain the time history outputs of the structure, as well as isolators and dampers, readers should refer to the viscous dampers chapter. In this chapter and this section, after the time history analysis of the structure along with the isolator, the hysteric curve of the damper and the energy diagrams will be plotted.

As observed from Figure 7.23, a high percentage of the input earthquake energy is dissipated by the lead rubber bearings, which proves the high efficiency of the isolators and the correct design process. Figure 7.24 illustrares hysteretic curve of lead rubber bearing No. 6.

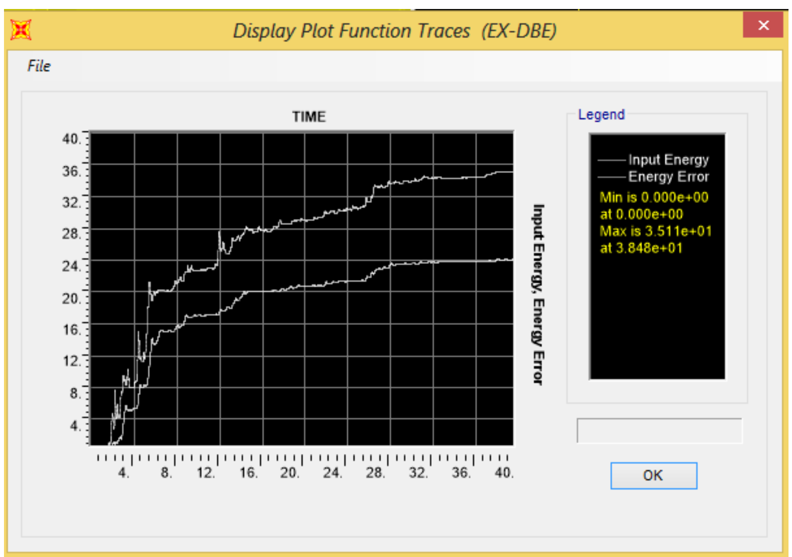

Figure 7.23 Time-history plot window for input energy and dissipated energy

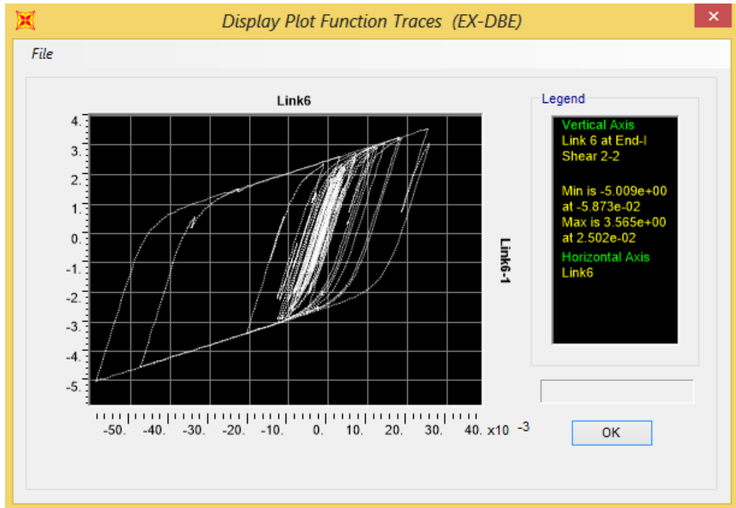

Figure 7.24 Hysteresis plot window of Lead-Rubber Bearing (LRB) No. 6

7.10. Modeling Lead Rubber Bearing in OpenSees

To model the lead rubber bearing in OpenSees software, the **KikuchiAikenLRB** material, developed by Mr. Kikuchi, is used. For this purpose, it is sufficient to define two nodes under each column with identical coordinates and then define a **zeroLength** element between these two nodes under the structure and assign this material to it. In the two-dimensional case, the vertical and rotational directions must also be adjusted accordingly. For this purpose, the **AxialSp** material is used in the vertical direction and the **Elastic** material is used in the rotational direction (Refer to the example). Figure 7.25 shows the hysteretic curve of the model developed using this material. The properties of Materials **KikuchiAikenLRB** and **AxialSp** are as follows. Figure 7.26 also shows the modeled vertical behavior curve of the isolator using the **AxialSp** material.

uniaxialMaterial **KikuchiAikenLRB** $matTag $type $ar $hr $gr $ap $tp $alph $beta <-T $temp> <-coKQ $rk $rq> <-coMSS $rs $rf>

$matTag	Unique material tag
$type	The type identifier of the isolator, which for the lead core is equal to 1, is applicable up to a maximum shear strain of 400%. If the shear strain exceeds 400%, you will encounter the "Response value exceeded limited strain" error.

$ar	Rubber area [unit: m^2]
$hr	Total rubber thickness [unit: m]
$gr	Shear modulus of rubber [unit: N/m^2]
$ap	Area of the lead core [unit: m^2]
$tp	Yield stress of the lead core [unit: N/m^2]
$alph	Shear modulus of the lead core [unit: N/m^2]
$beta	Ratio of initial stiffness to post-yield stiffness
$temp	Temperature [unit: C°]
$rk $rq	Post-yield stiffness reduction rate ($rk) and force at zero displacement ($rq)
$rs $rf	Stiffness reduction rate ($rs) and force ($rf)

The important point regarding this material is that the SI system is used for calculations and all input values must be entered in the format of [m], [m^2], [N/m^2].

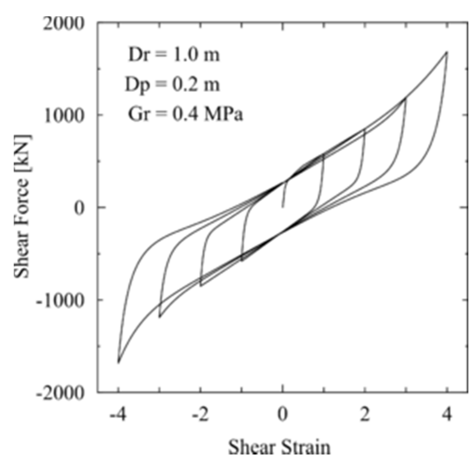

Figure 7.25 Hysteretic curve of the isolator modeled using the KikuchiAikenLRB material

uniaxialMaterial **AxialSp** $matTag $sce $fty $fcy <$bte $bty $bcy $fcr>	
$matTag	Unique material tag
$sce	Compressive modulus of the isolator
$fty $fcy	Yield stress in tension ($fty) and yield stress in compression ($fcy)
$bte $bty $bcy	Reduction rate for the elastic tensile range ($bte), tensile yield ($bty), and compressive yield ($bcy)
$fcr	Target stress point

The input parameters must satisfy the following conditions:

$fcy < 0.0 < $fty

0.0 ≤ $bty < $bte ≤ 1.0

0.0 ≤ $bcy ≤ 1.0

$fcy ≤ $fcr ≤ 0.0

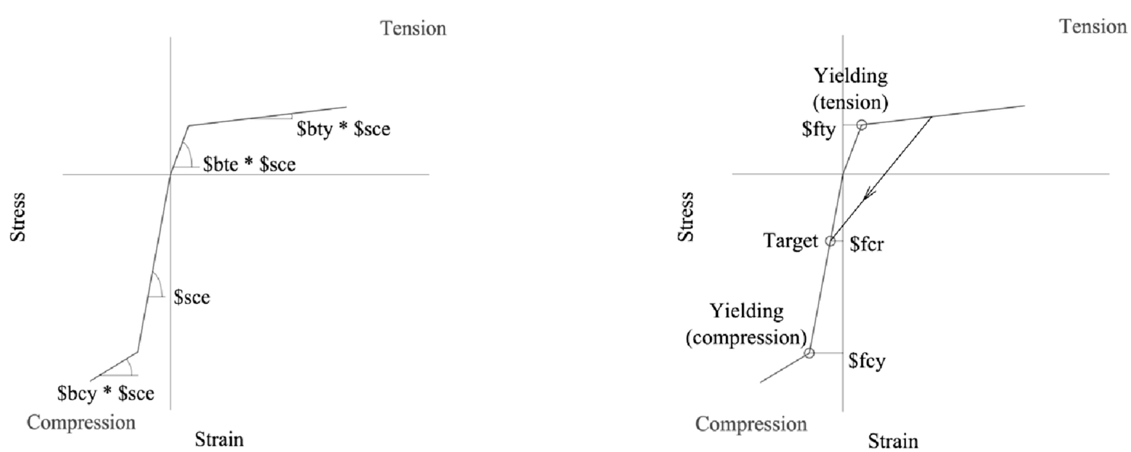

Figure 7.26 Axial (vertical) response curve of the isolator modeled with the AxialSp material

The following is a simple code written for the isolated frame in Figure 7.27. The isolator used has the following specifications: